Testing General Relativity in the Strong-field Dynamical Regime

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<u>20th century themes</u>

- High precision technology (clocks, space)
- Frameworks for comparing and testing theories
- Theory-experiment synergy

<u>21st century themes - Beyond Einstein</u>

- Strong-field gravity
- Gravitational waves
- Extreme-range gravity

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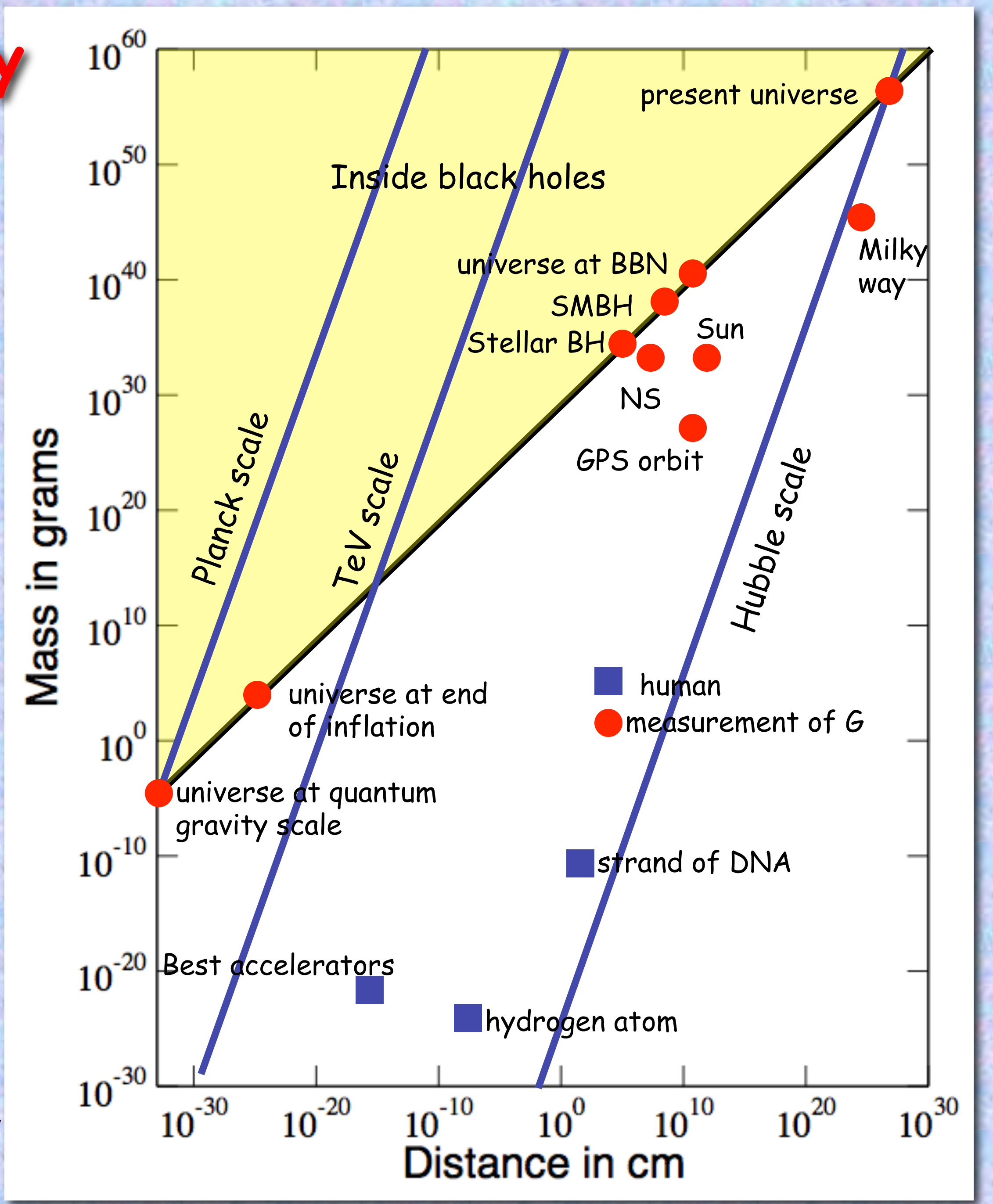
- Introduction - what is "strong"?
- Cosmic barbers: Are black holes really bald?
- Counting hair using gravitational waves
  - EMRIS
  - Ringdown radiation
- Counting hair at the galactic center
- Other strong-field tests

Strong Gravity
Weak Gravity

$$\frac{c^2 R}{GM} \sim 1$$

$$\frac{c^2 R^3}{GM} \sim \ell^2$$



Adapted from original figure by CMW
Used in 1999 NRC Decadal Survey of Gravitational Physics
Used in *Gravity*, by James Hartle

Cosmic Barbers: Are black holes really bald?

$1.6 \times 10^8 M_{sun}$

J. Michell (1784):

If there should really exist in nature any bodies whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us… we could have no information from sight; yet if any other luminous bodies should happen to revolve about them we might still [infer] the existence of the central ones….

P. S. Laplace (1796):

… the attractive force of a heavenly body could be so large that light could not flow out of it.

Cosmic Barbers:
Are black holes really bald?

