



So5 – Limb Darkening

ISSP workshop – 31 May 2023 N. Ebrahimkutty, D. Mourard, A. Domiciano

Limb Darkening

Limb Darkening (LD) is the variation of brightness distribution of a star from the centre to the limb of the star.

LD occurs due to the variation in Density and temperature from from centre to the limb.

Understanding LD is important to study stellar atmosphere and its structure.

Still there are only very few studies currently on LD



Credit: NASA

Importance of Limb Darkening

For accurate estimation on the fundamental parameters,

Such as T_{eff} , log g, radius etc.



Credit: NASA

Provides constraints on the stellar atmosphere models

Help understand atmosphere structure



Analysing the transiting object, such as Exoplanets.

Crucial in missions like PLATO



Credit: NASA

With not much studies on the topic.

It is important to have a better understanding of limb darkening across the HR diagram.

CHARA/SPICA

Interferometric Survey of Stellar Parameter (ERC funded program)

We employ 6T interferometry with CHARA/SPICA to observe a large number of stars.

From this we get the angular diameter and (bottom table) the limb darkening of hundreds of stars.

Dwarfs	fs Challouf		uf	Salsi-1			Salsi-2			Giants	Challouf			Salsi-1			Salsi-2		
SpTy	0	BO	A0	F5	G7	K4	MO	M3	M4	SpTy	0	BO	A0	F5	G7	К4	MO	M3	M4
V // V-K	-2	-1	0	1	2	3	4	5	6	V // V-K	-2	-1	0	1	2	3	4	5	6
0	0,10	1,00	3,35	6,28	11,82	22,25	39,94	70,70	125,14	0	0,24	1,09	3,16	6,72	11,79	20,68	36,41	62,26	106,46
1	0,06	0,63	2,11	3,96	7,46					1	0,15	0,69	1,99	4,24	7,44				67,17
2		0,40	1,33	2,50	4,71	8,86				2	0,10	0,44	1,26	2,68	4,69	8,23			42,38
3		0,25	0,84	1,58	2,97	5,59	10,03	17,76		3		0,27	0,79	1,69	2,96	5,20	9,15		26,74
4		0,16	0,53	0,99	1,87	3,53	6,33	11,20		4		0,17	0,50	1,07	1,87	3,28	5,77	9,87	16,87
5		0,10	0,33	0,63	1,18	2,23	3,99	7,07		5		0,11	0,32	0,67	1,18	2,07	3,64	6,23	10,65
6			0,21	0,40	0,75	1,40	2,52	4,46		6			0,20	0,42	0,74	1,30	2,30	3,93	6,72
7			0,13	0,25	0,47	0,89	1,59	2,81	4,98	7			0,13	0,27	0,47	0,82	1,45	2,48	4,24
8				0,16	0,30	0,56	1,00	1,78	3,14	8			0,08	0,17	0,30	0,52	0,91	1,56	2,67
9				0,10	0,19	0,35	0,63	1,12	1,98	9			0,05	0,11	0,19	0,33	0,58	0,99	1,69
10				0.06		0.22	0.40	0.71		10						0.21	0.36	0.62	1.06
	Challouf				The Contract of Co	Local Sections													
Dwarfs	С	hallo	uf		Salsi-			Salsi-2	2	Giants	(hallou	f		Salsi-1			Salsi-2	
Dwarfs SpTy	C O	hallo B0	uf A0	F5	Salsi-: G7	K4	MO	Salsi-2 M3	2 M4	Giants SpTy	0	Challou B0	f A0	F5	Salsi-1 G7	K4	MO	Salsi-2 M3	M4
Dwarfs SpTy V // V-K	C 0 -2	hallo B0 -1	uf A0 0	F5 1	Salsi-: G7 2	K4 3	M0 4	Salsi-2 M3 5	2 M4 6	Giants SpTy V // V-K	0 -2	Challou B0 -1	f A0 0	F5 1	Salsi-1 G7 2	K4 3	M0 4	Salsi-2 M3 5	M4 6
Dwarfs SpTy V // V-K 0	C 0 -2 0,10	hallo B0 -1 1,00	uf A0 0 3,35	F5 1 6,28	Salsi-: G7 2 11,82	K4 3 22,25	M0 4 39,94	Salsi-2 M3 5 70,70	2 M4 6 1,25,14	Giants SpTy V // V-K 0	0 -2 0,24	Challou B0 -1 1,09	f A0 0 3,16	F5 1 6,72	Salsi-1 G7 2 11,79	K4 3 20,68	M0 4 36,41	Salsi-2 M3 5 62,26	M4 6 106,46
Dwarfs SpTy V // V-K 0 1	C 0 -2 0,10 0,06	hallo B0 -1 1,00 0,63	uf A0 0 3,35 2,11	F5 1 6,28 3,96	Salsi-2 G7 2 11,82 7,46	K4 3 22,25 14,04	M0 4 39,94 25,20	Salsi-2 M3 5 70,70 44,61	2 M4 6 1.25,14 78,96	Giants SpTy V // V-K 0 1	0 -2 0,24 0,15	Challou B0 -1 1,09 0,69	f A0 0 3,16 1,99	F5 1 6,72 4,24	Salsi-1 G7 2 11,79 7,44	K4 3 20,68 13,05	M0 4 36,41 22,97	Salsi-2 M3 5 62,26 39,28	M4 6 106,46 67,17
