A role for magnetic reconnection in anisotropic plasma turbulence

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M. W. Kunz (Princeton University)

Transalpine Workshop on Magnetic Reconnection and Turbulence in Space and Fusion Plasmas
Nice, 15–16 May 2024

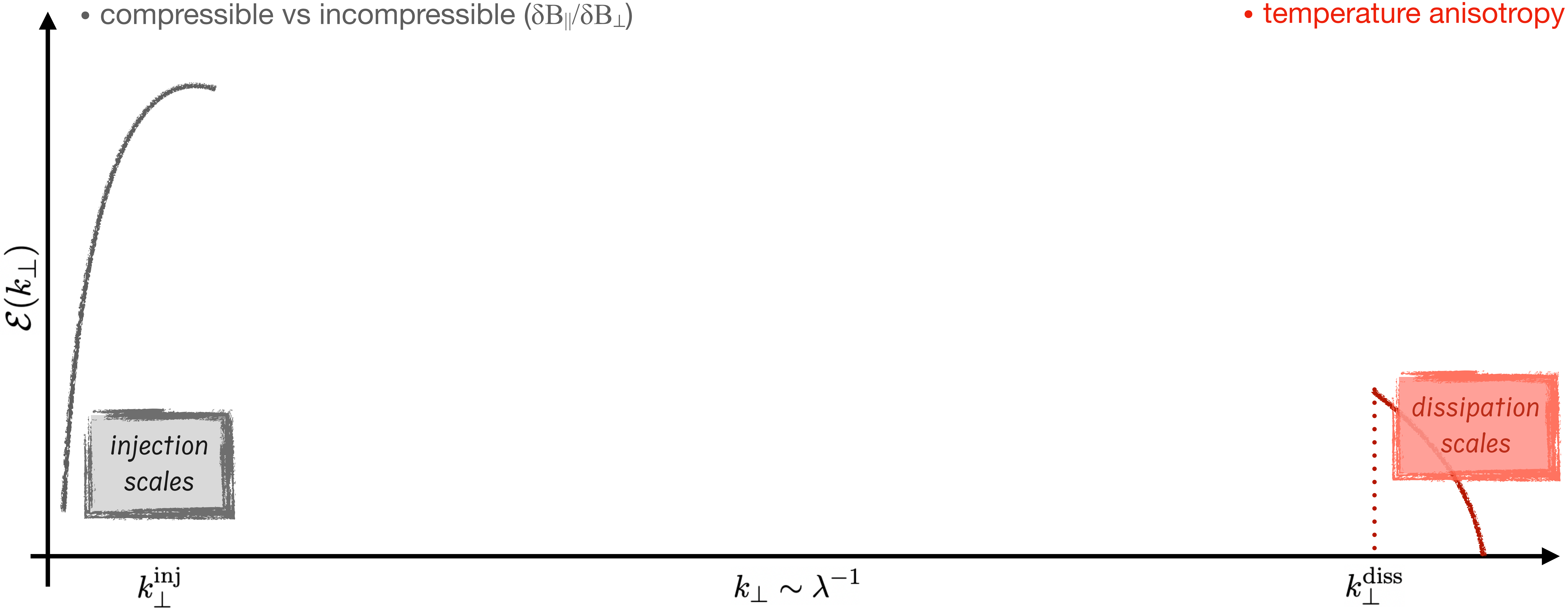
Motivation

Injection

- fluctuations' amplitude ($\delta B/B_0$)
- isotropic vs anisotropic ($L_{\parallel}/L_{\perp}$)
- compressible vs incompressible ($\delta B_{\parallel}/\delta B_{\perp}$)

Dissipation

- scale separation (L/λ_{diss}, $L/\rho_{i,e}$)
- plasma beta (β_i , β_e)
- temperature anisotropy ($T_{\perp,s}/T_{\parallel,s}$)



Motivation

Turbulent Cascade

Injection → **Dissipation**

Injection
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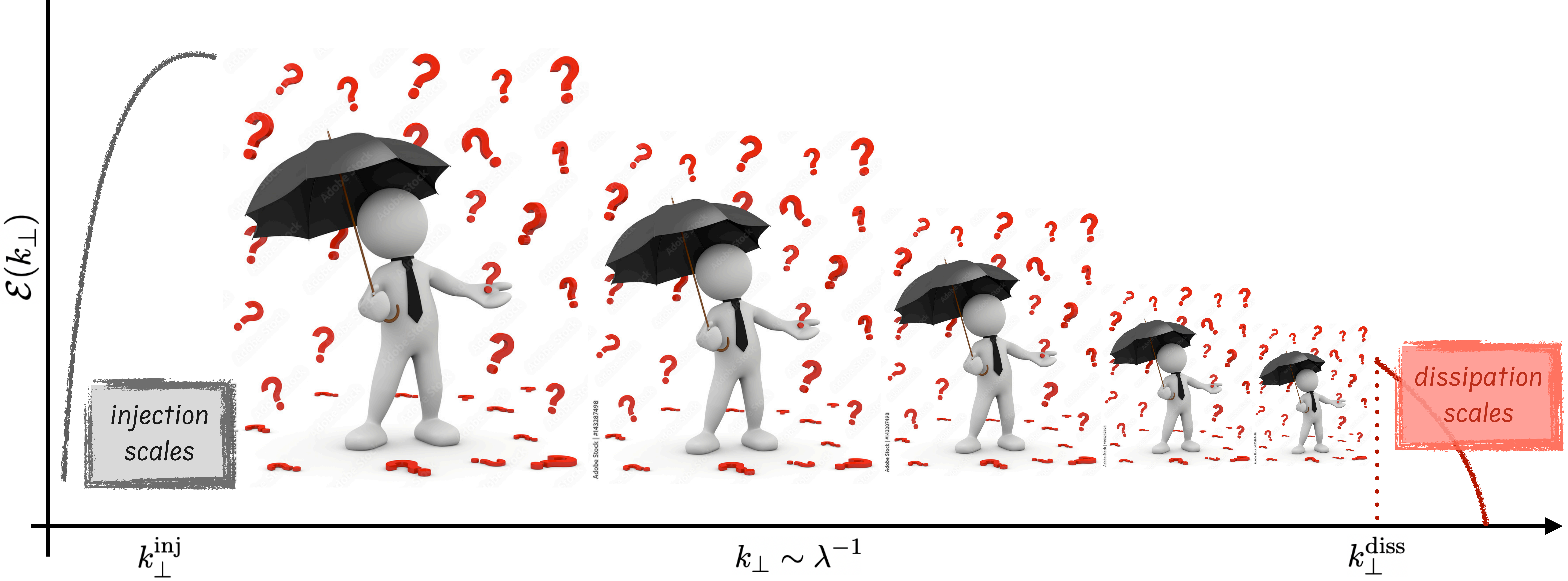

Introduction

[Borovsky, JGRA (2008)]

Alfvénic Turbulence

☞ incompressible MHD in the Elsässer formulation ($\eta = \nu$):

$$\boldsymbol{z}^{\pm} \doteq \boldsymbol{u} \pm \frac{\boldsymbol{B}}{\sqrt{4\pi\rho_0}}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{z}^{\pm} = 0$$

$$\frac{\partial \boldsymbol{z}^{+}}{\partial t} + (\boldsymbol{z}^{-} \cdot \boldsymbol{\nabla})\boldsymbol{z}^{+} = -\boldsymbol{\nabla}\widetilde{P}_{\rm tot} + \eta\nabla^2\boldsymbol{z}^{+}$$

$$\frac{\partial \boldsymbol{z}^{-}}{\partial t} + (\boldsymbol{z}^{+} \cdot \boldsymbol{\nabla})\boldsymbol{z}^{-} = -\boldsymbol{\nabla}\widetilde{P}_{\rm tot} + \eta\nabla^2\boldsymbol{z}^{-}$$

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Alfvén waves traveling "up" or "down" the magnetic field **B**

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non-linear interaction only between counter-propagating Alfvén waves

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non-linear interaction only between counter-propagating Alfvén waves

Alfvén waves traveling "up" or "down" the magnetic field **B**

Alfvénic turbulence ~ interaction of counter-propagating AWs

Alfvénic Turbulence

☞ split into "background + Alfvénic fluctuations":

$$\boldsymbol{B} = \boldsymbol{B}_0 + \delta\boldsymbol{B}_\perp$$

$$\boldsymbol{u} = \cancel{\boldsymbol{u}}_0 + \delta\boldsymbol{u}_\perp$$

$$\boldsymbol{z}^\pm = \boldsymbol{z}_0^\pm + \delta\boldsymbol{z}_\perp^\pm$$

$$\boldsymbol{z}_0^\pm = \pm\boldsymbol{B}_0/\sqrt{4\pi\rho_0} = \pm\boldsymbol{v}_{\mathrm{A},0}$$

$$\delta\boldsymbol{z}_\perp^\pm = \delta\boldsymbol{u}_\perp \pm \delta\boldsymbol{B}_\perp/\sqrt{4\pi\rho_0}$$

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$$\boldsymbol{z}_0^\pm = \pm \boldsymbol{B}_0 / \sqrt{4\pi\rho_0} = \pm \boldsymbol{v}_{\mathrm{A},0}$$

$$\boxed{\delta \boldsymbol{z}_\perp^\pm = \delta \boldsymbol{u}_\perp \pm \delta \boldsymbol{B}_\perp / \sqrt{4\pi\rho_0}}$$

🛑 *CAVEAT!*

purely transverse fluctuations

(w.r.t. a mean field <B>)

Alfvénic Turbulence

☞ split into "background + Alfvénic fluctuations":

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$$\boldsymbol{z}_0^{\pm} = \pm \boldsymbol{B}_0 / \sqrt{4\pi\rho_0} = \pm \boldsymbol{v}_{\mathrm{A},0}$$

$$\delta \boldsymbol{z}_{\perp}^{\pm} = \delta \boldsymbol{u}_{\perp} \pm \delta \boldsymbol{B}_{\perp} / \sqrt{4\pi\rho_0}$$

$$\left(\frac{\partial}{\partial t} \mp \underbrace{v_{\mathrm{A},0} \nabla_{||}}_{\omega_{\mathrm{lin}}^{\pm} \sim k_{||}^{\pm} v_{\mathrm{A},0}} + \underbrace{\delta \boldsymbol{z}_{\perp}^{\mp} \cdot \boldsymbol{\nabla}_{\perp}}_{\omega_{\mathrm{nl}}^{\pm} \sim k_{\perp}^{\pm} \delta z_{\perp}^{\mp}} - \underbrace{\eta \nabla^2}_{\omega_{\mathrm{diss}}^{\pm} \sim \eta k^{\pm 2}} \right) \delta \boldsymbol{z}_{\perp}^{\pm} = - \frac{\boldsymbol{\nabla} \delta P_{\mathrm{tot}}}{\rho_0}$$

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LINEAR frequency ($\sim 1/\tau_A$)

NON-LINEAR frequency ($\sim 1/\tau_{NL}$)

DISSIPATION frequency ($\sim 1/\tau_{diss}$)

Alfvénic Turbulence

$$\left(\frac{\partial}{\partial t} \mp \underbrace{v_{A,0}\,\nabla_{\|}}_{\omega^{\pm}_{\rm lin} \sim k^{\pm}_{\|}\, v_{A,0}} + \underbrace{\delta z^{\mp}_{\perp} \cdot \boldsymbol{\nabla}_{\perp}}_{\omega^{\pm}_{\rm nl} \sim k^{\pm}_{\perp}\, \delta z^{\mp}_{\perp}} - \underbrace{\eta\, \nabla^{2}}_{\omega^{\pm}_{\rm diss} \sim \eta\, k^{\pm\,2}}\right) \delta z^{\pm}_{\perp} = -\frac{\boldsymbol{\nabla}\delta P_{\rm tot}}{\rho_0}$$

LINEAR frequency ($\sim 1 / \tau_A$)

NON-LINEAR frequency ($\sim 1 / \tau_{NL}$)

$$\chi^{\pm} \sim \frac{|(\delta z^{\mp}_{\perp} \cdot \boldsymbol{\nabla}_{\perp})\, \delta z^{\pm}_{\perp}|}{|(v_{A,0}\, \nabla_{\|})\, \delta z^{\pm}_{\perp}|} \sim \frac{\omega^{\pm}_{\rm nl}}{\omega^{\pm}_{\rm lin}} \sim \frac{\tau^{\pm}_{\rm A}}{\tau^{\pm}_{\rm nl}} \sim \frac{k^{\pm}_{\perp}\, \delta z^{\mp}_{k}}{k^{\pm}_{\|}\, v_{A,0}}$$

NON-LINEAR PARAMETER

☞ in the following, I will consider ***balanced turbulence*** and forget about "±" for simplicity

Alfvénic Turbulence

$$\Bigg(\frac{\partial}{\partial t} \mp \underbrace{v_{A,0}\,\nabla_{\|}}_{\omega^{\pm}_{\rm lin} \sim k^{\pm}_{\|}\, v_{A,0}} + \underbrace{\delta z^{\mp}_{\perp} \cdot \nabla_{\perp}}_{\omega^{\pm}_{\rm nl} \sim k^{\pm}_{\perp}\, \delta z^{\mp}_{\perp}} - \underbrace{\eta\,\nabla^{2}}_{\omega^{\pm}_{\rm diss} \sim \eta\, k^{\pm 2}}\Bigg)\delta z^{\pm}_{\perp} = -\frac{\nabla \delta P_{\rm tot}}{\rho_0}$$

LINEAR frequency (~ 1 / τ_A)

NON-LINEAR frequency (~ 1 / τ_{NL})

non-linear parameter: $$\chi \doteq \frac{\omega_{\rm nl}}{\omega_{\rm A}} = \frac{k_{\perp}\delta z}{k_{\|} v_{\rm A}}$$

→ << 1 (“WEAK”)

→ ~ 1 (“STRONG”)

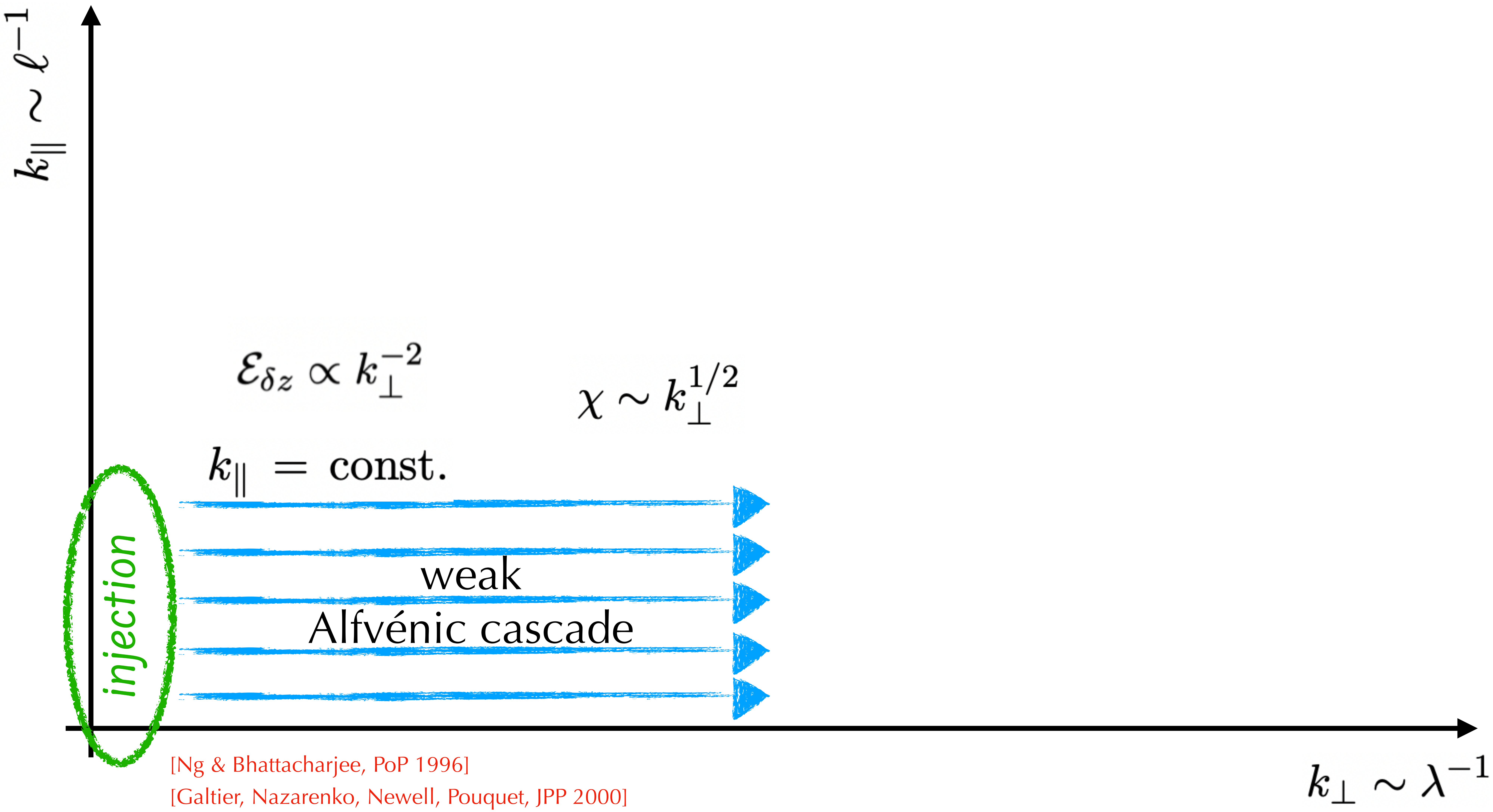
cascade time: $$\tau_{\rm casc} \sim N\,\tau_{\rm A} \sim \frac{\tau_{\rm nl}^2}{\tau_{\rm A}} = \frac{\tau_{\rm nl}}{\chi}$$

“Classical” phenomenology of Alfvénic Turbulence

Energy flux in k space

"Classical" phenomenology of Alfvénic Turbulence

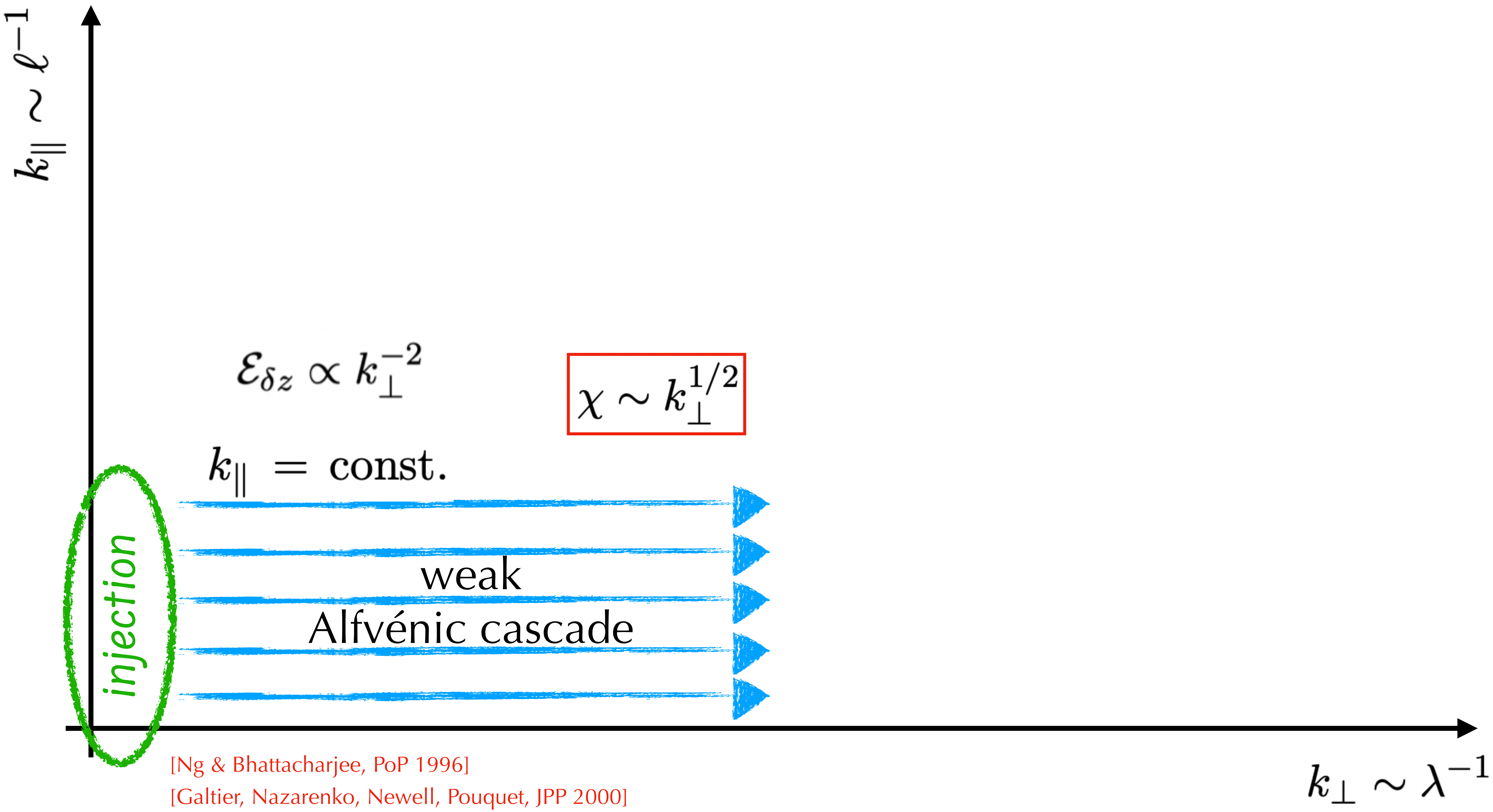
Energy flux in k space



[Ng & Bhattacharjee, PoP 1996]
[Galtier, Nazarenko, Newell, Pouquet, JPP 2000]