The 3 Stooges: Moe, Curly & Larry (1934 -46)

Rotating black holes in general relativity

The Schwarzschild solution (1916)

- unique static, spherical asymptotically flat vacuum solution
- matches smoothly to matter interior - star
- non-singular event horizon
- non-rotating black hole

The Kerr solution (1963)

- unique stationary axisymmetric, asymptotically flat vacuum solution with non-singular event horizon
- no reasonable fluid interior solution ever found
- rotating black hole if $J \leq GM^2/c$

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External potentials of charge and mass distributions

Electromagnetism (axisymmetric body)

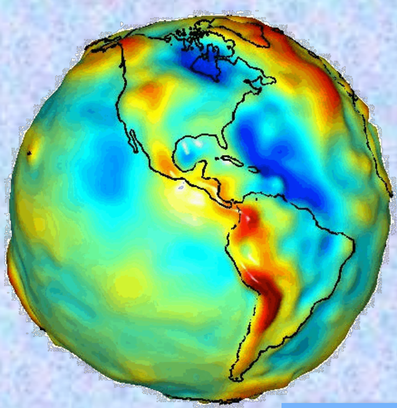
$$\Phi : \frac{e}{r} + \frac{D P_1(\cos\theta)}{r^2} + \frac{Q_2 P_2(\cos\theta)}{r^3} + \ldots$$

$$A^i : \frac{\mu^i}{r^2} + \frac{M_2 \tilde{P}_2^{\,i}(\cos\theta)}{r^3} + \ldots$$

Newtonian gravity (axisymmetric body)

$$U : \frac{M}{r} + \frac{Q_2 P_2(\cos\theta)}{r^3} + \frac{Q_3 P_3(\cos\theta)}{r^4} + \ldots$$

$$Q_\ell = M R^\ell j_\ell$$



Earth: $j_2 = 10^{-3}$, $j_3 = -2 \times 10^{-6}$, $j_4 = -1.5 \times 10^{-6}$, …

Grace, CHAMP: ……. j_{360}

Black holes have no hair

Exterior geometry of Kerr

$$g_{00} : \frac{M}{r} + \frac{Q_2 P_2(\cos\theta)}{r^3} + \frac{Q_4 P_4(\cos\theta)}{r^5} + \ldots$$

$$g_{0\varphi} : \frac{J}{r^2} + \frac{J_3 \tilde{P}_3(\cos\theta)}{r^4} + \frac{J_5 \tilde{P}_5(\cos\theta)}{r^6} + \ldots$$