Dwarfs SpTy V // V-K 0 1 2	C 0 -2 0,10 0,06 0,04	hallor B0 -1 1,00 0,63 0,40	uf A0 0 3,35 2,11 1,33	F5 1 6,28 3,96 2,50	Salsi-: G7 2 11,82 7,46 4,71	K4 3 22,25 14,04 8,86	M0 4 39,94 25,20 15,90	Salsi-2 M3 5 70,70 44,61 28,14	2 M4 6 1.25,14 78,96 49,82	Giants SpTy V // V-K 0 1 2	0 -2 0,24 0,15 0,10	Challou B0 -1 1,09 0,69 0,44	f A0 0 3,16 1,99 1,26	F5 1 6,72 4,24 2,68	Salsi-1 G7 2 11,79 7,44 4,69	K4 3 20,68 13,05 8,23	M0 4 36,41 22,97 14,49	Salsi-2 M3 5 62,26 39,28 24,79	M4 6 106,46 67,17 42,38
Dwarfs SpTy V // V-K 0 1 2 3	C 0 -2 0,10 0,05 0,04 0,02	hallor B0 -1 1,00 0,63 0,40 0,25	uf A0 0 3,35 2,11 1,33 0,84	F5 1 6,28 3,96 2,50 1,58	Salsi-: G7 2 11,82 7,46 4,71 2,97	K4 3 22,25 14,04 8,86 5,59	M0 4 39,94 25,20 15,90 10,03	Salsi-2 M3 5 70,70 44,61 28,14 17,76	2 M4 6 1.25,14 78,96 49,82 31,43	Giants SpTy V // V-K 0 1 2 3	0 -2 0,24 0,15 0,10 0,06	Challou B0 -1 1,09 0,69 0,44 0,27	f A0 0 3,16 1,99 1,26 0,79	F5 1 6,72 4,24 2,68 1,69	Salsi-1 G7 2 11,79 7,44 4,69 2,96	K4 3 20,68 13,05 8,23 5,20	M0 4 36,41 22,97 14,49 9,15	Salsi-2 M3 5 62,26 39,28 24,79 15,64	M4 6 106,46 67,17 42,38 26,74
Dwarfs SpTy V // V-K 0 1 2 3 4	C O -2 0,10 0,06 0,04 0,02	hallou B0 -1 1,00 0,63 0,40 0,25 0,16	A0 0 3,35 2,11 1,33 0,84	F5 1 6,28 3,96 2,50 1,58 0,99	Salsi-: G7 2 11.82 7,46 4,71 2,97 1,87	K4 3 22,25 14,04 8,86 5,59 3,53	M0 4 39,94 25,20 15,90 10,03 6,33	Salsi-2 M3 5 70,70 44,61 28,14 17,76 11,20	2 M4 6 125,14 78,96 49,82 31,43 19,83	Giants SpTy V // V-K 0 1 2 3 4	0 -2 0,24 0,15 0,10 0,06 0,04	Challou B0 -1 1,09 0,69 0,44 0,27 0,17	f A0 0 3,16 1,99 1,26 0,79 0,50	F5 1 6,72 4,24 2,68 1,69 1,07	Salsi-1 G7 2 11,79 7,44 4,69 2,96 1,87	K4 3 20,68 13,05 8,23 5,20 3,28	M0 4 36,41 22,97 14,49 9,15 5,77	Salsi-2 M3 5 62,26 39,28 24,79 15,64 9,87	M4 6 106,46 67,17 42,38 26,74 16,87
Dwarfs SpTy V // V-K 0 1 2 3 4 5	C 0 -2 0,06 0,04 0,02 0,02 0,01	hallor B0 -1 1,00 0,63 0,40 0,25 0,16 0,10	uf A0 3,35 2,11 1,33 0,84 0,53 0,33	F5 1 6,28 3,96 2,50 1,58 0,99 0,63	Salsi-: G7 2 11,82 7,46 4,71 2,97 1,87 1,18	K4 3 22,25 14,04 8,86 5,59 3,53 2,23	M0 4 39,94 25,20 15,90 10,03 6,33 3,99	Salsi-2 M3 5 70,70 44,61 28,14 17,76 11,20 7,07	2 M4 6 125,14 78,96 49,82 31,43 19,83 12,51	Giants SpTy V // V-K 0 1 2 3 4 5	0 -2 0,24 0,15 0,10 0,06 0,04 0,02	Challou B0 -1 1,09 0,69 0,44 0,27 0,17 0,11	f A0 0 3,16 1,99 1,26 0,79 0,50 0,32	F5 1 6,72 4,24 2,68 1,69 1,07 0,67	Salsi-1 G7 2 11,79 7,44 4,69 2,96 1,87 1,18	K4 3 20,68 13,05 8,23 5,20 3,28 2,07	M0 4 36,41 22,97 14,49 9,15 5,77 3,64	Salsi-2 M3 5 62,26 39,28 24,79 15,64 9,87 6,23	M4 6 106,46 67,17 42,38 26,74 16,87 10,65
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Dwarfs SpTy V // V-K 0 1 2 3 4 5 6 7	C 0 -2 0,06 0,04 0,02 0,02 0,01 0,01 0,00	hallor B0 -1 1,00 0,63 0,40 0,25 0,16 0,10 8,06 0,04	uf A0 3,35 2,11 1,33 0,84 0,59 0,33 0,21 0,13	F5 1 6,28 3,96 2,50 1,58 0,99 0,63 0,40 0,25	Salsi-: G7 2 11.82 7,46 4,71 2,97 1,87 1,18 0,75 0,47	K4 3 22,25 14,04 8,86 5,59 3,53 2,23 1,40 0,89	M0 4 39,94 25,20 15,90 10,03 6,33 3,99 2,52 1,59	Salsi-2 M3 5 70,70 44,61 28,14 17,76 11,20 7,07 4,46 2,81	2 M4 6 125,14 78,96 49,82 31,43 19,83 12,51 7,90 4,98	Giants SpTy V // V-K 0 1 2 3 4 5 6 7	0 -2 0,24 0,15 0,10 0,06 0,04 0,02 0,02 0,01	Challou B0 -1 1,09 0,69 0,44 0,27 0,17 0,11 0,07 0,04	f A0 3,16 1,99 1,26 0,79 0,50 0,32 0,20 0,13	F5 1 6,72 4,24 2,68 1,69 1,07 0,67 0,42 0,27	Salsi-1 G7 2 11,79 7,44 4,69 2,96 1,87 1,18 0,74 0,47	K4 3 20,68 13,05 8,23 5,20 3,28 2,07 1,30 0,82	M0 4 36,41 22,97 14,49 9,15 5,77 3,64 2,30 1,45	Salsi-2 M3 5 62,26 39,28 24,79 45,64 9,87 6,23 3,93 2,48	M4 6 106,46 67,17 42,38 26,74 16,87 10,65 6,72 4,24
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Mourard et al.2022