"Classical" phenomenology of Alfvénic Turbulence

Energy flux in k space



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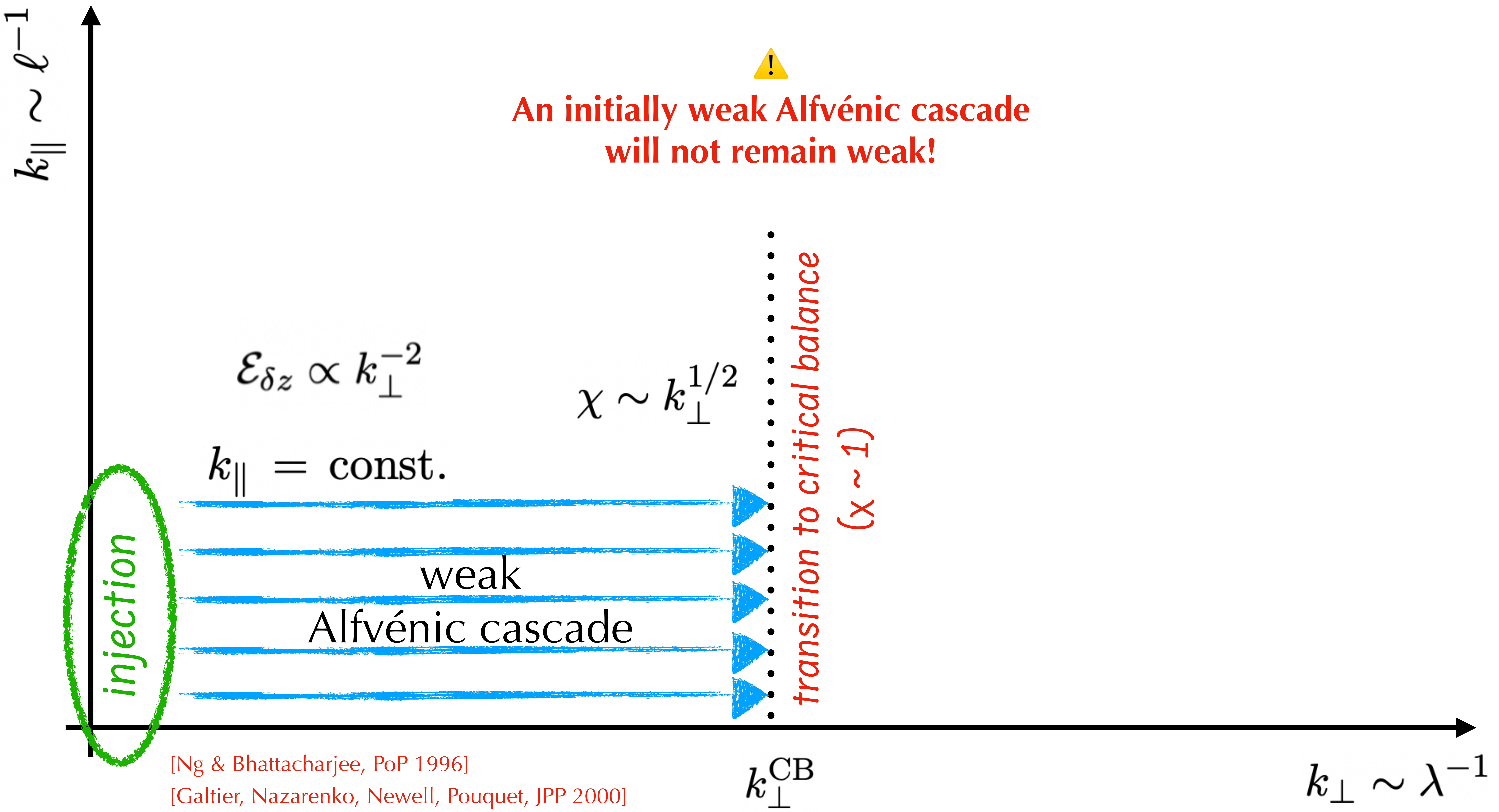

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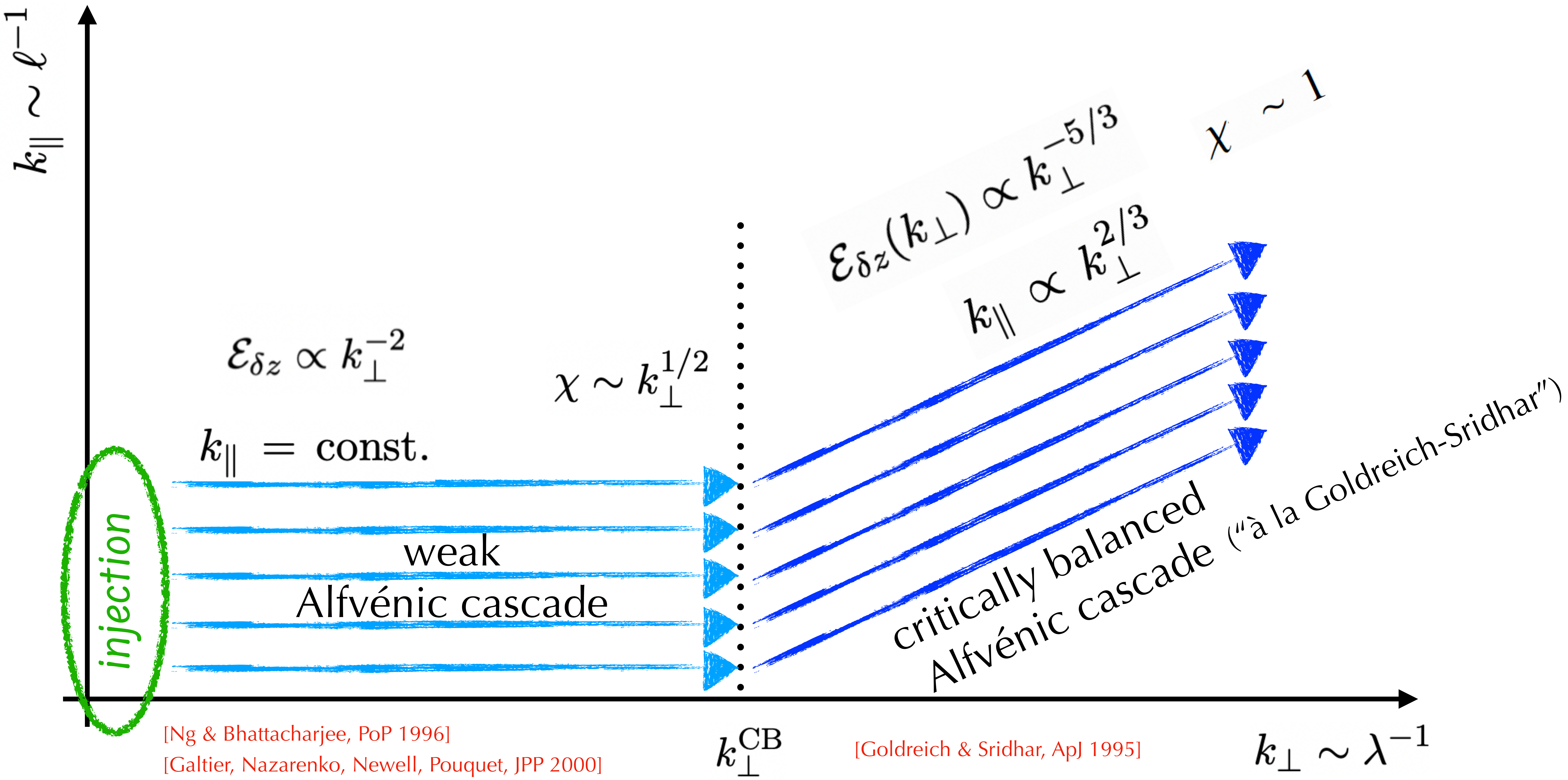


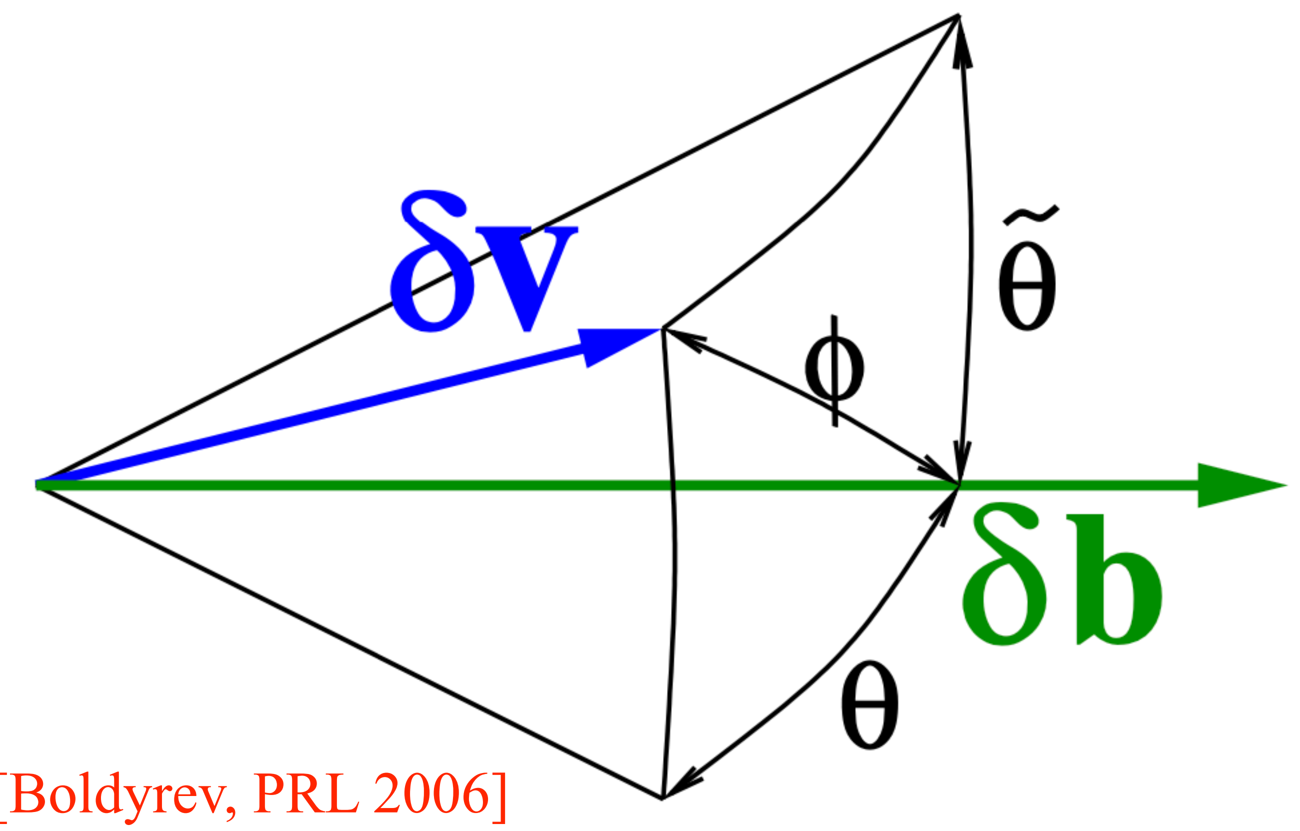
[Ng & Bhattacharjee, PoP 1996]
[Galtier, Nazarenko, Newell, Pouquet, JPP 2000]
[Goldreich & Sridhar, ApJ 1995]

Further advances in phenomenology of strong Alfvénic Turbulence

critical balance + **dynamic alignment**

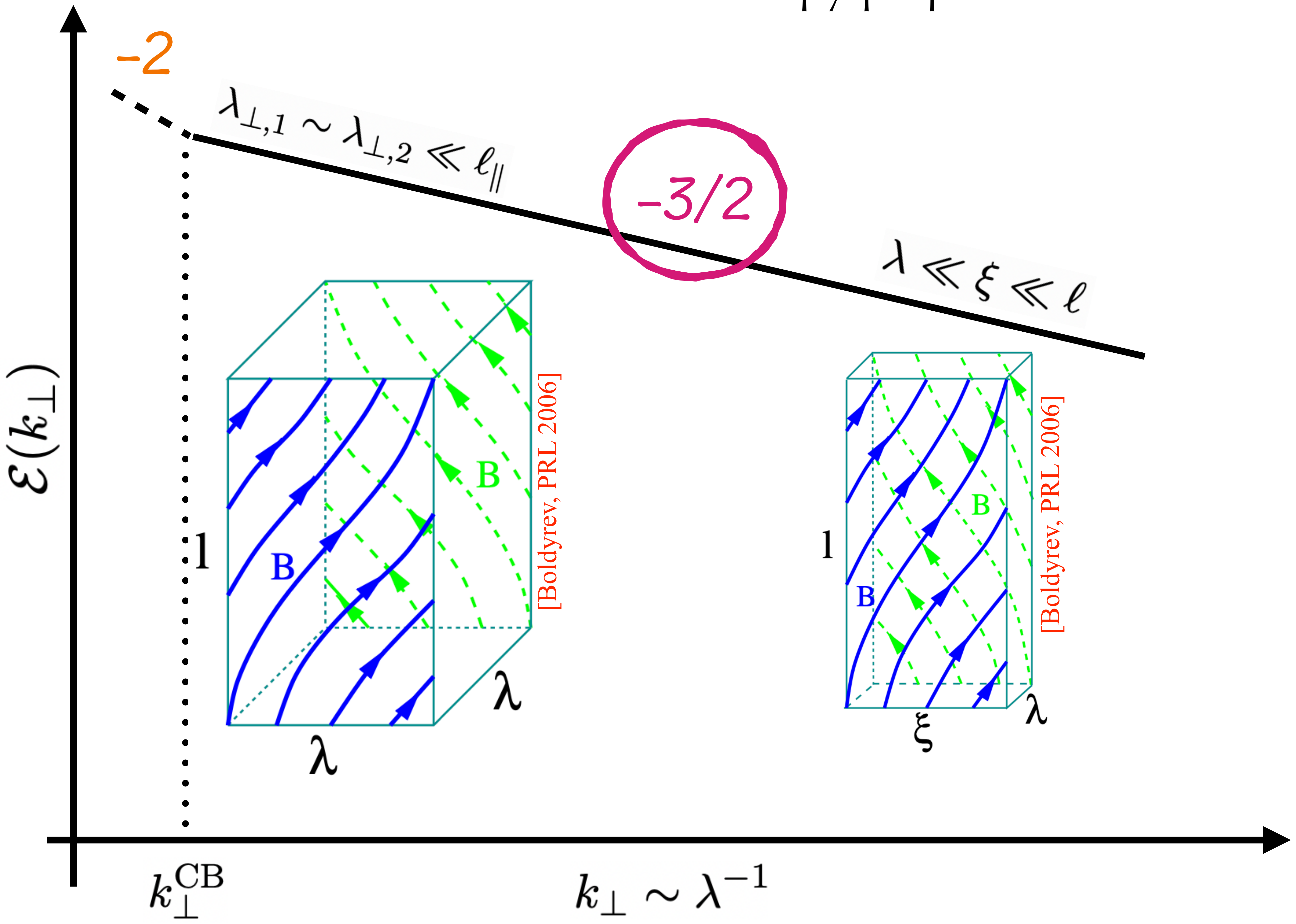
[Boldyrev, PRL 2006]
[Chandran, Schekochihin, Mallet, ApJ 2015]

1. weakening of nonlinearities
2. induce anisotropy perpendicular to <**B**> (3D anisotropy)