No hair theorem

$$Q_\ell + iJ_\ell = M(ia)^\ell$$

$$Q_0 = M$$

$$J_1 = J$$

$$a = J / M$$

Hansen 1974

$$Q_2 = -Ma^2 = -J^2 / M$$

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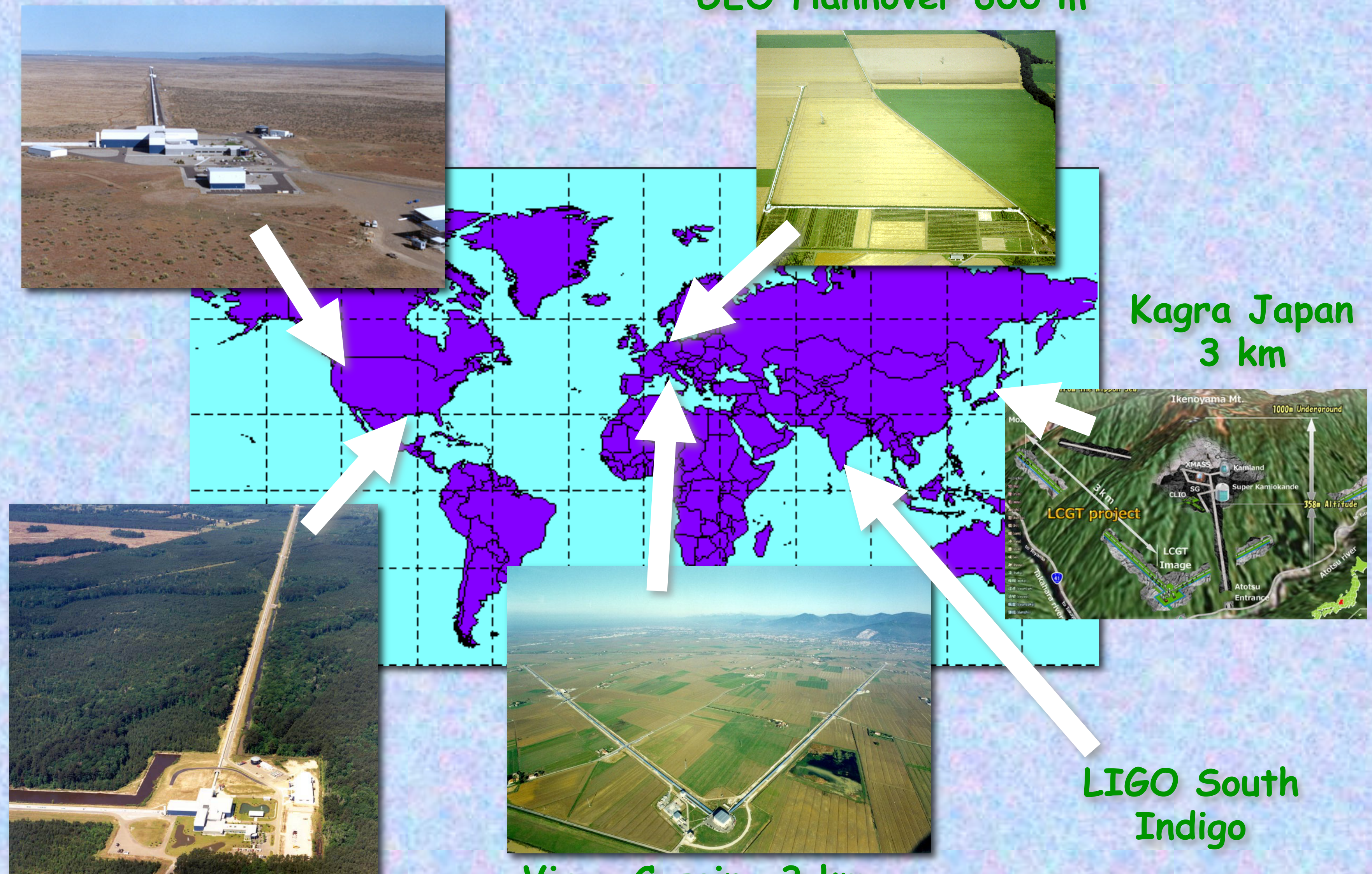
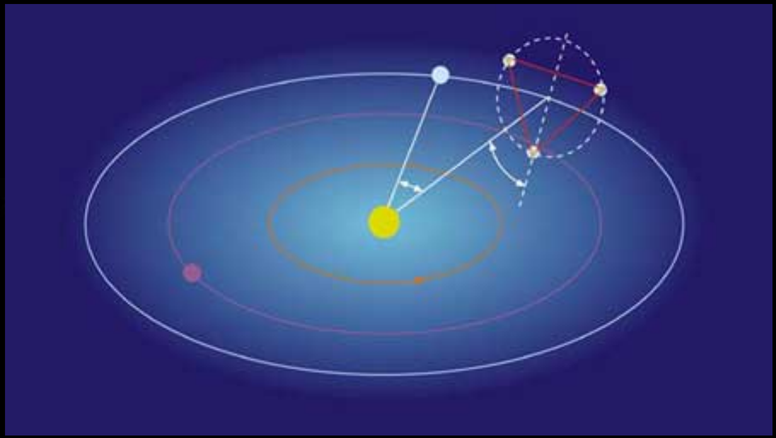
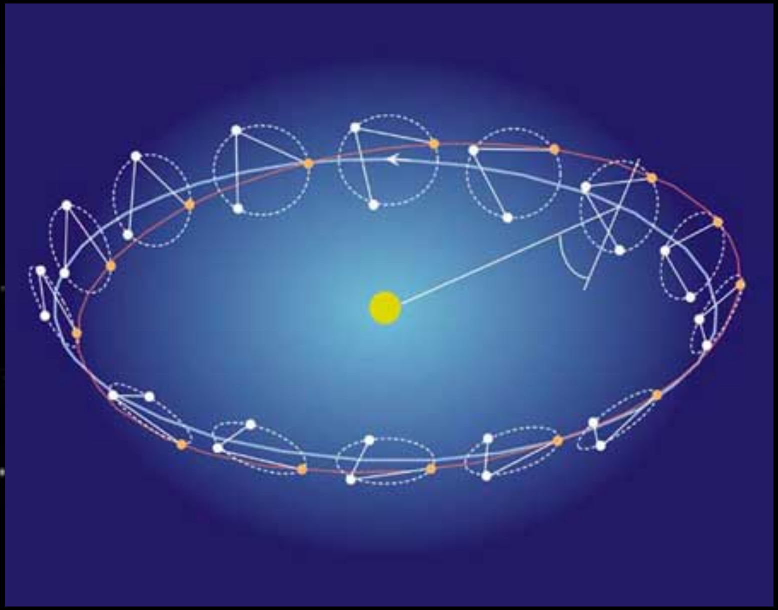
A Global Network of Interferometers



with maybe a teeny bit of US $$





Inspiralling Compact Binaries - Strong Gravity GR Tests?