S05 – Limb Darkening

With CHARA/SPICA, we aim to observe a large sample of stars using the CHARA/SPICA over the span of 3 years.

162 stars from SpTy: B0 - M3 and SpC: III-V

All stars are selected with:

- No activity
- DEC > -30⁰
- Vmag< 6
- 0.7< angular diameter < 11 mas

Overall goal is to produce a **catalog** of directly measured intensity profiles, angular diameter (1% precision) and other fundamental parameters.



Catalog

In the catalog we have multiple star in each stellar types from **dwarfs**, **giants** and **main-sequence**.

Most stars are without stellar activity to isolate the LD.

Stars with different priorities of observations with highest priority to cover most of the HR diagram.

We couldn't find much stars without stellar activities in these region. We are still looking for these.



Observation strategy

We use 60 spectral channels covering 600-900 nm to take multiple observation, with each taking 40 min.



We get the squared visibility of the star from we can get the angular diameter and other parameters.



For each star we will make at least 2 observation each.

Pipeline

- We have developed a pipeline to combine observations from **Spectroscopy**, **Interferometry** and **Photometry** to give an accurate estimation of the fundamental parameters.
- Within the **SAPP (PLATO)** framework an artificial neural network has been developed to make a model fitting and estimate the parameter from this fitting. We employ similar technique in our work.
- The idea is to use the parameters from the **Spectroscopy** module similar to **SAPP** to initiate and guide the model fitting in the **Interferometry** and them combine this result with the **Photometry** to get an accurate estimation of stellar parameters.