[Boldyrev, PRL 2006]

$$\theta_{k_\perp} \propto k_\perp^{-1/4}$$

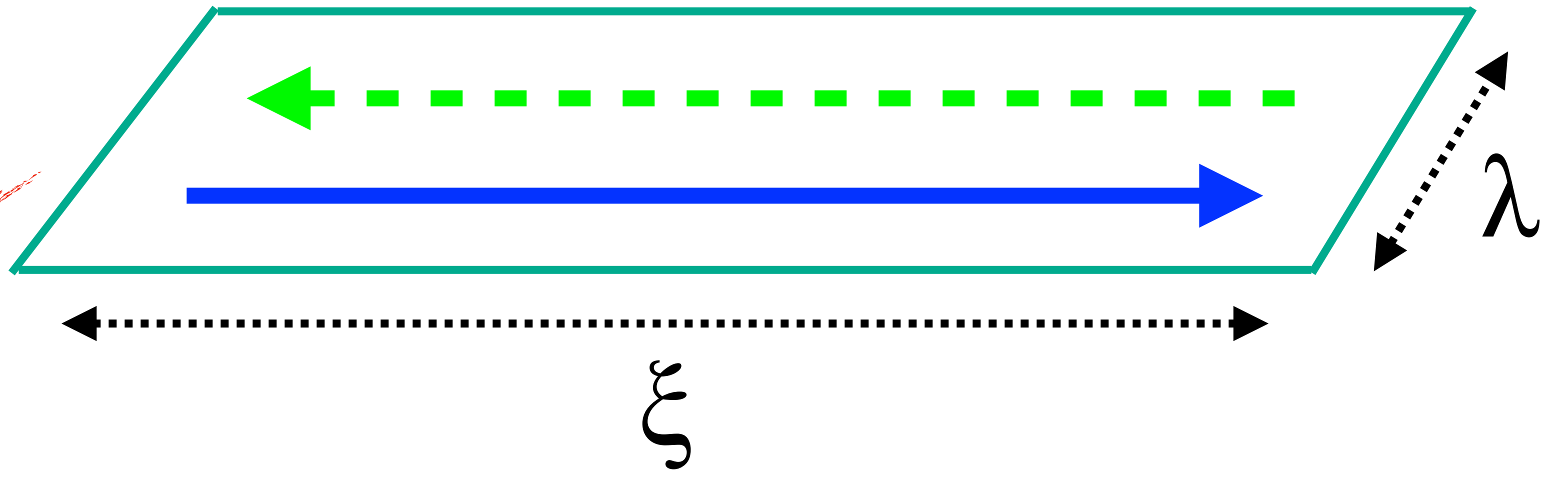


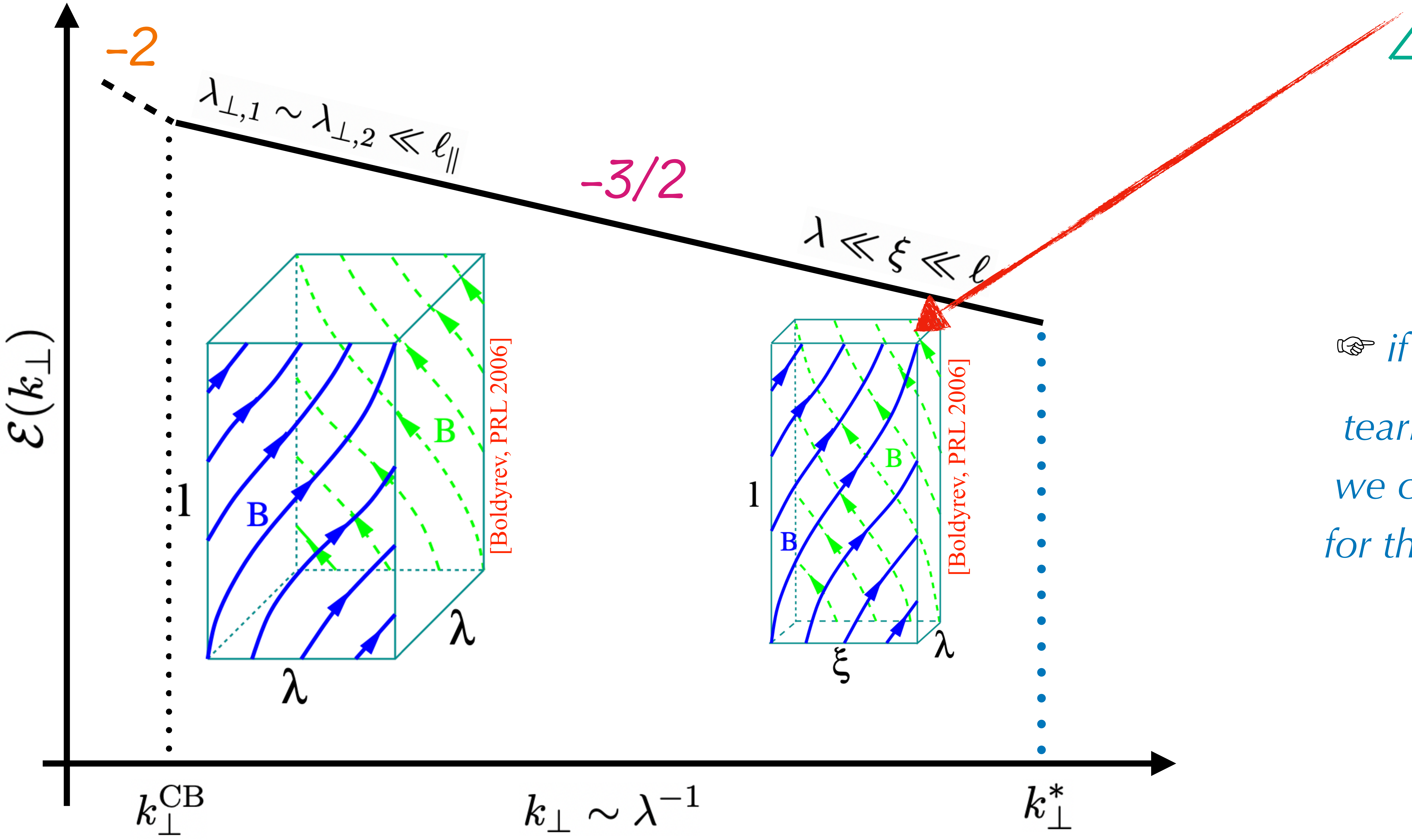
$$\mathcal{E}(k_\perp) \propto k_\perp^{-3/2} \qquad k_\parallel \propto k_\perp^{1/2}$$

spectrum of
dynamically aligned, strong Alfvénic turbulence

Further advances in phenomenology of strong Alfvénic Turbulence

…due to dynamic alignment the turbulent eddies look like
a current sheet in the plane perpendicular to B!





☞ *if the eddies at a scale live "long enough" for the tearing instability (i.e., reconnection) to grow, then we can imagine that this process will be responsible for the production of small-scale magnetic fluctations*

$$\gamma^{\rm rec}\,\tau_{\rm nl} \sim 1 \quad \text{at} \quad \frac{\lambda_*}{\ell_0} \sim S_0^{-4/7}$$

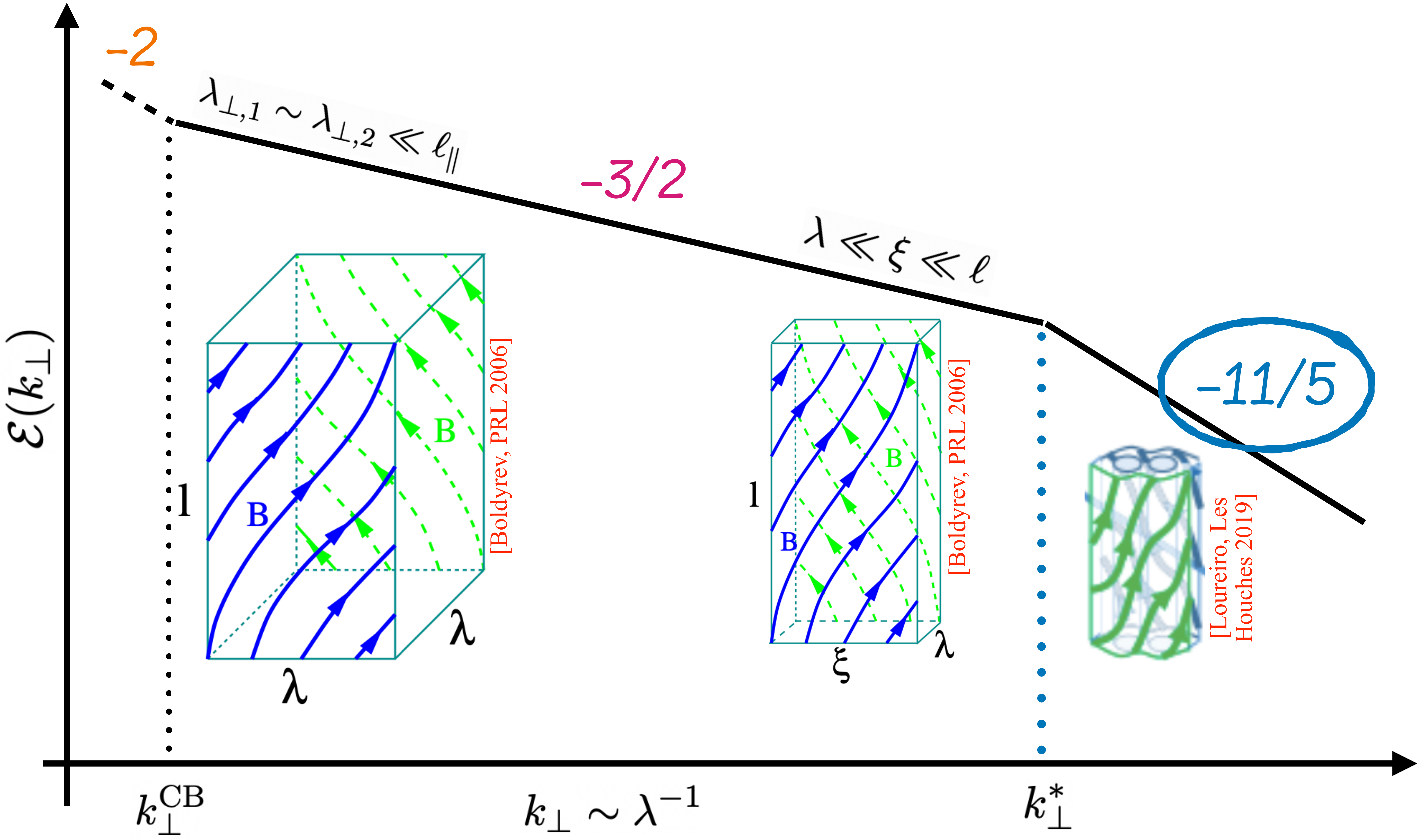
$S_0 = v_A\, l_0 / \eta$ (Lunquist number)

Further advances in phenomenology of strong Alfvénic Turbulence

dynamic alignment → → → → → reconnection-mediated regime

[Boldyrev, PRL 2006]
[Chandran, Schekochihin, Mallet, ApJ 2015]

[Boldyrev & Loureiro, ApJ 2017]
[Mallet, Schekochihin, Chandran, MNRAS 2017]



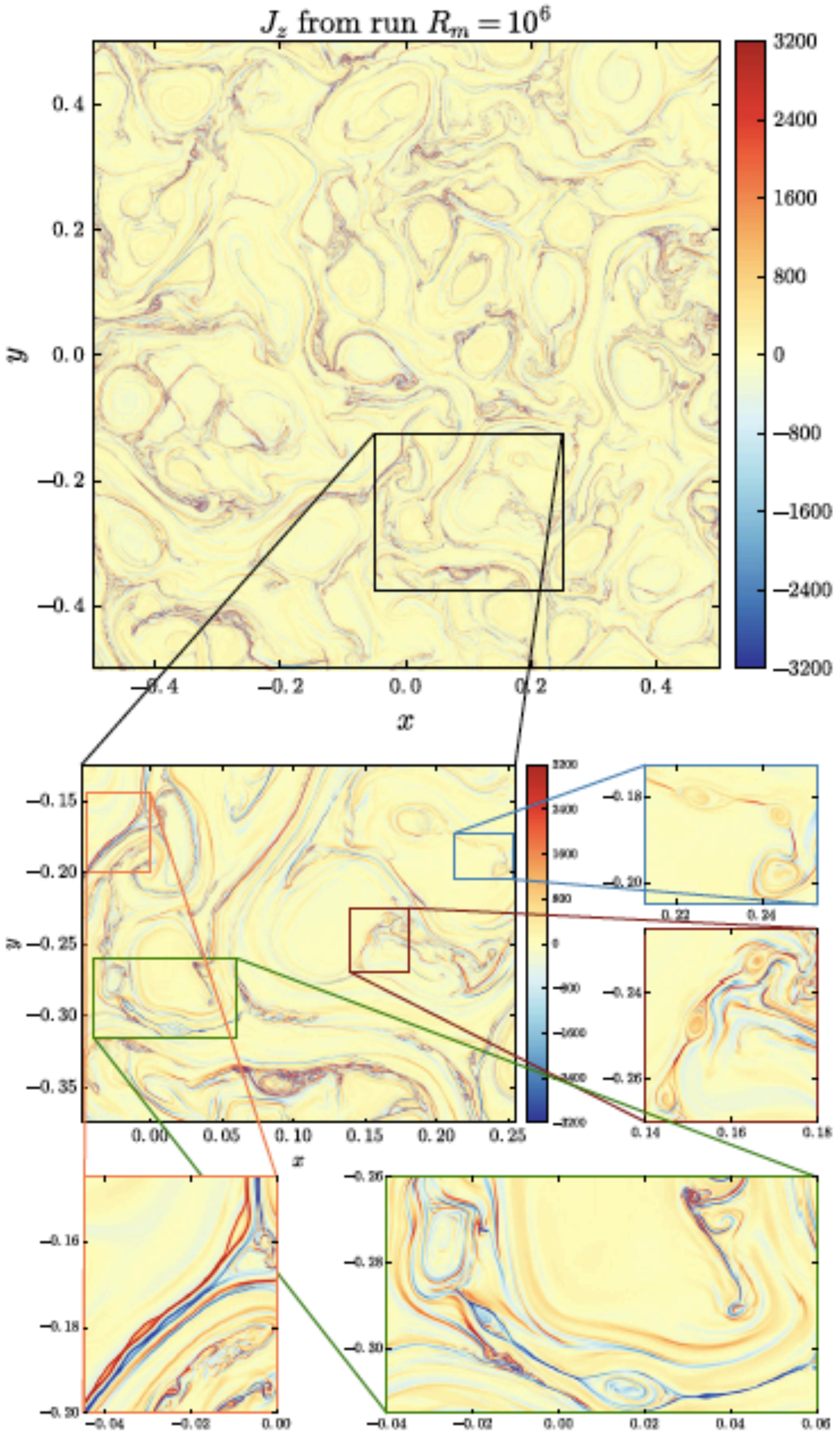
$$\mathcal{E}(k_\perp) \propto k_\perp^{-11/5}$$

$$\theta_{k_\perp} \propto k_\perp^{4/5}$$

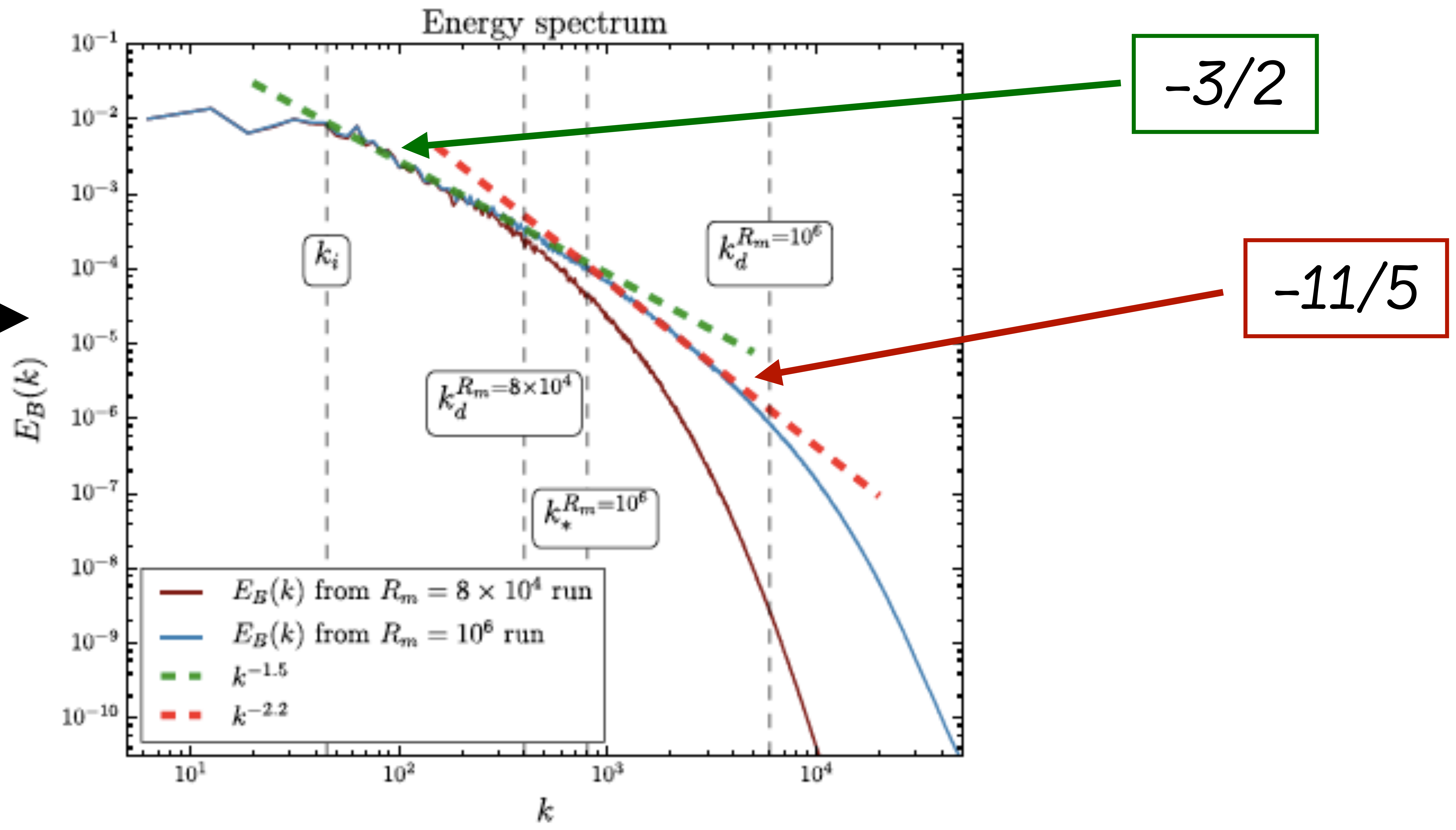
$$k_\| \propto k_\perp^{6/5}$$

spectrum of reconnection-mediated turbulence

Simulations of tearing-mediated turbulence (at MHD scales)



[Dong et al., PRL (2018)]



☞ for some time, this was the only evidence for the realization of a reconnection-mediated regime <u>at MHD scales</u> (i.e., not at kinetic scales)

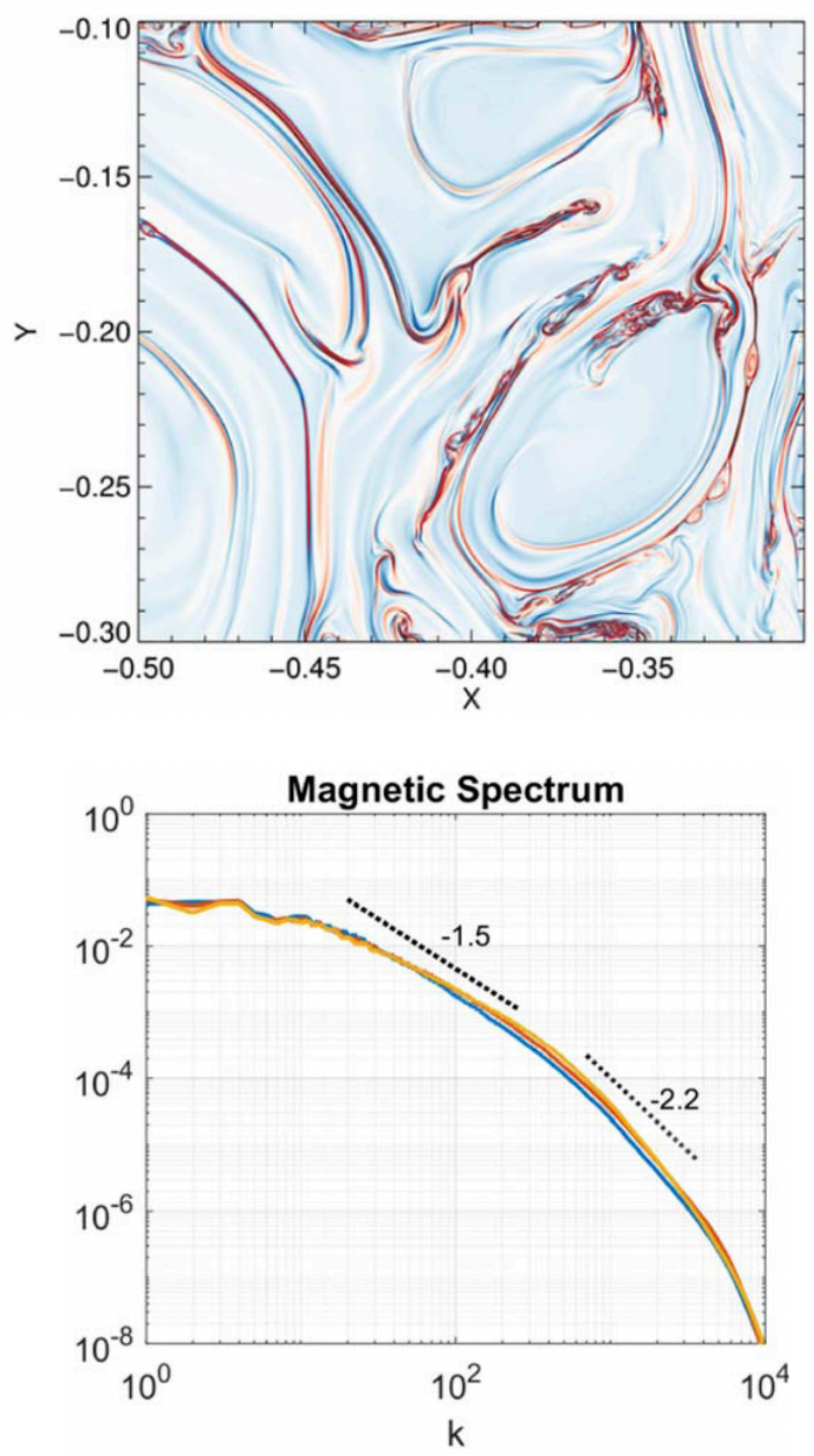
- only in **2D geometry**
- requires ***extremely large numerical grids*** (64000^2 !!!)