- Fate of the binary pulsar in 100 My
- GW energy loss drives pair toward merger

<u>Ground-Based (hectahertz)</u>

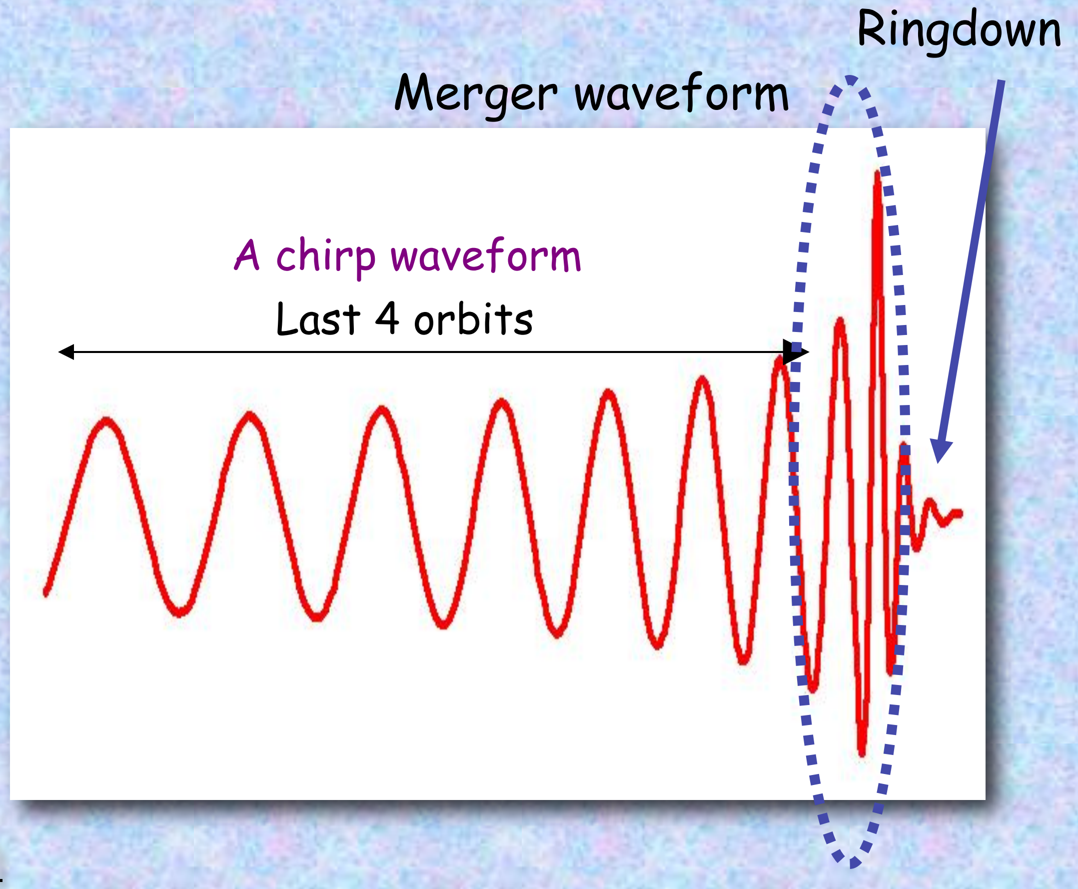
- Last few minutes (10K cycles) for NS-NS
- 40 - 700 per year by 2016
- BH inspirals could be more numerous

<u>Space-Based (millihertz)</u>

- MBH pairs (10^5 - 10^7 M_s) in galaxies to large Z
- EMRIs

<u>Pulsar Timing Arrays (nanohertz)</u>

- MBH pairs



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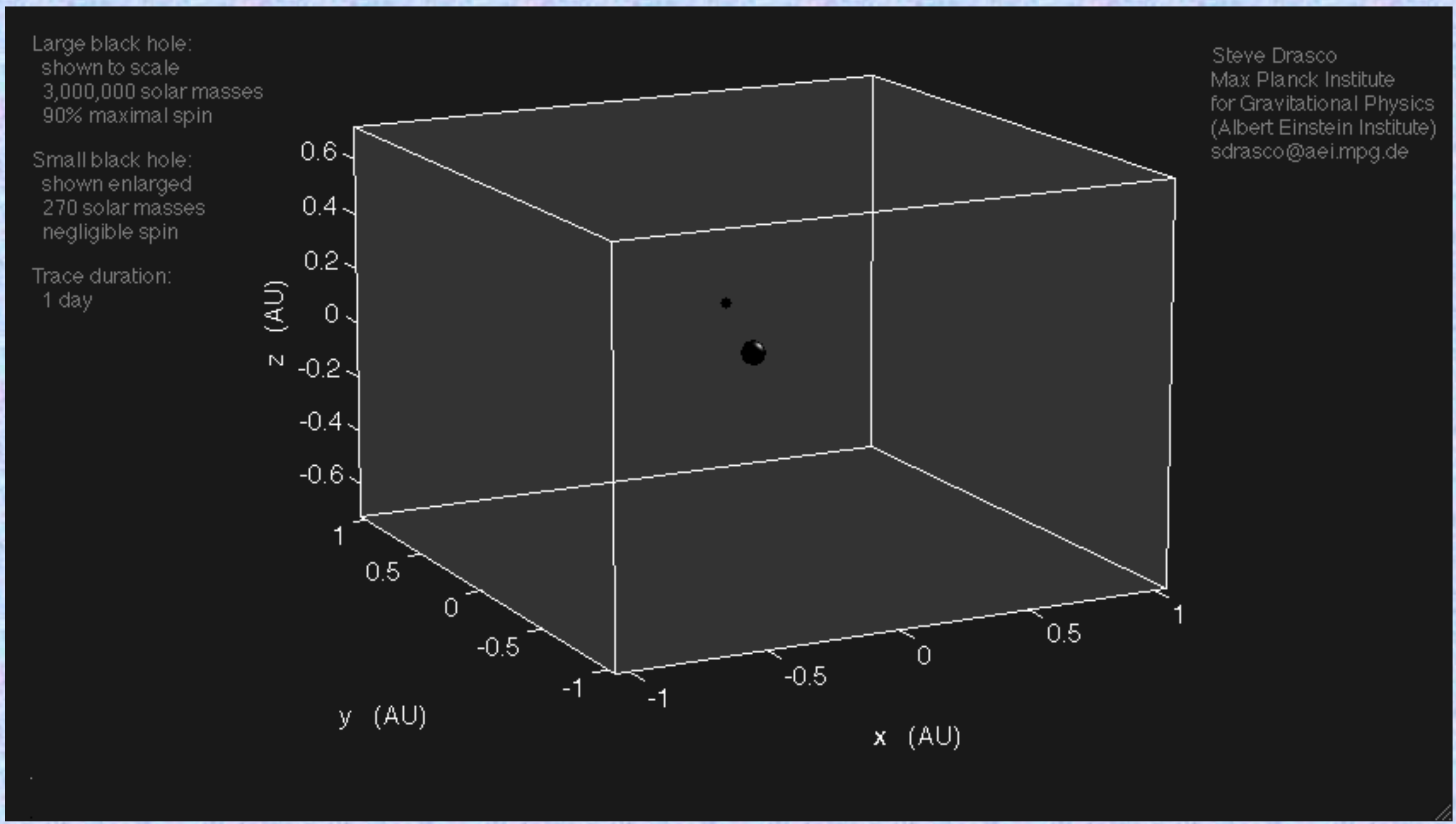
Hair counting using GW from EMRIs