Interferometric Analysis



Spectroscopy



Photometry



Pipeline



Simulating Interferometric Data



Additional Data

Spectroscopic Data

For the spectral analysis we use Gaia RVS spectra available in Gaia archive.

18 Sco Gaia RVS 1.0 0.9 Normalized Flux 0.8 0.7 0.6 0.5 0.4 535 540 545 550 555 560 wavelength (nm)

It is also possible to analyse any given spectra with this module.

Photometry

For the photometric analysis, we are currently making use of stellar track model to generate synthetic photometric data.

Using the luminosity from this we calculate the bolometric flux.

But soon we will replace it with photometric observations of the targets from various archives and catalogs.

Results

Different stars were simulated with the reference parameters sample of benchmark stars Table 1. In Gent et al. 2022.

Star	T _{eff} (Model) (K)	log g (model) (dex)	Angular Diameter (mas)(model)	T _{eff} (K)	log g (dex)	Angular Diameter _(mas)	[Fe/H]	Luminosity (L _o)	Radius (R _o)
18 Sco	5810	4.44	0.665	5826.36 ±104.01	4.408 ± 0.053	0.6653 ± .0005	-0.0886 ± .2746	1.08 ± 0.03	1.009 ± 0.034
δ Eri	5022	3.76	2.405	5019.33 ± 94.94	3.737 ± 0.077	2.400 ± 0.0004	0.111 ± 0.246	3.12 ± 0.08	2.333 ± 0.111
η Βοο	6099	3.79	2.179	6336.60 ± 122.40	3.892 ± 0.057	2.1760 ± 0.0003	0.158 ± 0.266	7.81 ± 0.30	2.671 ± 0.105
Procyon	6554	4.0	5.426	6619.08 ± 223.73	4.00 ± .086	5.4191 ± 0.0003	0.0795 ± 0.364	6.98 ± 0.39	2.047 ± 0.153

1D vs 3D

Comparison with Maxted 2023

 $h'_1 = I\left(\frac{1}{3}\right)$



 $h'_2 = I\left(\frac{1}{3}\right) - I\left(\frac{2}{3}\right)$



7000

Link with PLATO

- The work explained is in close association with the PLATO WP 122 and this will be integrated with the MStesci1 pipeline in PLATO. This will be part of creating the benchmark star and getting their fundamental parameters.
- Within the PLATO framework an article is under preparation which will be submitted later this year.
- In addition, SPICA will also play part in WP145300m in the PLATO Ground Observation Program. In which we propose to do follow-up observations with possible interesting targets that will be detected by PLATO.

Conclusion

- By the end of the program we will have a catalog of stars across HR diagram to giving a better understanding of stellar LD.
- By combining **Spectrometry**, **Interferometry** and **Photometry**, we were able to estimate nad constrain the stellar parameters.
- We will keep improving the pipeline to include 3D models for the model-fitting and replace synthetic photometry with observations
- Also, an article (Ebrahimkutty et al. 2023) is under preparation which will be submitted this year.

Model Fitting



To get the best fit between the model and the observation we use Chi-square minimisation method.

This done by changing the values of free parameters (Teff, log g and angular diameter) before each iteration and valuation chi square.

In addition we use MCMC on the fitting to get better sampling over the results.







Neural network using Pytorch

Learning Rate = 1e-5 Epoch = 10000 Loss <.01%

The blue line shows the loss from the neural network training at each epoch.

The orange line depicts the loss from in the evaluation of the NN training at each epoch



(left) true values (red) of intensity overplotted on the predicted values (blue) from the test set (taken in random) for 12 different Radau points as a function of wavelength and (right) same intensities for different wavelength as function of Radau points.

Star Model

With the intensity profile, we can get the square visibility of the star by taking the Fourier transform.

As it has spherical symmetry, we can easily replace the Fourier transform with Hankel transform and square it to get the square visibility.



η Boo

For n Boo



Angular Diameter - 2.179 mas, Teff - 6099 K, log g - 3.79 dex





Spectroscopy



Draft paper in progress

Optimized use of spectroscopy, interferometry, and stellar atmosphere models for the determination of fundamental parameters of stars

Paper to be submitted within the framework of PLATO, with the below contents

- 1. General introduction on stellar parameters and the proposed method
- 2. From the MARCS grid to the intensity profiles with ML (discussion of models is needed)
- 3. Fit of synthetic interferometric data (hopefully with actual data before the summer)

 I limitations
- Inclusion of the interferometric pipeline in the general SAPP pipeline, and combination of spectroscopy and photometry for an optimized determination of R, Teff, log

18 Sco