Simulations of tearing-mediated turbulence (at MHD scales)

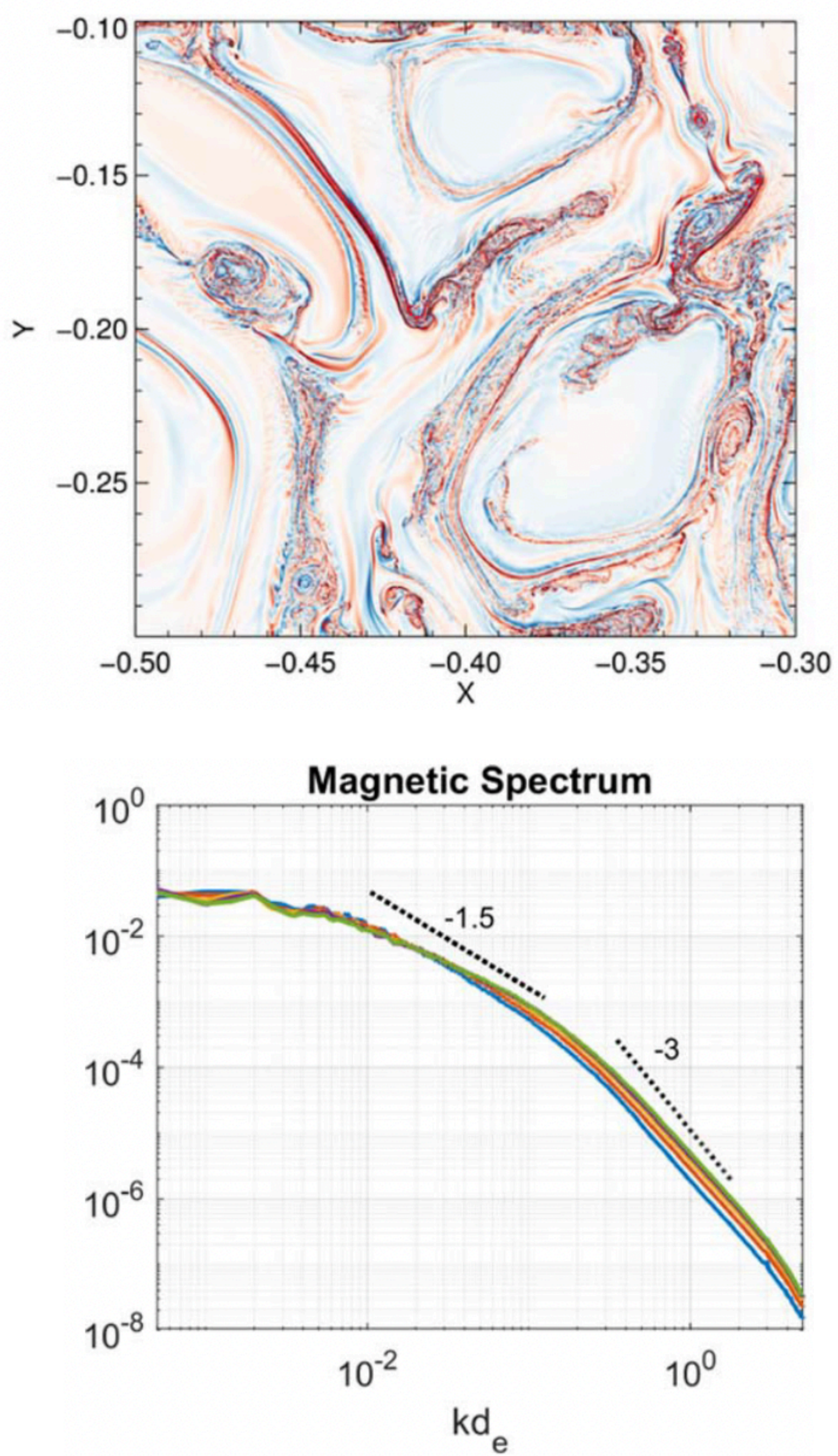
☞ more recent 2D simulations (resistive vs collisionless):

[Borgogno et al., ApJ (2022)]

resitive case



collisionless case



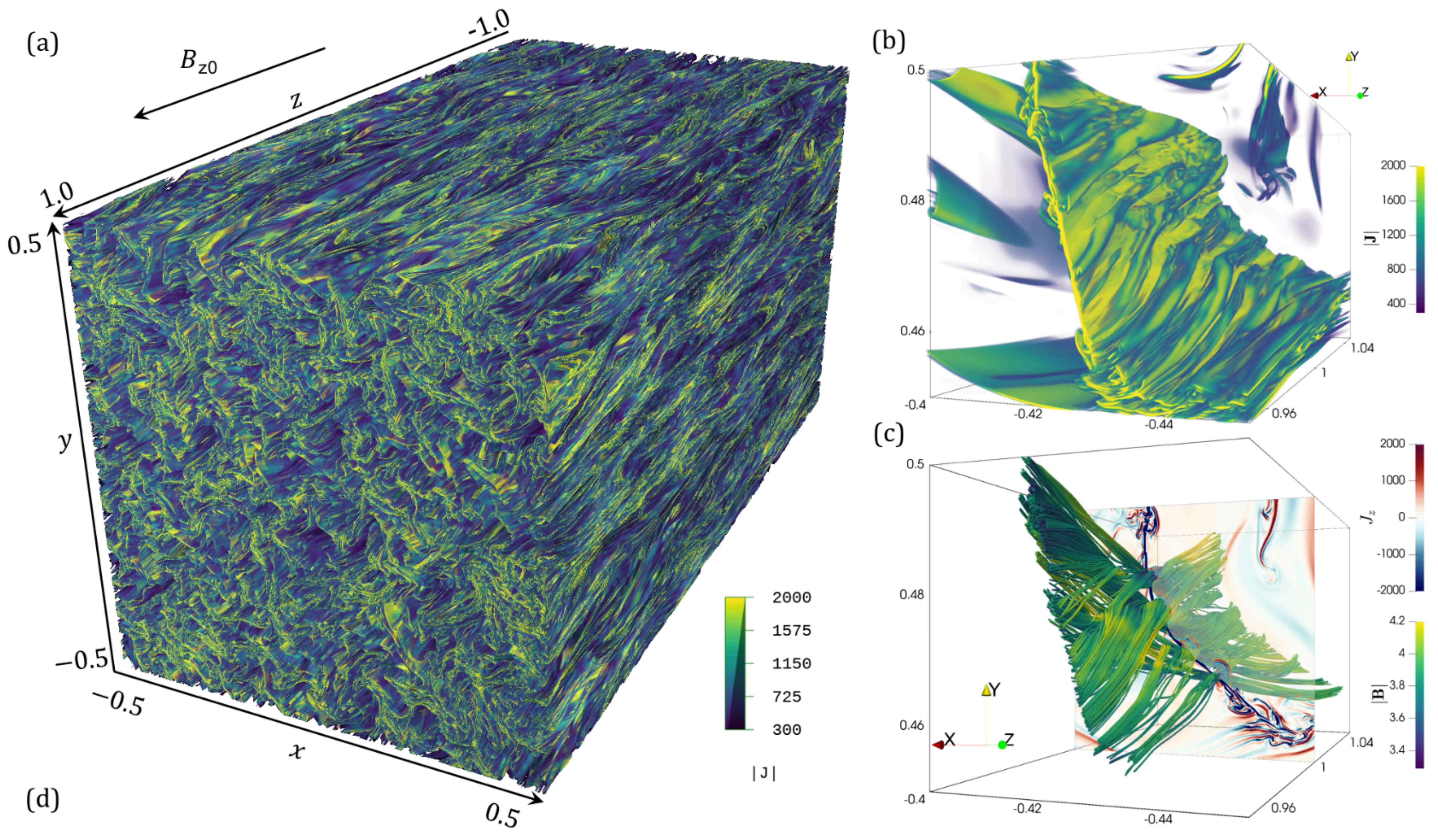
- gyrofluid model
- include electron inertia (for collisionless rec.)
  ☞ "kinetic regime" (will come back to this…)
- reconnection <—> Kelvin-Helmholtz
  ☞ see also Kowal et al., ApJ (2017, 2020)

(talk by Dario Borgogno)

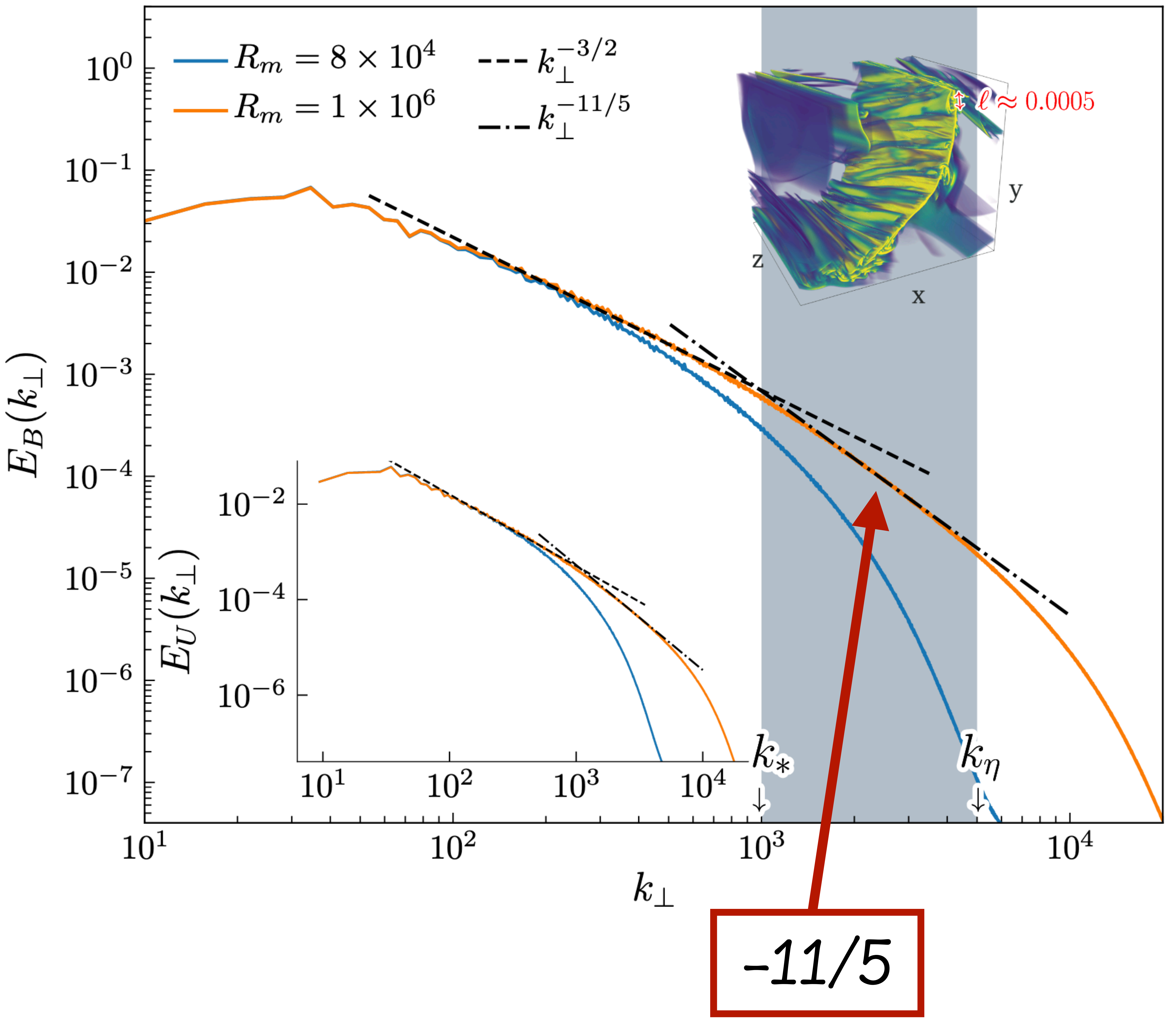
Simulations of tearing-mediated turbulence (at MHD scales)

☞ latest news from 3D simulations:

[Dong et al., Sci. Adv. (2022)]



10.000 x 10.000 x 5.000 (!!!)



⚠️ However, despite its extremely high resolution,
the simulation by Dong et al. still shows only a limited -11/5 range…

Simulations of tearing-mediated turbulence (at MHD scales)

Simulatenously, we adopted **a different approach**, still in 3D:

$$\gamma^{\rm rec}\,\tau_{\rm nl} \sim 1$$

Simulations of tearing-mediated turbulence (at MHD scales)

Simulatenously, we adopted **a different approach**, still in 3D:

$$\gamma^{\mathrm{rec}}\tau_{\mathrm{nl}} \sim 1$$

Usual approach:

increase the separation between the transition scale λ_* and the actual dissipation scale λ_{diss}

ONLY by achieving very large S: requires extreme resolution!

Simulations of tearing-mediated turbulence (at MHD scales)

Simulatenously, we adopted **a different approach**, still in 3D:

$$\gamma^{\rm rec}\tau_{\rm nl} \sim 1$$

Our approach:

increase ALSO the lifetime of turbulent eddies, so that tearing becomes relevant at even larger scales!

(and this is done by considering a smaller non-linear parameter, $\chi < 1$)

Our Approach

- Look at the problem of tearing-mediated turbulence from a fundamental standpoint…

  ☞ *interaction of counter-propagating Alfvén-wave (AW) packets in 3D*

- Enable tearing to "easily" grow on top of (3D-anisotropic) turbulent eddies…

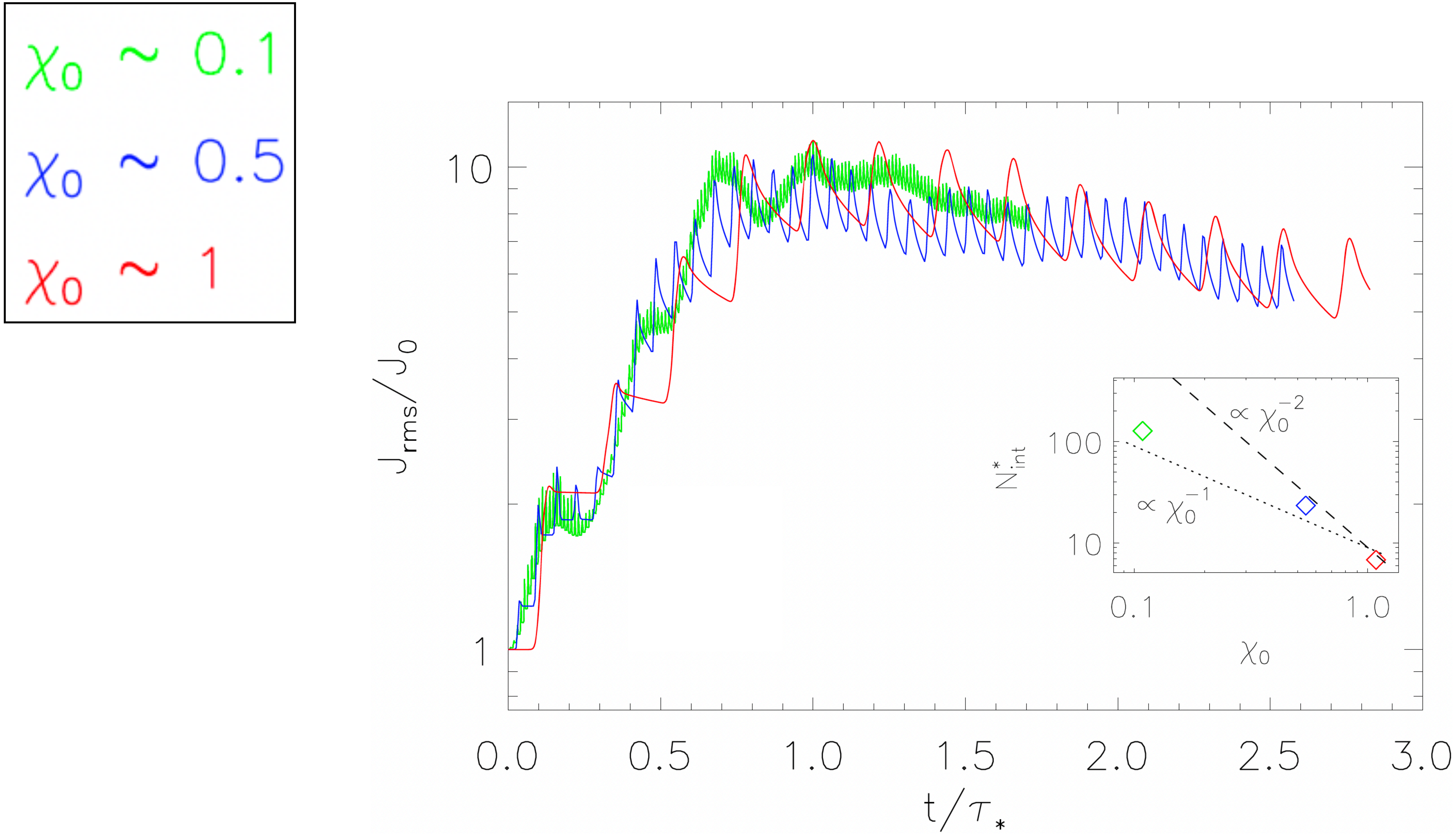
  ☞ *increase the eddy lifetime time by decreasing the strength of nonlinearities*

- Study a purely Alfvénic cascade, without interaction with other MHD modes…

  ☞ *2-fields gyro-fluid model (~ Reduced-MHD) in order to keep only Alfvénic dynamics*

3D simulations of colliding AW packets

[Cerri et al. ApJ 2022]

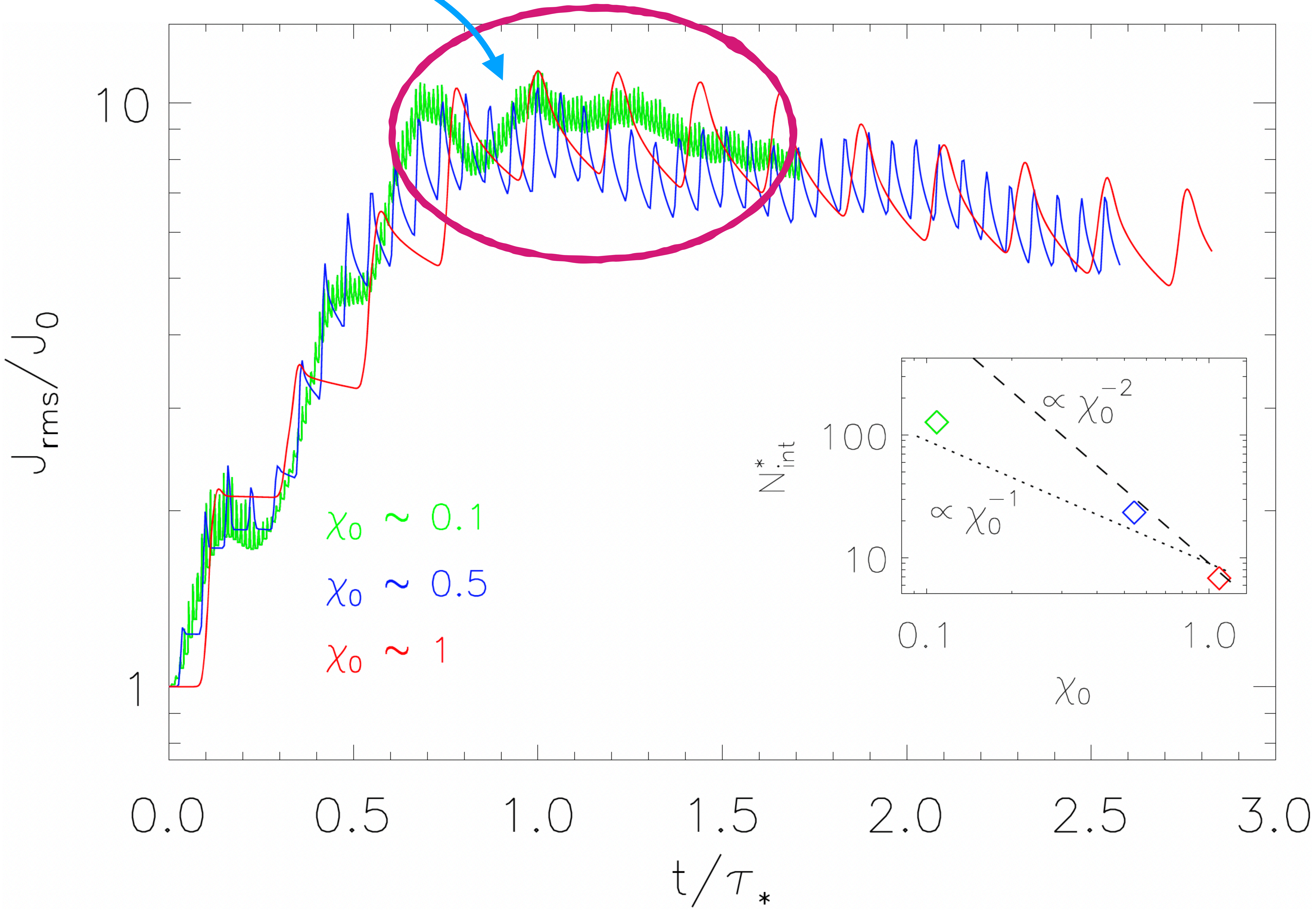


3D simulations of colliding AW packets

[Cerri et al. ApJ 2022]

turbulence peak activity

time-averaged properties (unless specified...)