Hair counting using GW from EMRIs

- EMRI: extreme mass-ratio inspiral
- GW source for LISA
- particle probes strong-field BH geometry
- accurate template waveforms needed
- “test” body motion (geodesics) no longer adequate
- calculate the “self-force” of body’s own field
- “Capra program”

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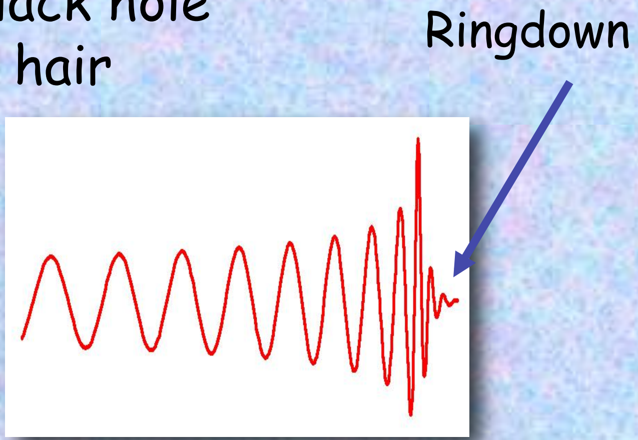
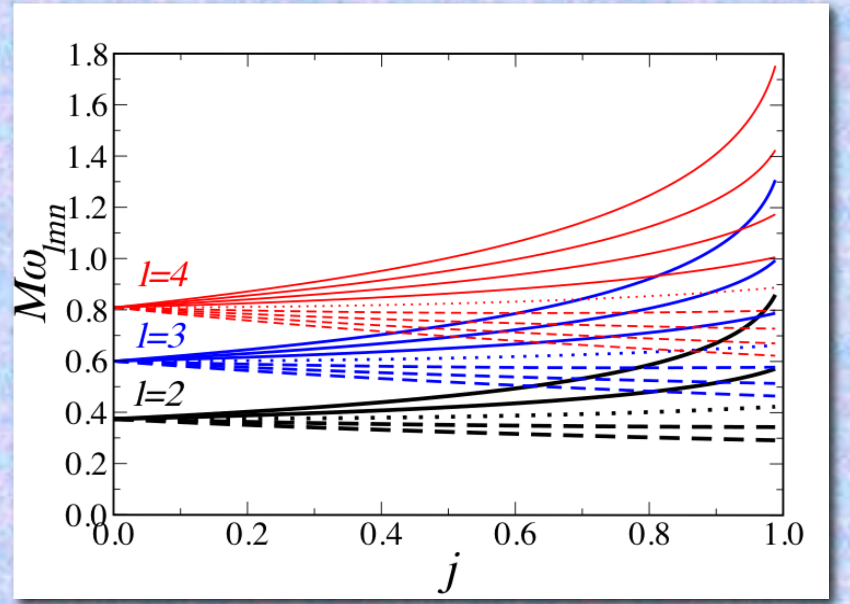
Temporary hair: Perturbed black holes

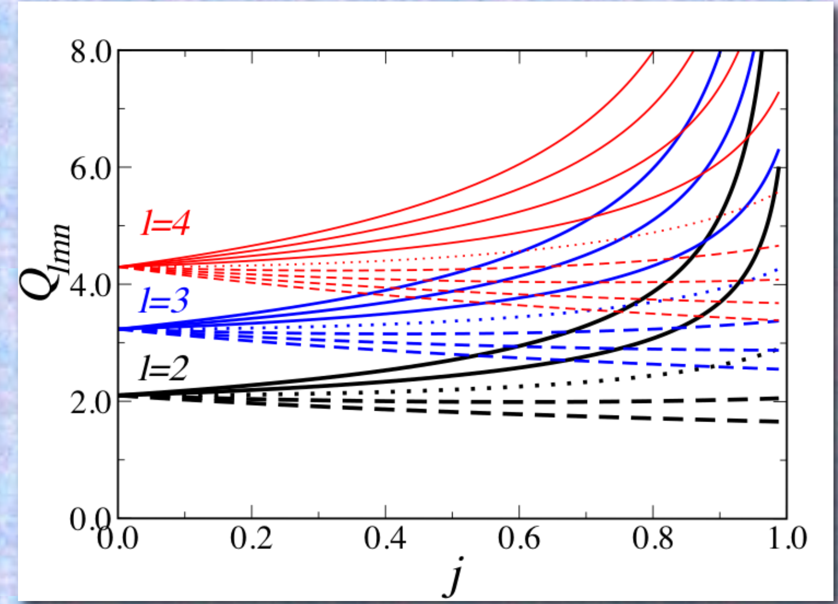
- collapse or merger produces distorted black hole
- hole radiates "ringdown'' waves to shed hair
- final state a stationary Kerr black hole
- quasi-normal modes

