Towards higher order HBT astronomical interferometry

Vinay Malvimat
Indian Institute of Technology, Kanpur, India
Olaf Wucknitz
Max-Plank Institute for Radio-astronomy and Argelander Institute for Astronomy, Bonn, Germany
Prasenjit Saha
Institute for Theoretical Physics, University of Zurich, Switzerland

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Theory of HBT effect

- Hanbury Brown and Twiss (1950s): Wave Optics
- Roy J Glauber, E.C.G. Sudarshan(1960s) : Photon Statistics

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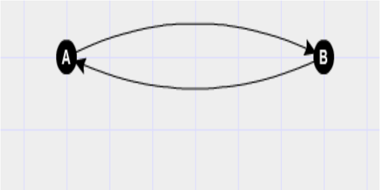
$$G^{(n)}(x_1, \ldots, x_N, x_N, \ldots, x_1) = \sum_{\mathcal{P}} \prod_{k=1}^{N} G(x_k, \mathcal{P}x_k)$$

Standard HBT

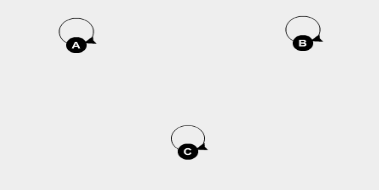
$G_{AA}G_{BB}$

$+$



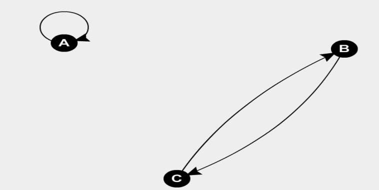
$G_{AB}G_{BA}$ (G = spatial FT of source)

Three-point HBT



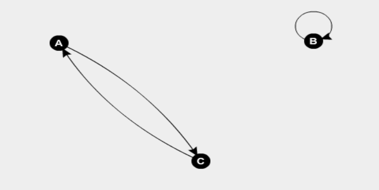
$$G_{AA} G_{BB} G_{CC}$$

$$+$$



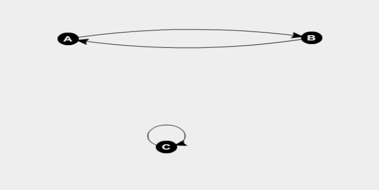
$$G_{AA} G_{BC} G_{CB}$$

Three-point HBT



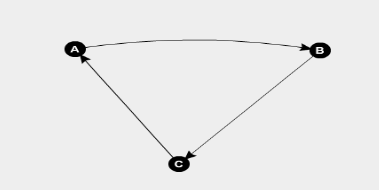
$$G_{AC}\, G_{BB}\, G_{CA}$$

$$+$$



$$G_{AB}\, G_{BA}\, G_{CC}$$

Three-point HBT



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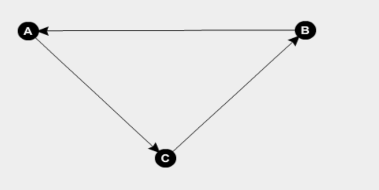
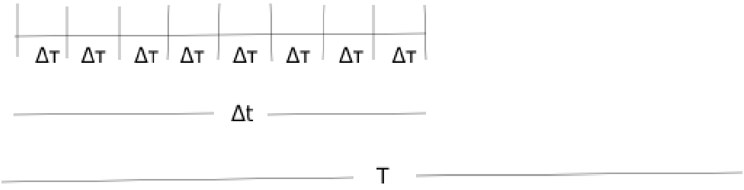


$G_{AC} G_{BA} G_{CB}$ (phase of G not lost)

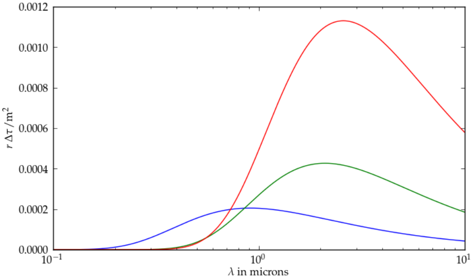
Important time scales



$$\Delta\tau = \text{coherence time}$$
$$\Delta t = \text{resolution time of the detectors}$$
$$T = \text{total observation time}$$