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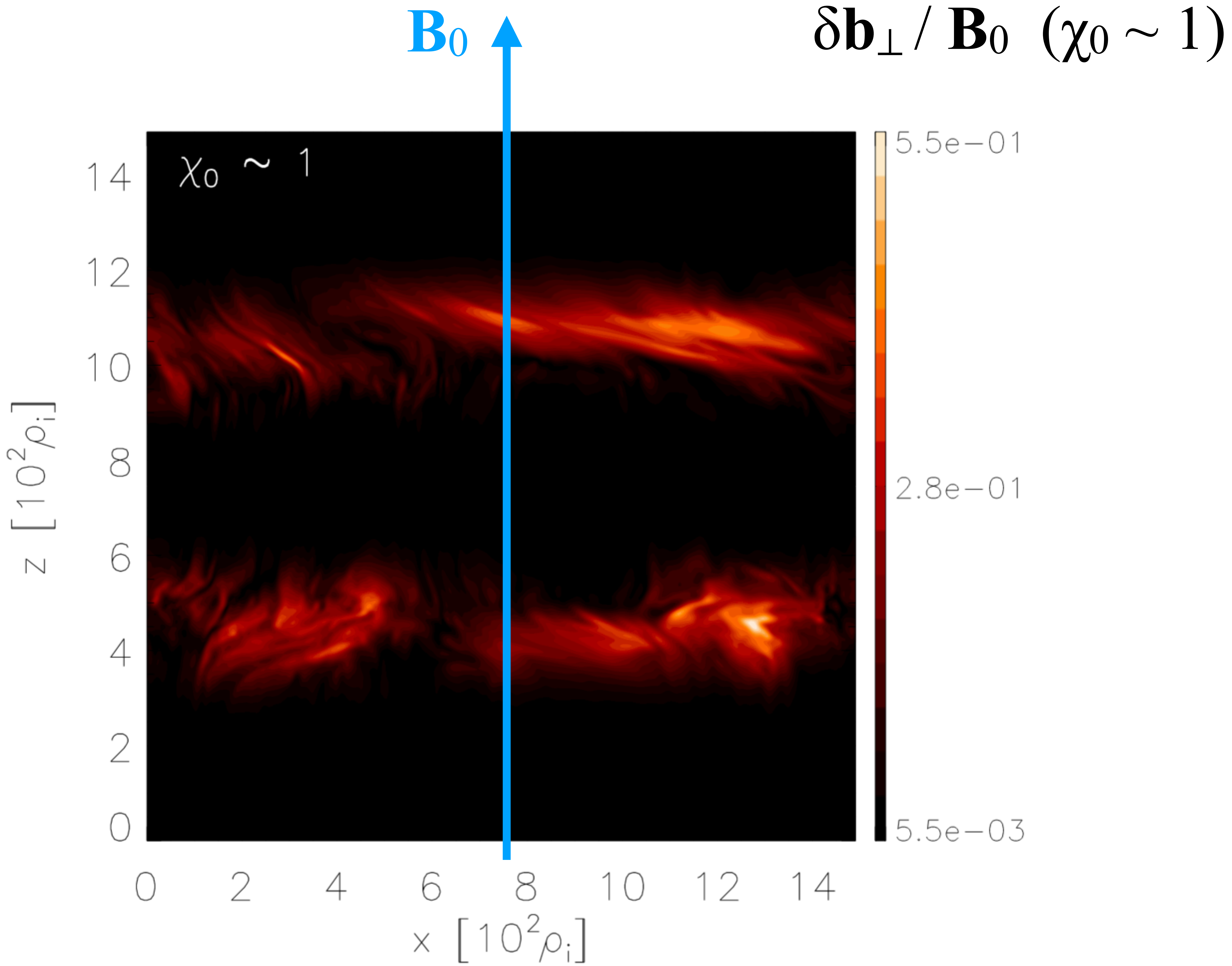

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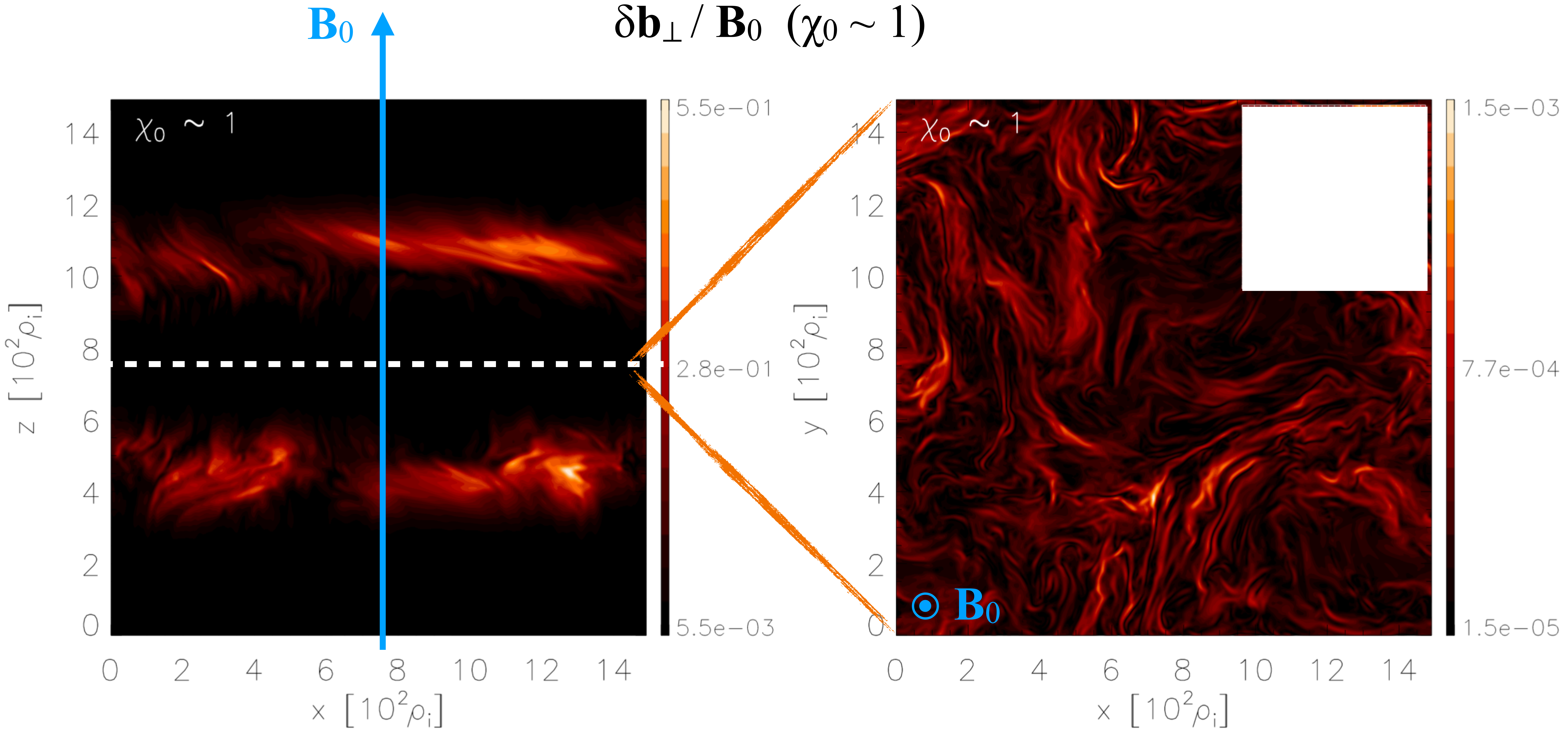

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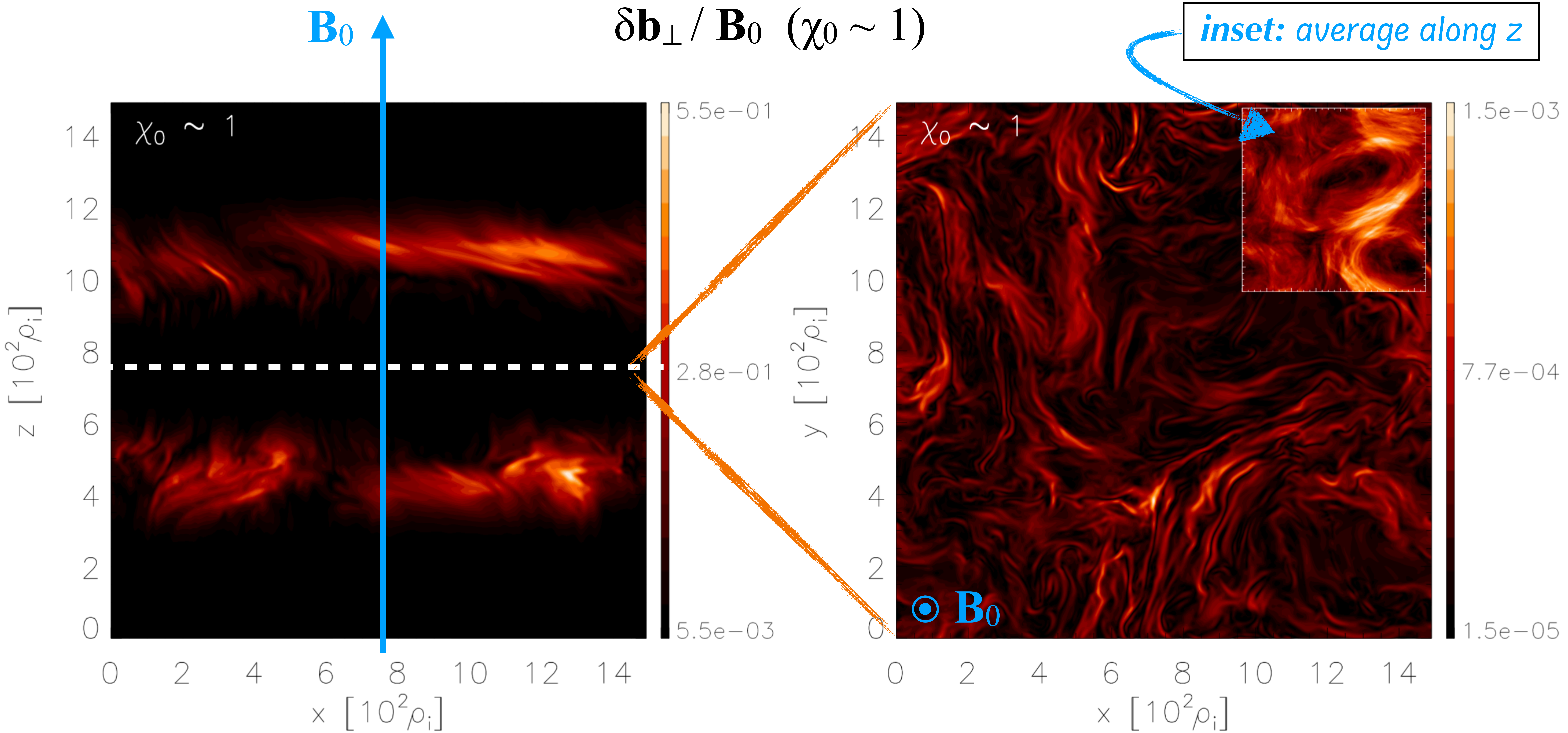
$\delta \mathbf{b}_\perp / \mathbf{B}_0 \ (\chi_0 \sim 1)$



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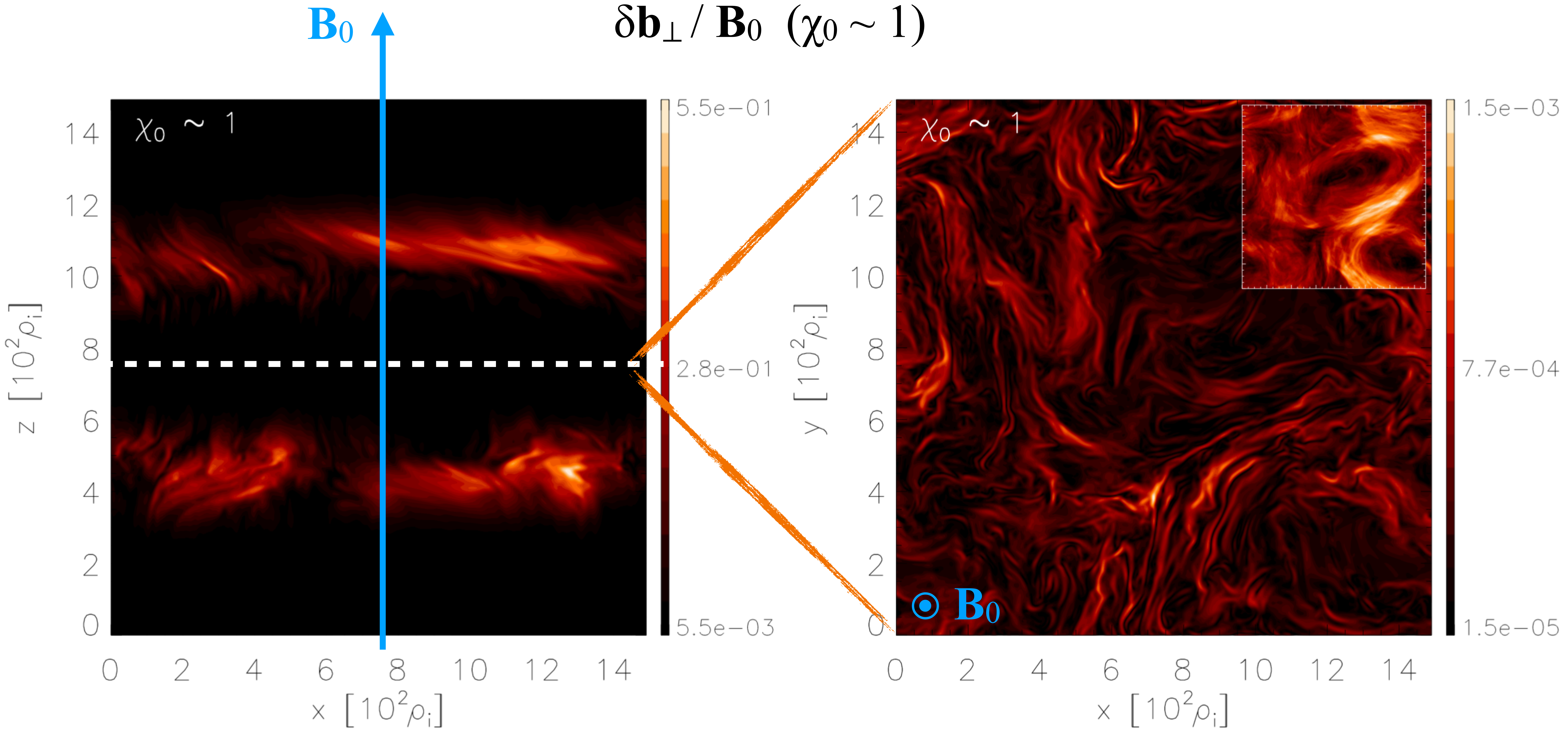
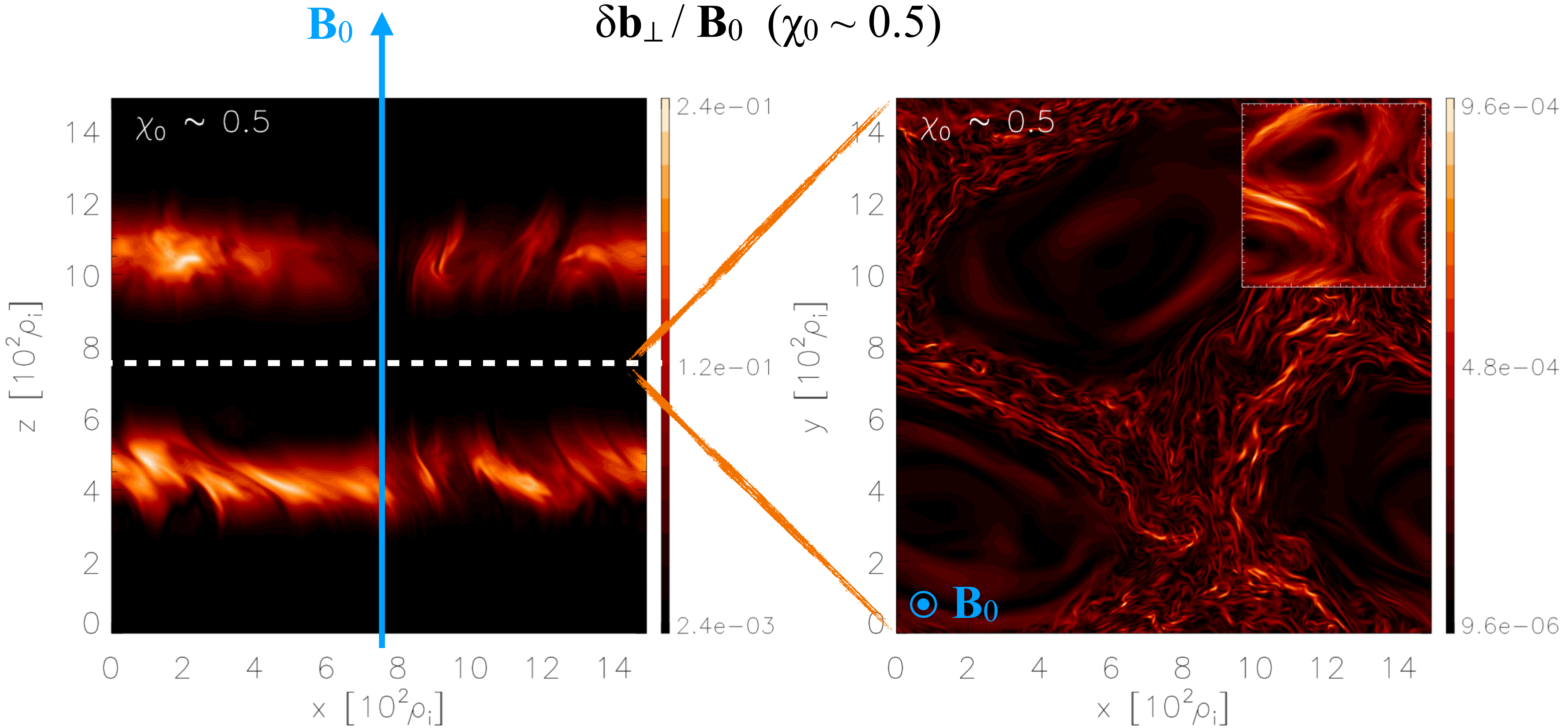


3D simulations of colliding AW packets

[Cerri et al. ApJ 2022]