$$\omega = \omega_{\ell mn} + i\left(\frac{\pi\omega_{\ell mn}}{2Q_{\ell mn}}\right)$$



Berti, Cardoso & CMW (2006)





j = J/M^2

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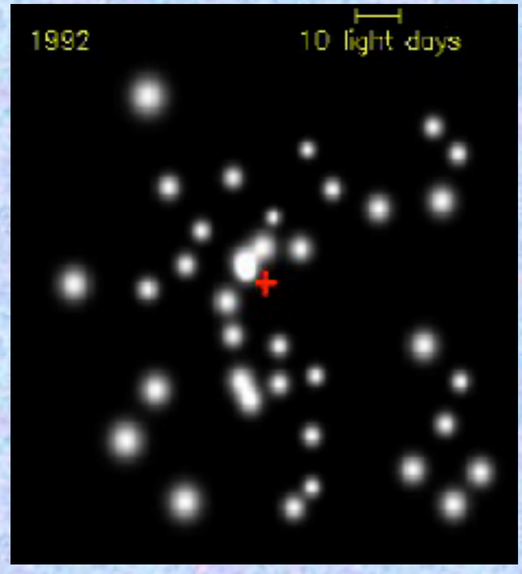
Counting hairs on the galactic center black hole SgrA*

- No hair theorems:

$$M_L + iJ_L = M(ia)^L$$

- $J = Ma$; $Q = -Ma^2$
- relativistic effects:
  periholion advance, redshift
  Doppler shifts, Shapiro delays
- Frame dragging (J) and
  quadrupole moment (Q)
  produce precessions of planes



SgrA* - a $4.3 \times 10^6\ M_{sun}$ rotating black hole

Counting hairs on the galactic center black hole SgrA*

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Orbital plane precessions as no-hair tests for SgrA*