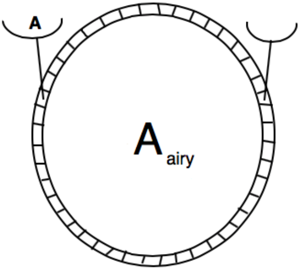
Counts per coherence time

$r\Delta\tau$ for Sirius, Arcturus and Betelgeuse



$$r\,\Delta\tau \sim \frac{\Omega A}{\lambda^2}\frac{1}{e^{1/(\lambda T)}-1}$$

Counts per coherence time



$$r\,\Delta\tau \sim \frac{A}{A_{\rm airy}}\,\frac{1}{e^{1/(\lambda T)}-1}$$

Signal to Noise

N-point HBT (after removing chance coincidences and lower order HBT).

$$\text{Signal} \sim \gamma_{1...N} \times (r\,\Delta\tau)^N\,(\Delta t/\Delta\tau)$$

$$\text{Poisson noise} \sim (r\Delta t)^{N/2} \qquad \text{if } r\Delta\tau \ll 1$$

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$$\mathrm{SNR}(N,\Delta t) \sim \gamma_{1...N} \times (r\,\Delta\tau)^{N/2}\,(\Delta t/\Delta\tau)^{N/2-1}$$

$$\mathrm{SNR}(N,T) \sim \mathrm{SNR}(N,\Delta t)\sqrt{T/\Delta t}$$

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$$\text{Super-Poisson noise} \sim (r\Delta t)^{N/2}\left(1+(r\Delta\tau)^{N/2}\right) \qquad \mathrm{SNR}(N,\Delta t) < 1$$

Numbers

- For visible light $\Delta\tau \sim 10^{-12}s$ and the best of the photon counter have a resolution-time $\Delta t \sim 10^{-10}s$ which gives us SNR

$$\begin{aligned} \mathrm{SNR}(T, N=2) &\sim \gamma_{12} \times (r\Delta\tau)\sqrt{\frac{T}{\Delta t}}\,, \\ \mathrm{SNR}(T, N=3) &\sim \gamma_{123} \times (r\Delta\tau)^{3/2}\frac{\sqrt{T\Delta\tau}}{\Delta t}\,. \end{aligned} \tag{1}$$

this gives a $SNR \sim 1$ in an hour for some of the bright stars

Conclusion

- Interesting graph theoretical representation exists for higher order HBT correlations (for thermal sources).
- Number of photons per coherence time can be derived from blackbody spectra.
- Standard HBT gives no information about phases. 3rd order HBT has the phase information.
- Using 4th or higher order correlations might or might not reveal more information, but it would be an interesting physics experiment to measure them.

Acknowledgements

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THANK YOU

Standard HBT

- In standard HBT first and second order correlations(also called 2 point and 4 point correlations) are as follows

$$G(x_1, x_2) \equiv \left\langle E^{(-)}(x_1)\, E^{(+)}(x_2) \right\rangle . \tag{2}$$

$$G^{(2)}(x_1, x_2, x_2, x_1) = G(x_1, x_1)\, G(x_2, x_2) + |G(x_1, x_2)|^2 . \tag{3}$$

- The first term corresponds to random coincidences where as the second one is the one responsible for HBT effect.

N point HBT

- For chaotic/thermal sources any nth order correlation can be split up into sum of product of n 1st order correlations

$$G^{(n)}(x_1, \ldots, x_N, x_N, \ldots, x_1) = \sum_{\mathcal{P}} \prod_{k=1}^{N} G(x_k, \mathcal{P}x_k)\,. \quad (4)$$

- Example
The normalized third order coincidence rate is then

$$\frac{G^{(3)}(x_1, x_2, x_3, x_3, x_2, x_1)}{G(x_1, x_1)G(x_2, x_2)G(x_3, x_3)} = 1 + \gamma_{12} + \gamma_{13} + \gamma_{23} + 2\Re\,\gamma_{123} \quad (5)$$

$$\gamma_{12} = \frac{G(x_1, x_2)G(x_2, x_1)}{G(x_1, x_1)G(x_2, x_2)}\,,$$
$$\gamma_{123} = \frac{G(x_1, x_2)G(x_1, x_2)G(x_1, x_2)}{G(x_1, x_1)G(x_2, x_2)G(x_3, x_3)}\,. \quad (6)$$