$\delta \mathbf{b}_\perp / \mathbf{B}_0 \ (\chi_0 \sim 0.5)$



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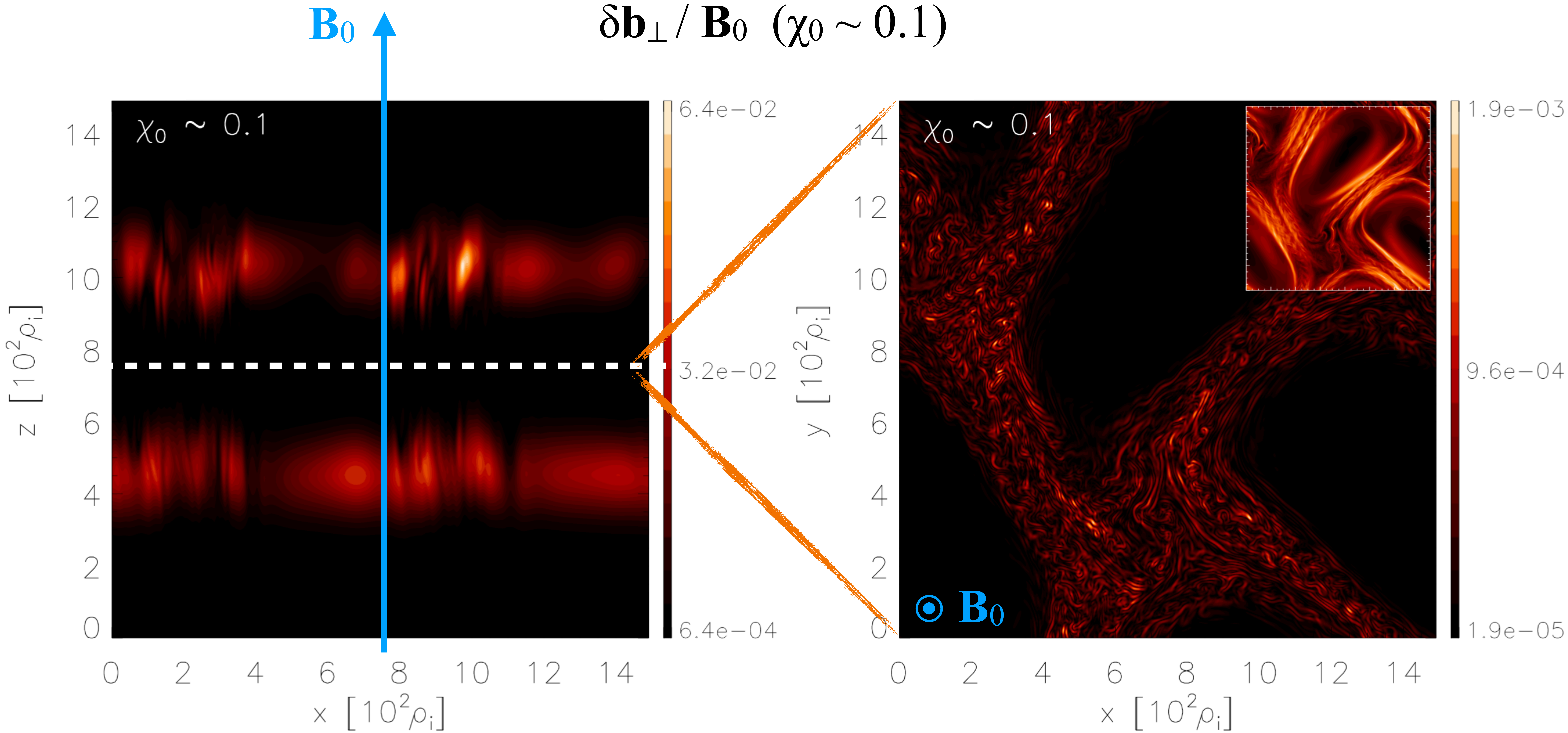

3D simulations of colliding AW packets

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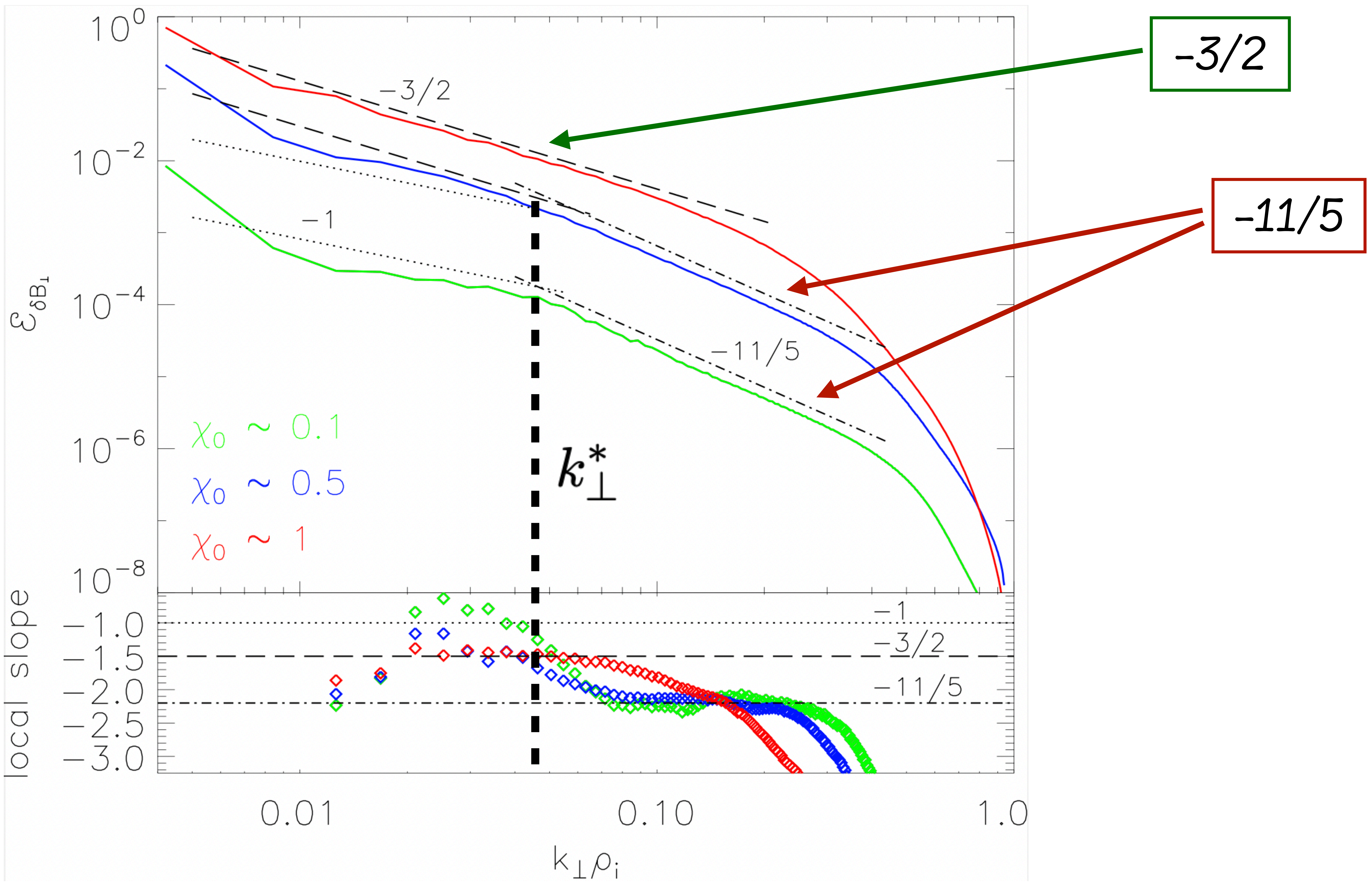

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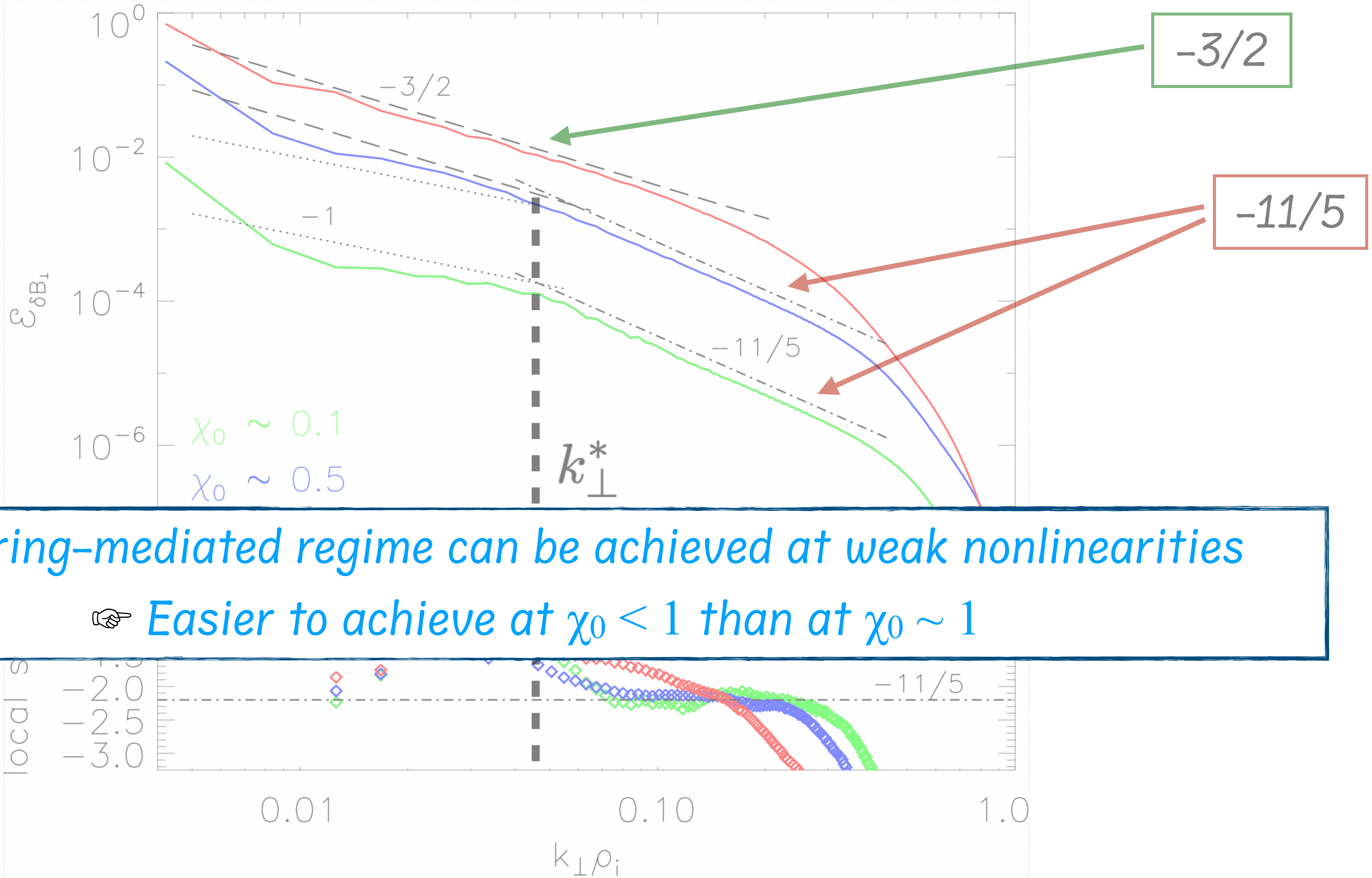


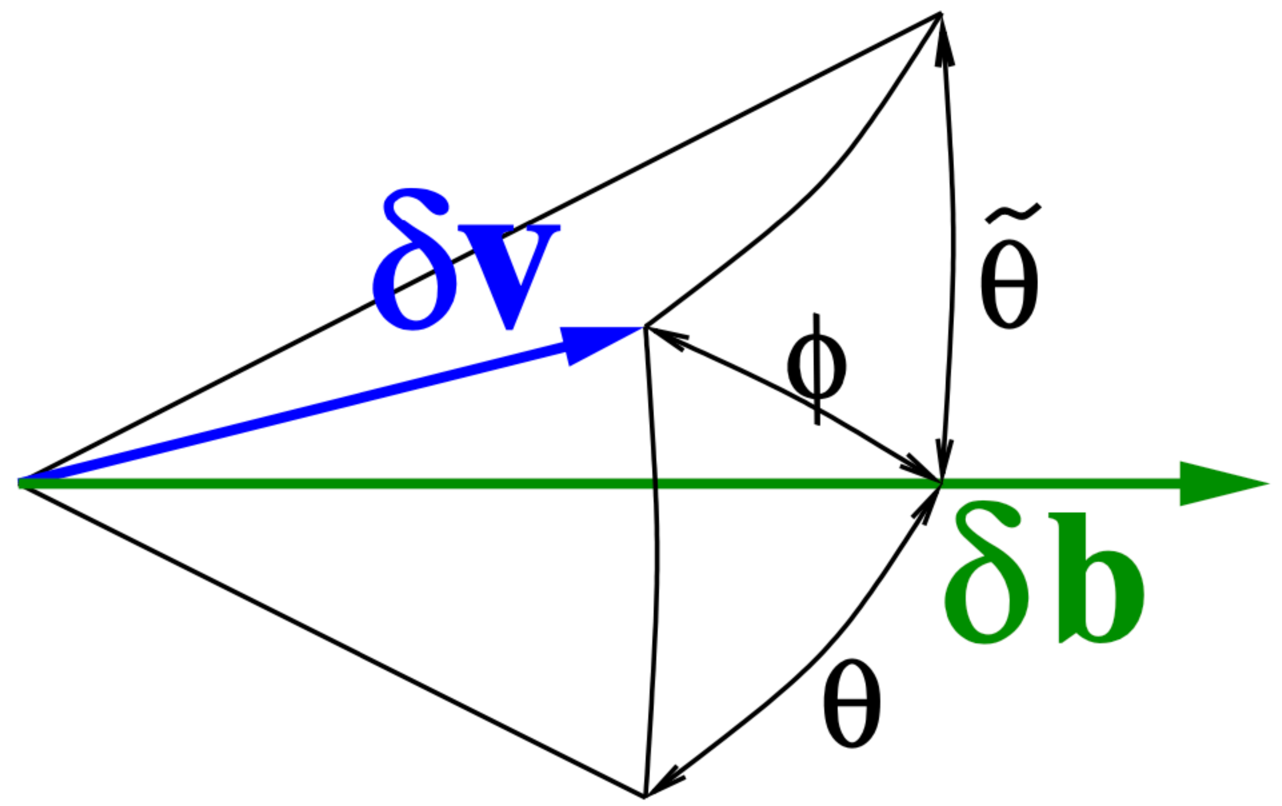
☞ *A tearing-mediated regime can be achieved at weak nonlinearities*

☞ *Easier to achieve at $\chi_0 < 1$ than at $\chi_0 \sim 1$*

3D simulations of colliding AW packets

[Cerri et al. ApJ 2022]

- Is this ***tearing-mediated turbulence***? if yes, it ***requires dynamic alignment!***



$$\sin\theta_{k_\perp} = \frac{\left\langle \sum_{k\leqslant k_\perp<k+1} |\delta \boldsymbol{u}_{\perp,\lambda} \times \delta \boldsymbol{b}_{\perp,\lambda}| \right\rangle}{\left\langle \sum_{k\leqslant k_\perp<k+1} |\delta \boldsymbol{u}_{\perp,\lambda}||\delta \boldsymbol{b}_{\perp,\lambda}| \right\rangle}$$

3D simulations of colliding AW packets

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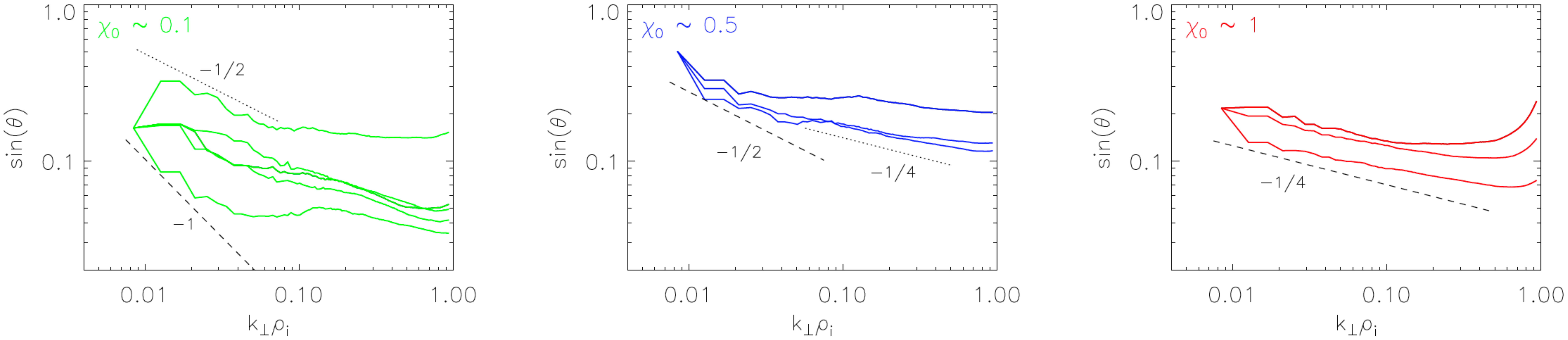
[Cerri et al. ApJ 2022]

during AW collisions
(scale-dependent alignment induced by AW shearing)



3D simulations of colliding AW packets

$$\sin\theta_{k_\perp} = \frac{\left\langle \sum_{k\leqslant k_\perp<k+1} |\delta \boldsymbol{u}_{\perp,\lambda} \times \delta \boldsymbol{b}_{\perp,\lambda}| \right\rangle}{\left\langle \sum_{k\leqslant k_\perp<k+1} |\delta \boldsymbol{u}_{\perp,\lambda}||\delta \boldsymbol{b}_{\perp,\lambda}| \right\rangle}$$

[Cerri et al. ApJ 2022]

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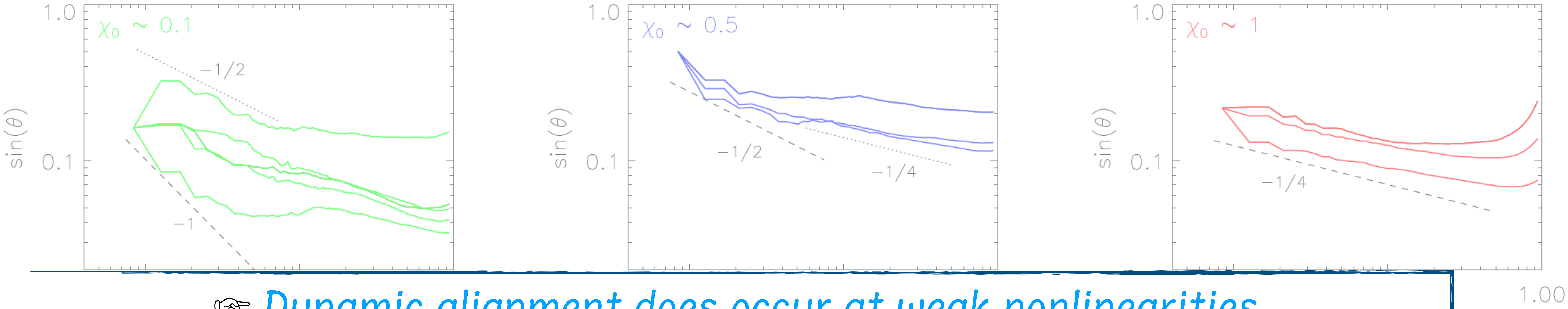

☞ *Dynamic alignment does occur at weak nonlinearities*

☞ *Scale-dependent alignment is stronger at $\chi_0 < 1$ than at $\chi_0 \sim 1$*