	ω	Ω	i
M	✔		
J	✔	✔	✔*
Q	✔	✔	✔
dirt	✔	✔	✔

$$A_M = 6\pi \frac{M}{\bar{a}(1-e^2)}$$

$$A_J = 4\pi \frac{J}{M^2}\left(\frac{M}{\bar{a}(1-e^2)}\right)^{3/2}$$

$$A_Q = 3\pi \frac{Q}{M^3}\left(\frac{M}{\bar{a}(1-e^2)}\right)^{2}$$

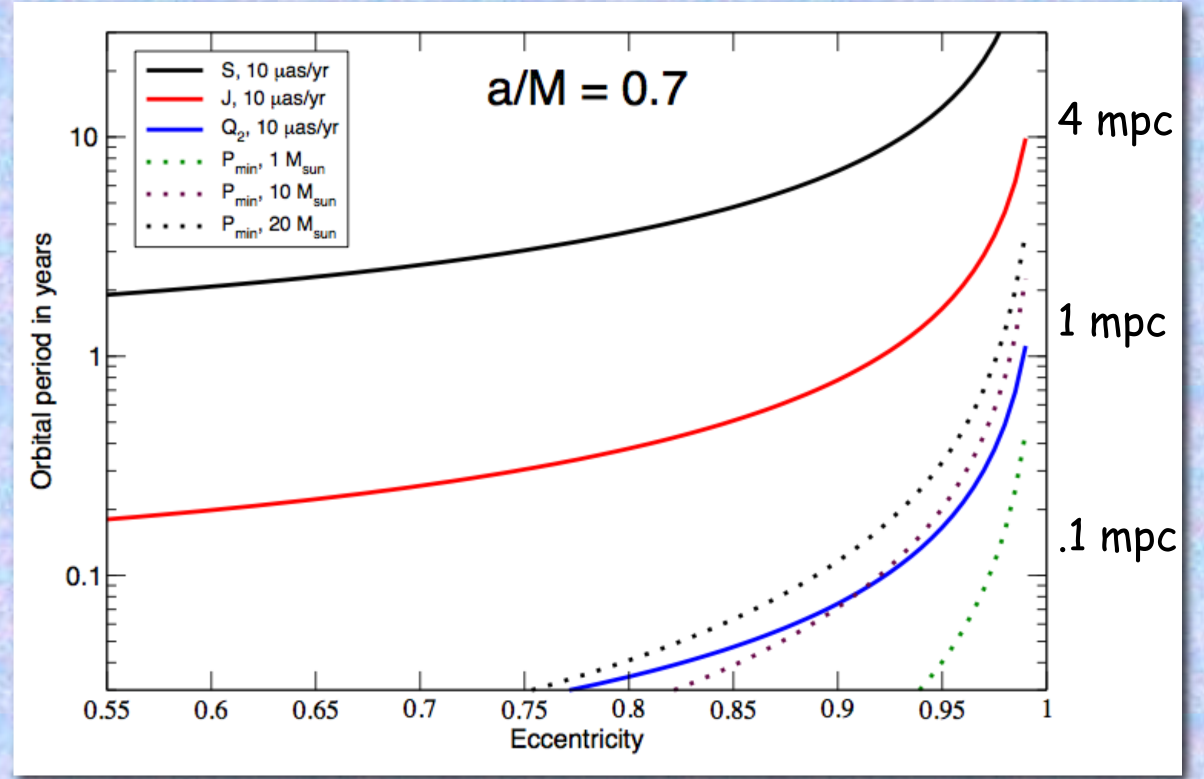
$a/M > 0.5$

$P \sim 0.1$ yr, $d < 10^{-3}$ pc, $e \sim 0.9$

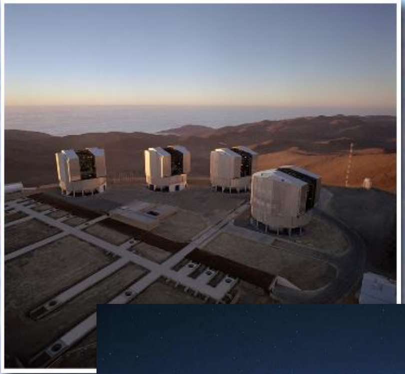
=> Precessions ~ 10 μas/yr

CMW, Ap J Lett. 647, L25 (2008)

The observational challenge

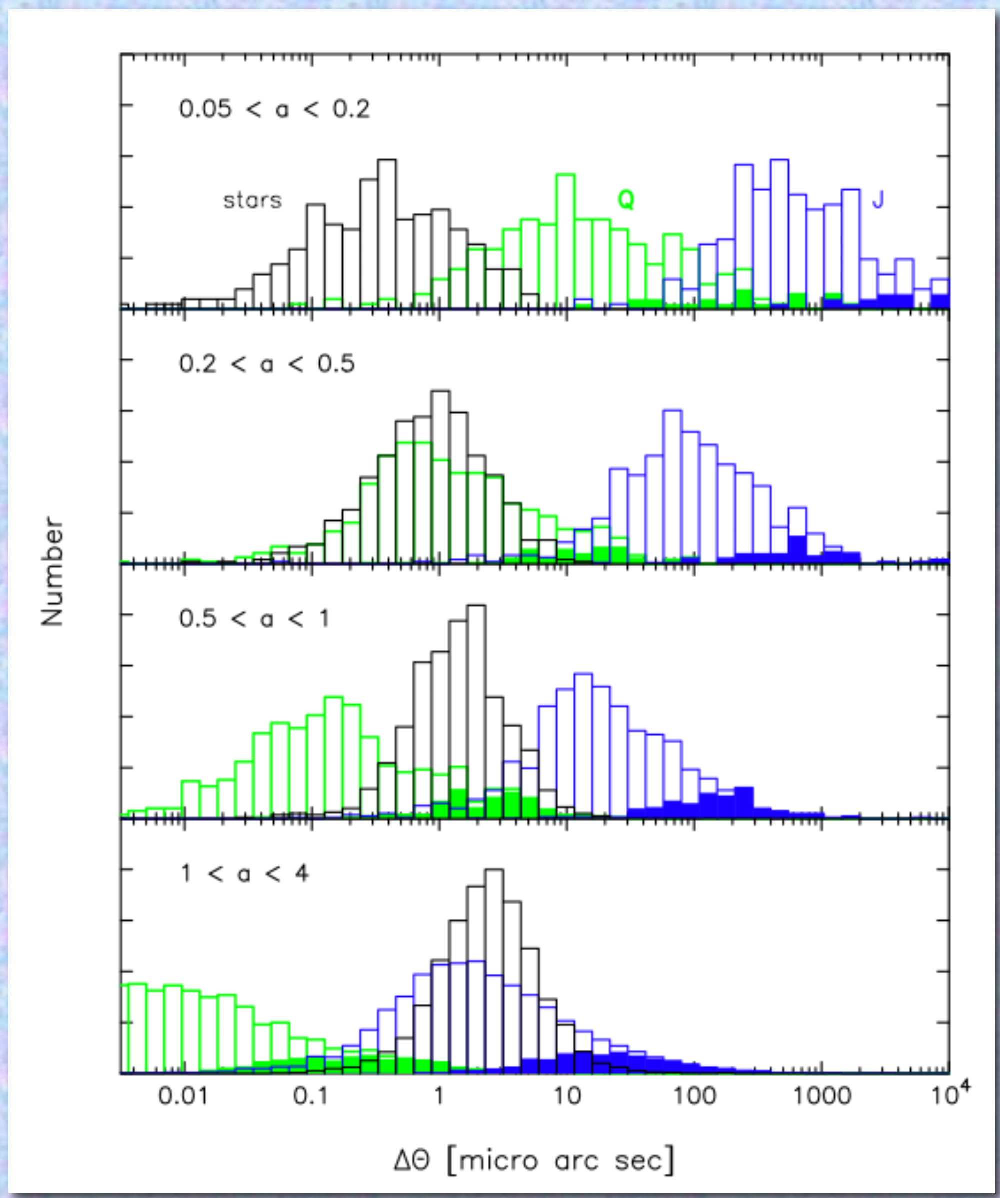


GRAVITY: near IR adaptive optics instrument for the Very Large Telescope Interferometer



ASTRA: extending the Keck interferometer

Effect of other stars/BH in the central mpc



- 10 stars ($1M_o$) & 11 BH ($10M_o$) within 4 mpc
- 100 realizations
- isotropic, mass segregated
- $J/M^2 = 1$

Numerical N-body simulations:
D. Merritt, T. Alexander, S. Mikkola, & CMW, PRD **81**, 062002 (2010)
Analytic orbit perturbation theory:
L. Sadeghian & CMW, CQG **28**, 225029 (2011)

Counting black hole hair at the galactic center

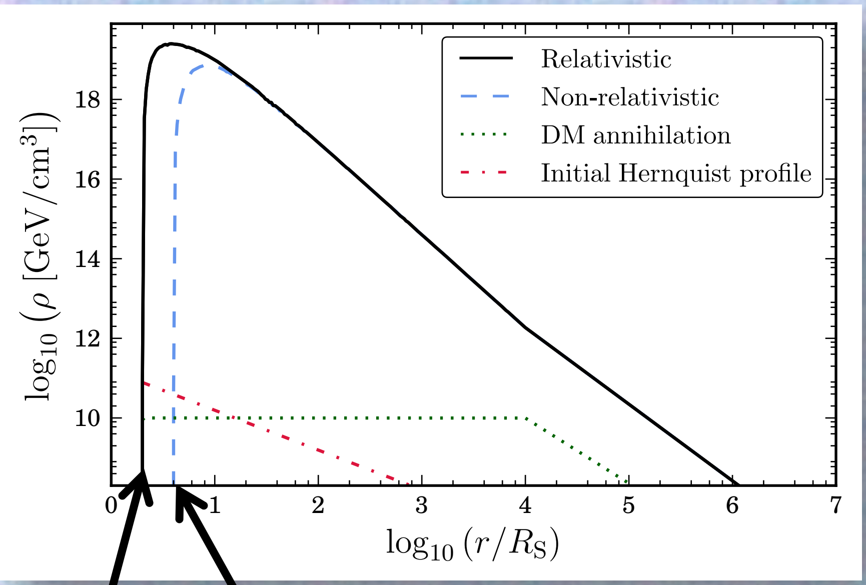
Other issues:

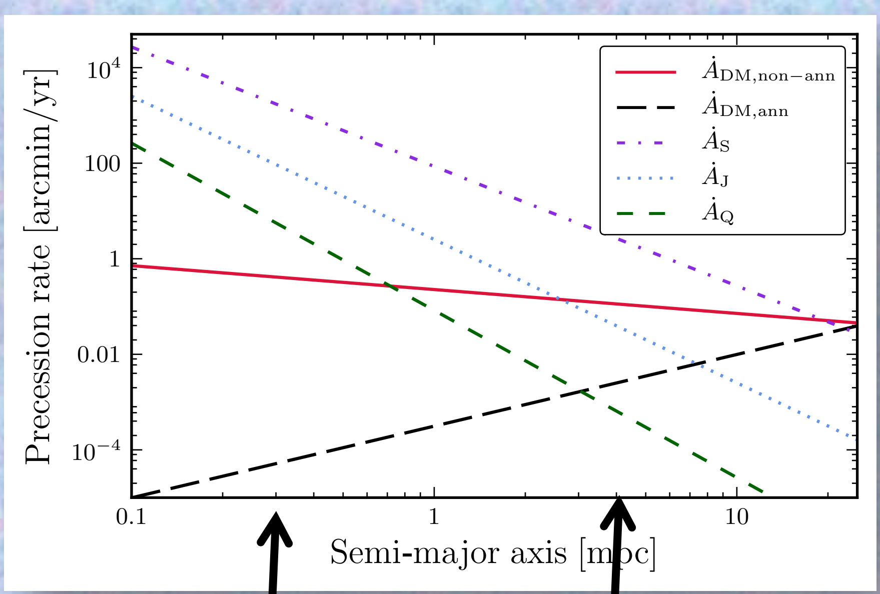
- effects of tidal distortions at close approach to the BH
  - ✓ *negligible*
- effects of a dark matter distribution
  - ✓ *Schwarzschild geometry*

    (Sadeghian, Ferrer & CMW, 1305.2619)

  - *extend to Kerr geometry*
- covariance analysis of actual astrometric observations of N candidate stars
- pulsars at the galactic center provide a complementary way to count hair

Dark matter at the galactic center





Gondolo & Silk 1999

Sadeghian, Ferrer & CMW 2013

no-hair target star

S2

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Strong-Field tests of general relativity

- neutron star masses and radii
- existence of event horizons with ADAFs
- measurement of maximum BH spins
- short gamma-ray bursts
- tidal disruption events
- direct imaging of nearby BH (SgrA*, M87)

Extreme-range tests of general relativity

- gravity at sub-mm and micron scales
- dark matter vs. modified gravity at galactic scales
- evolution of structure - dark energy vs. f(R)

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