3D simulations of colliding AW packets

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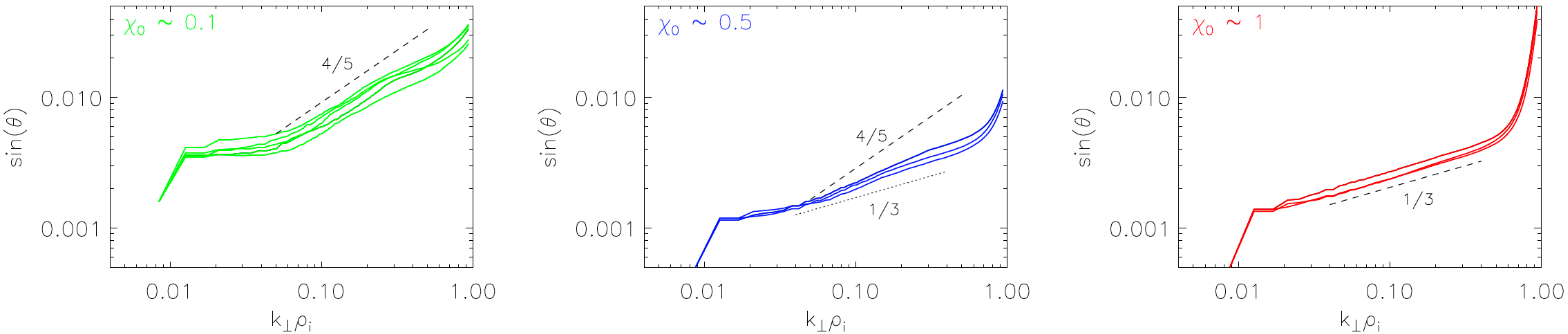
[Cerri et al. ApJ 2022]

while AW are far apart

(scale-dependent mis-alignment induced by reconnection)



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[Cerri et al. ApJ 2022]

☞ *The scale-dependent alignment angle exhitbis a “patchy” behaviour (in both space and time!)*

☞ *Should be taken into account when performing an ensemble average (in both simulations and spacecraft data!)*

Weak Alfvénic turbulence with dynamic alignment

[Cerri et al. ApJ 2022]

New phenomenological scalings

☞ Moderately weak regime ($\chi < 1$)

- $k_{||} = const.$
- $sin(\theta_k) \propto k_\perp^{-1/2}$
- $E_B(k_\perp) \propto k_\perp^{-3/2}$

- *transition to tearing-mediated turbulence competes with the usual transition to critical balance*

☞ Asymptotically weak regime ($\chi << 1$)

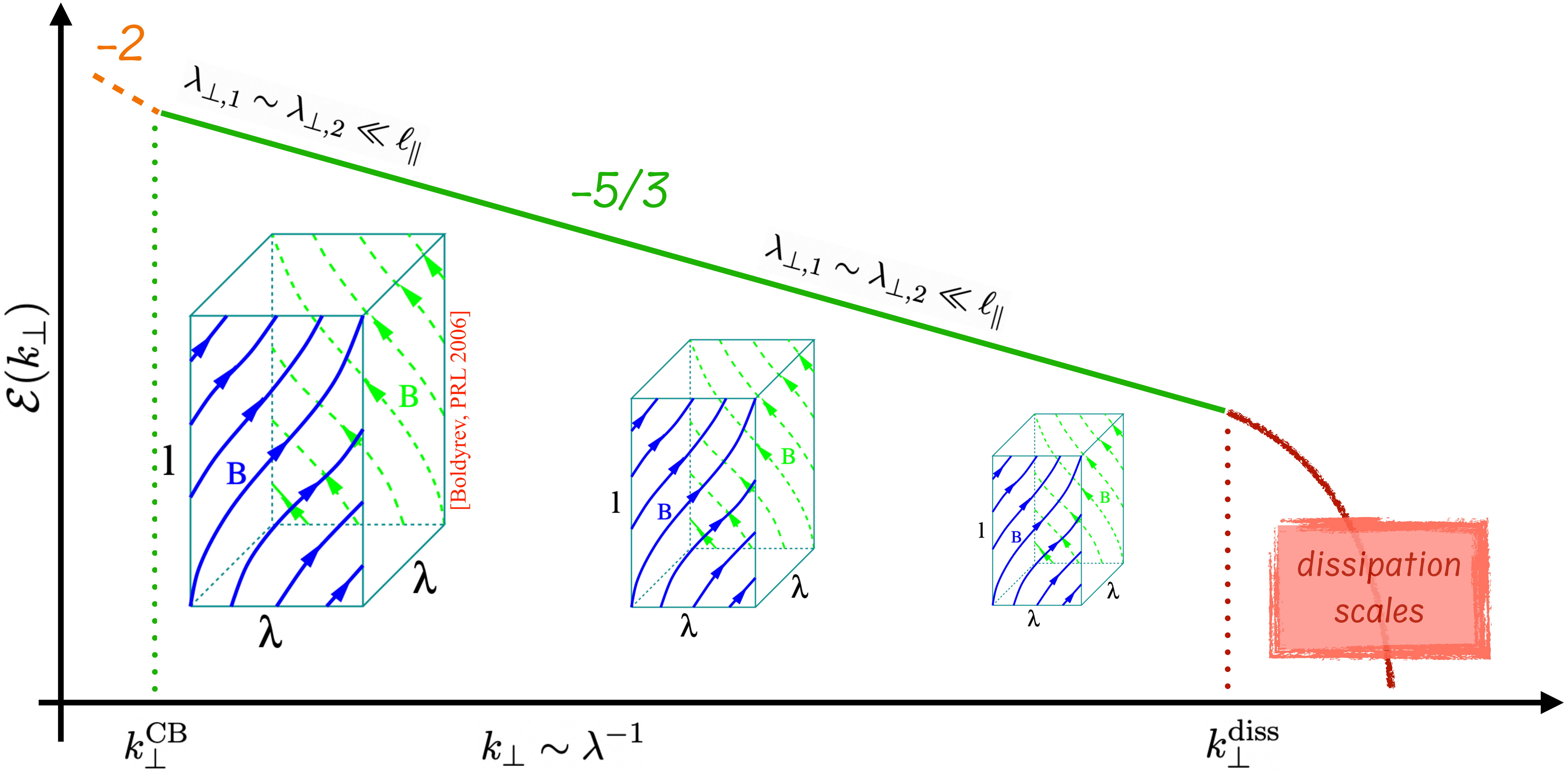
- $k_{||} = const.$
- $sin(\theta_k) \propto k_\perp^{-1}$
- $E_B(k_\perp) \propto k_\perp^{-1}$

- *no usual transition to critical balance possible, only transition to tearing-mediated turbulence*

The many faces of the Alfvénic cascade

(weak cascade) → CB cascade without dynamic alignment → actual dissipation range

[Goldreich & Sridhar, ApJ (1995)]



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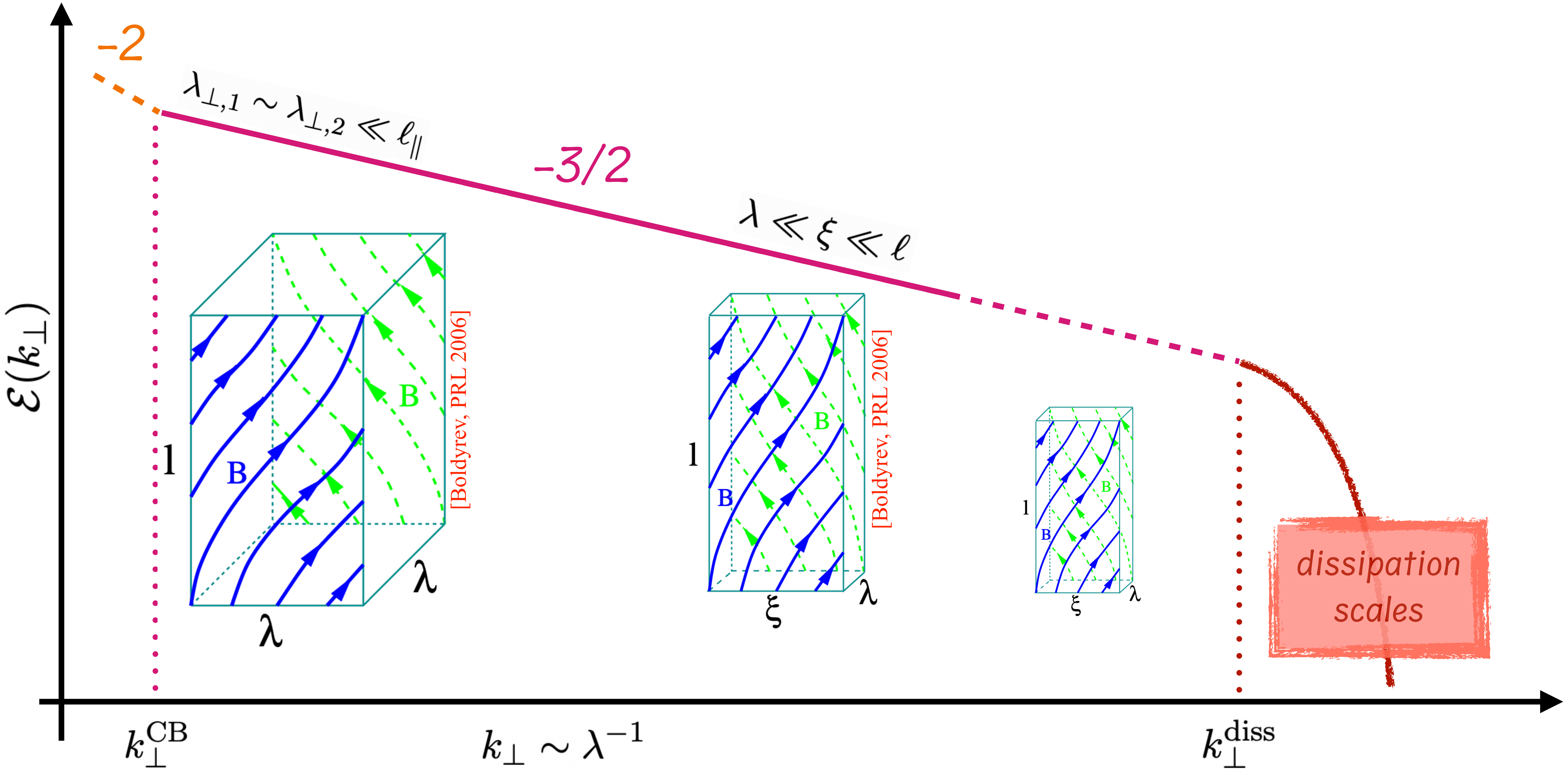


The many faces of the Alfvénic cascade

(weak cascade)

CB cascade without dynamic alignment

[Goldreich & Sridhar, ApJ (1995)]

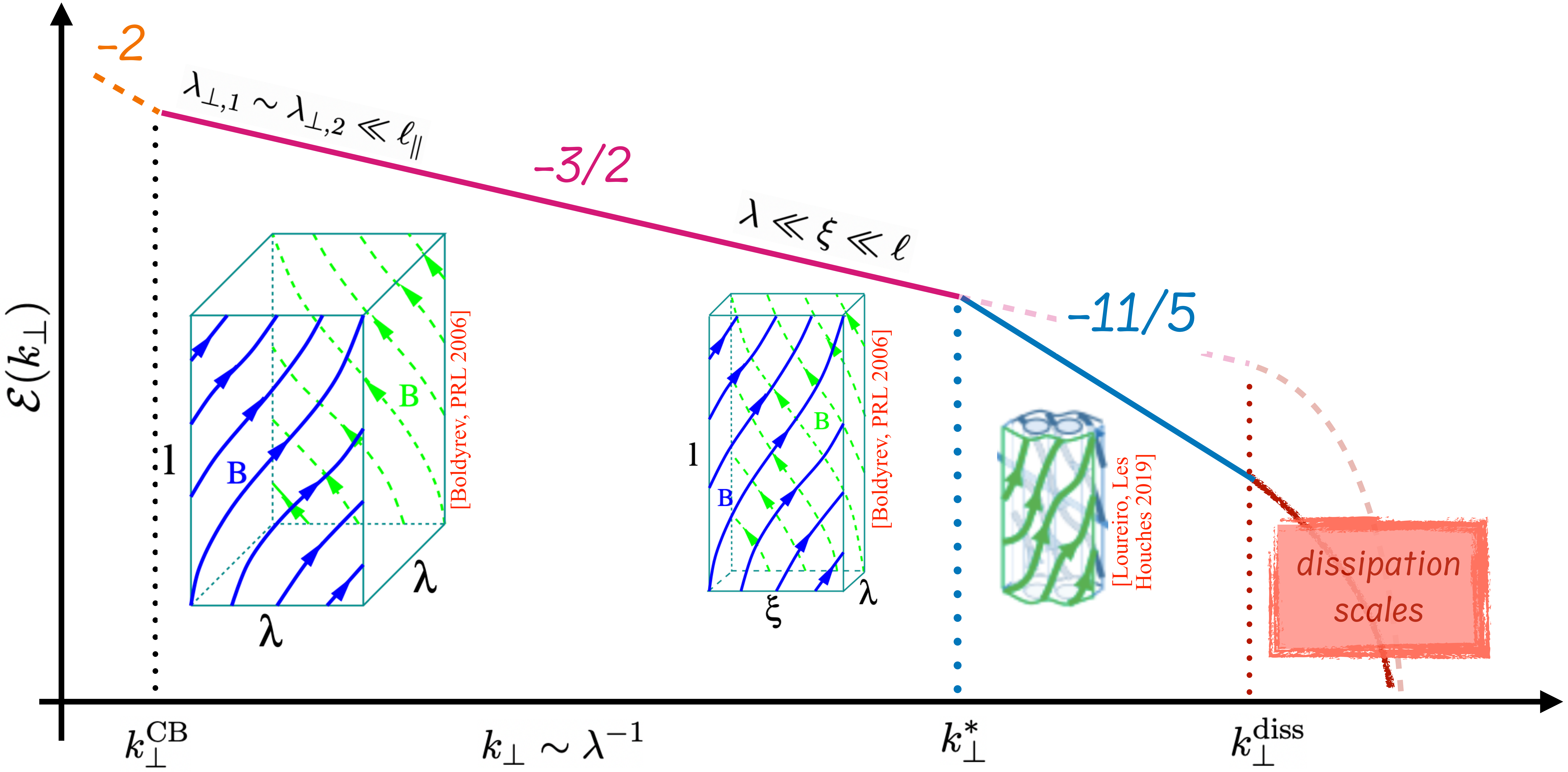
CB cascade with dynamic alignment

[Boldyrev, PRL (2006)]

actual dissipation range

reconnection-mediated regime

[Boldyrev & Loureiro, ApJ (2017)]



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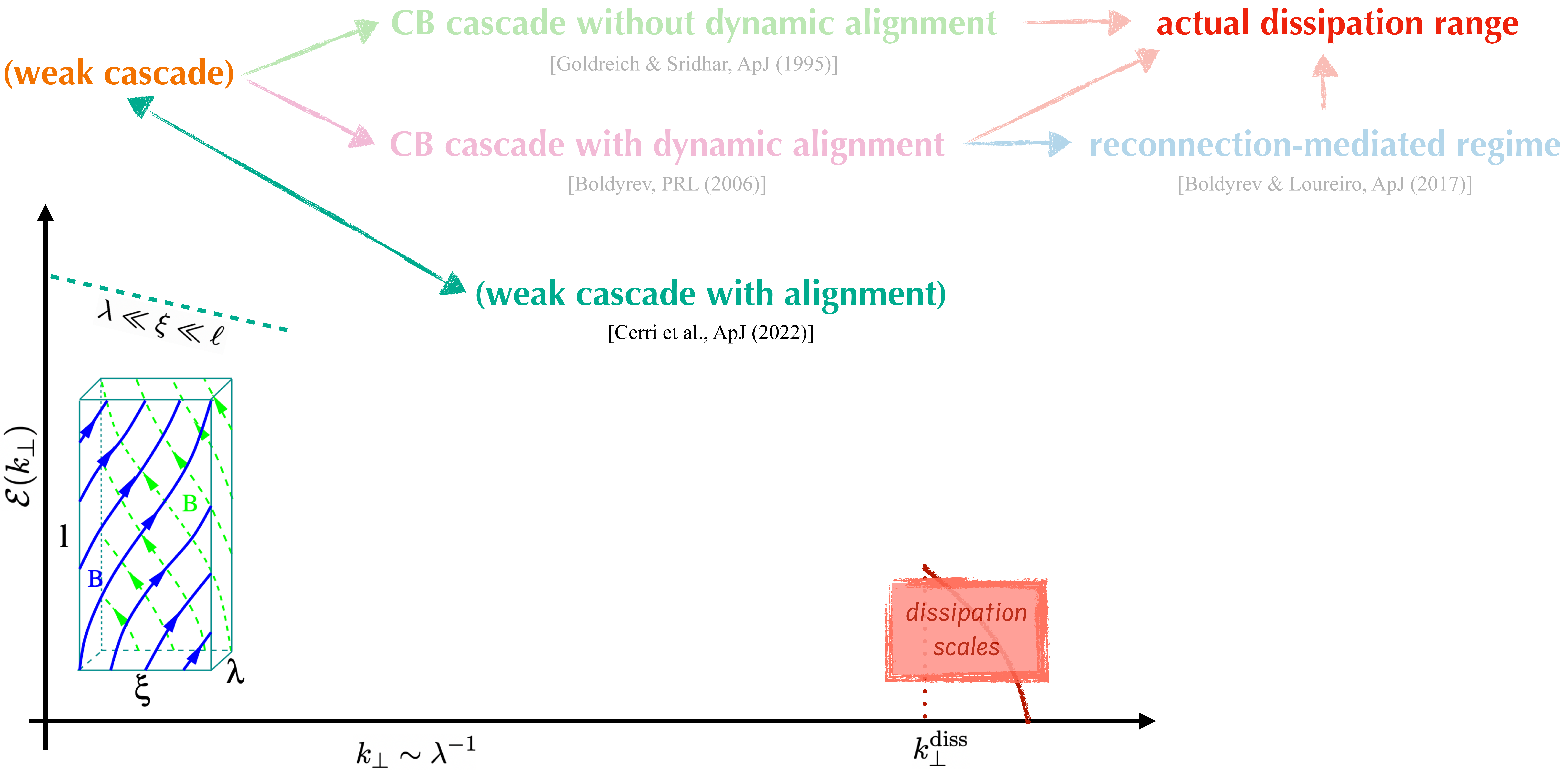

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The many faces of the Alfvénic cascade

The many faces of the Alfvénic cascade

(weak cascade) → CB cascade without dynamic alignment → actual dissipation range

[Goldreich & Sridhar, ApJ (1995)]

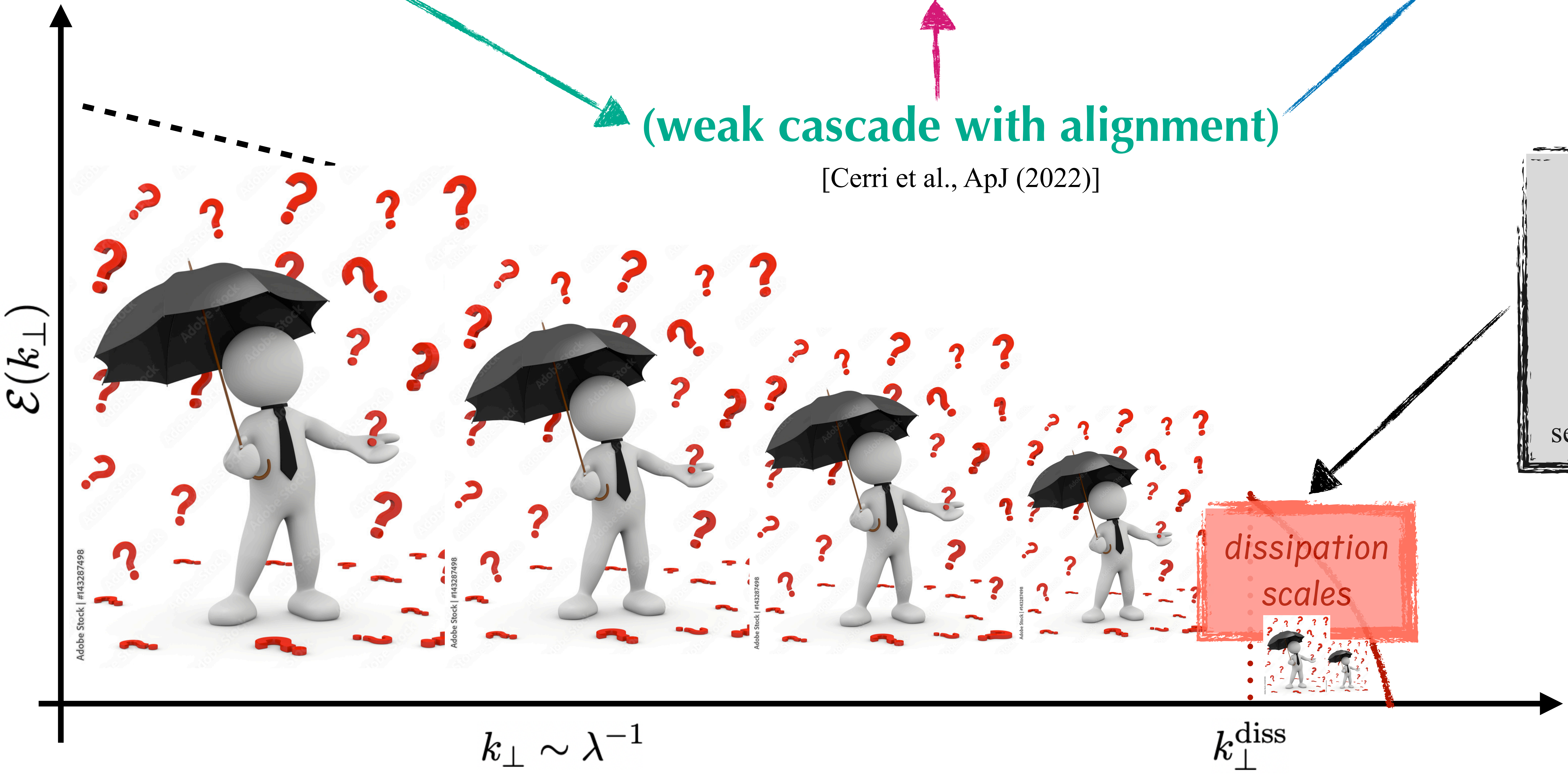
(weak cascade) → CB cascade with dynamic alignment → reconnection-mediated regime → actual dissipation range

[Boldyrev, PRL (2006)]

[Boldyrev & Loureiro, ApJ (2017)]

(weak cascade) ↔ (weak cascade with alignment) → CB cascade with dynamic alignment; (weak cascade with alignment) → reconnection-mediated regime

[Cerri et al., ApJ (2022)]



⚠️ *in "COLLISIONLESS" plasmas: transition to KINETIC-RANGE TURBULENCE*

see **Cerri, Groselj & Franci, FrASpS (2019)**

+

2D simulations of reconnection-mediated kinetic range:

☞ **Cerri & Califano, NJP (2017)**

☞ **Franci et al., ApJL (2017)**

Take-home message(s)

☞ ***New phenomenological scalings of weak turbulence with dynamic alignment***

- ➢ new transition scales depend on (χ , M_A , S) at injection scales
- ➢ at $\chi < 1$ the transition to tearing-mediated regime occurs a scales that can be larger than those predicted for a critically balanced cascade by several orders of magnitude
- ➢ transition to tearing-mediated regime may even supplant the usual weak-to-strong transition

☞ ***Dynamic alignment or mis-alignment states as "patchy" features in space and time***

- ➢ AW shear-induced dynamic alignment + tearing-induced dynamic mis-alignment

☞ ***A decade-long range of tearing-mediated regime in 3D Alfvénic turbulence***

- ➢ from RMHD simulations wtih a "first-principle" setup (AW-packets collisions)

➥ **FOR MORE DETAILS:** Cerri et al. ApJ 2022

Open issues that we could collaborate on

☞ ***Tearing-mediated vs Kelvin-Helmholtz-mediated turbulence*** *("fluid" approach possible)*

- ➢ parameter dependence (scale separation, species' beta, resistive vs collisionless, ...)
- ➢ role of ion/electron finite-Larmor-radius (FLR) effects
- ➢ role of species' temperature anisotropy
- ➢ see *Passot et al., arXiv:2401.03863*

☞ ***Reconnection & heating in sub-ion-scale turbulence*** *("kinetic" treatment necessary!)*

- ➢ role of ion-coupled vs electron-only reconnection
- ➢ role of different heating mechanisms (Landau damping, stochastic heating, ion-cyclotron, ...)
- ➢ see *talk by Camille!* *(tomorrow)*

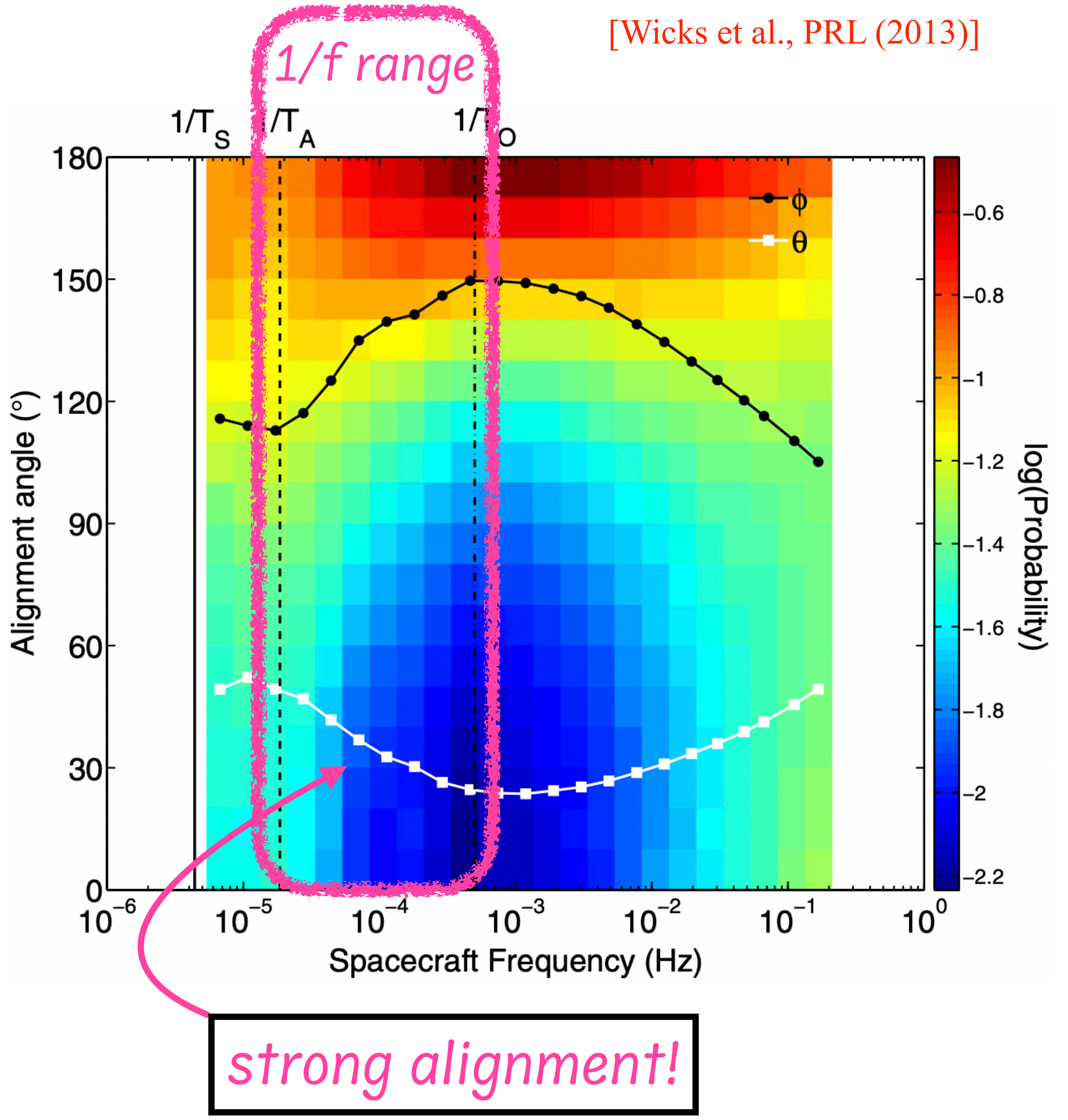
☞ ***The elephant in the room: balanced vs imbalanced turbulence...***

Thank you for your attention!

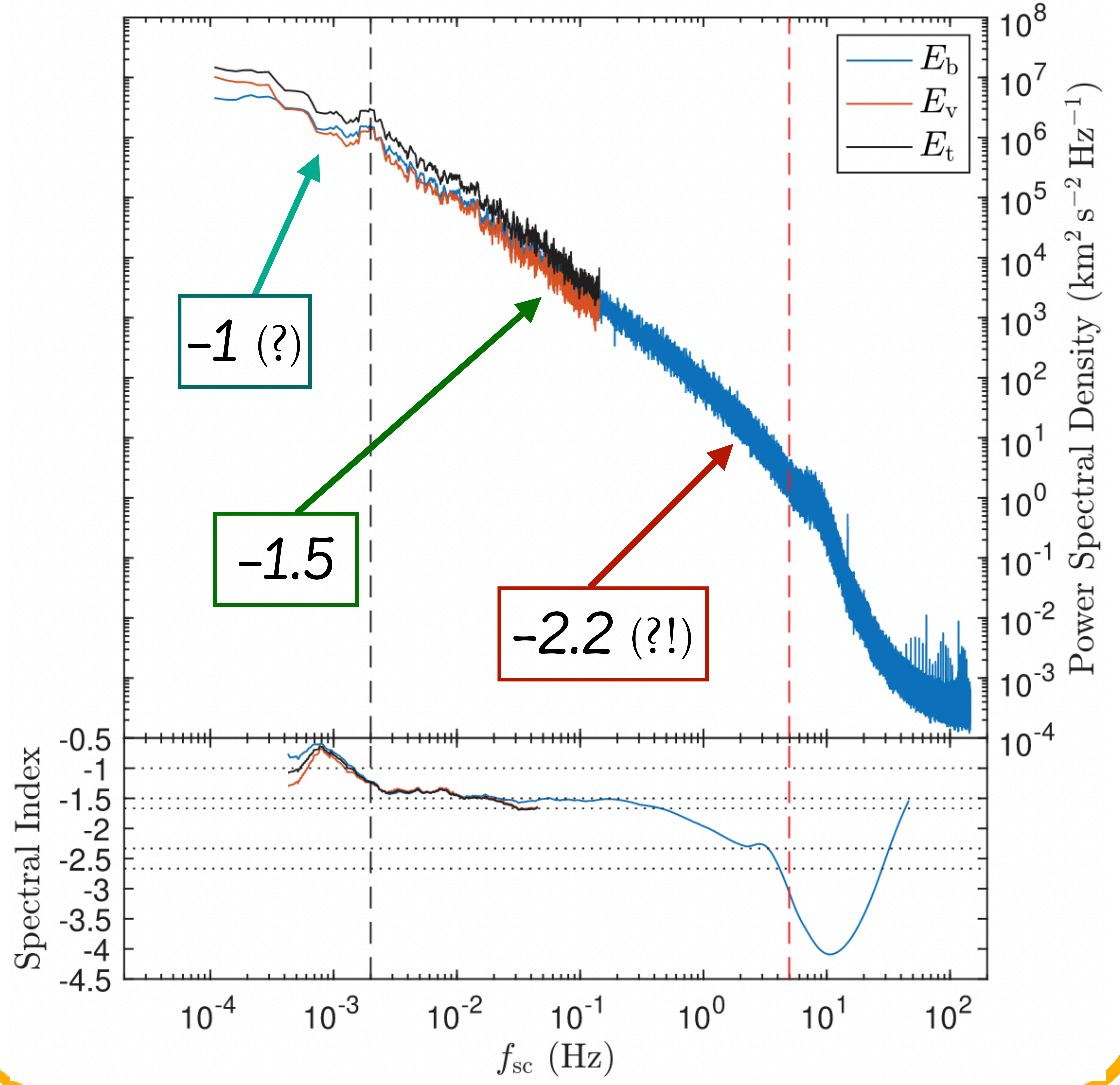
Backup Slides

Could solar-wind observations support these ideas?

[Wicks et al., PRL (2013)]



[Kasper et al., PRL (2021)]



Could solar-wind observations support these ideas?

☞ more recent analysis on PSP data (5-pts structure functions): [Sioulas et al., arXiv:2404.04055]

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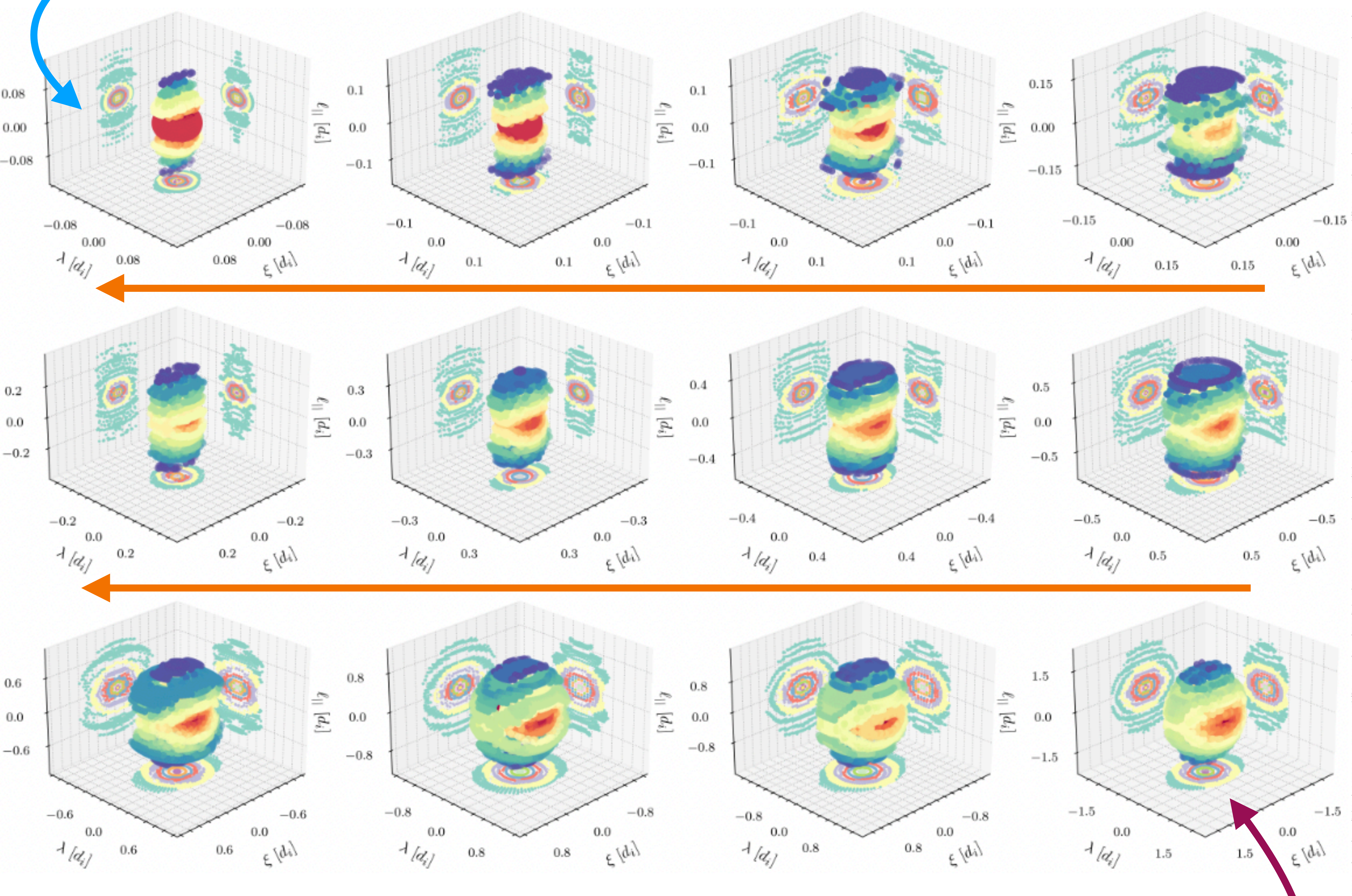
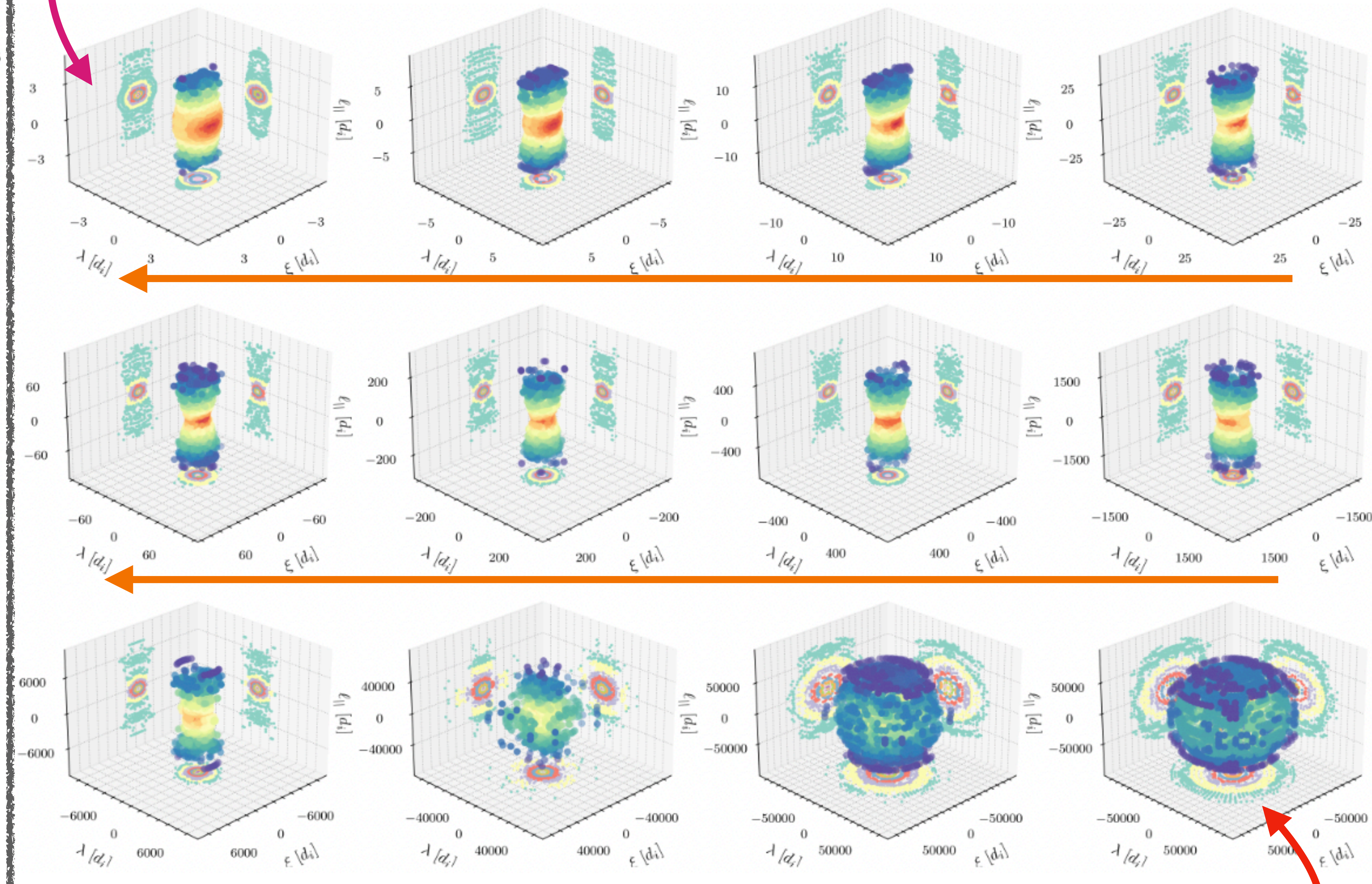
smallest scales: some 3D anisotropy develops again!



towards smaller scales

intermediate/transition scales:
decreasing anisotropy! (due to reconnection?)

intermediate scales: 3D anisotropy developed
due to scale-dependent alignment



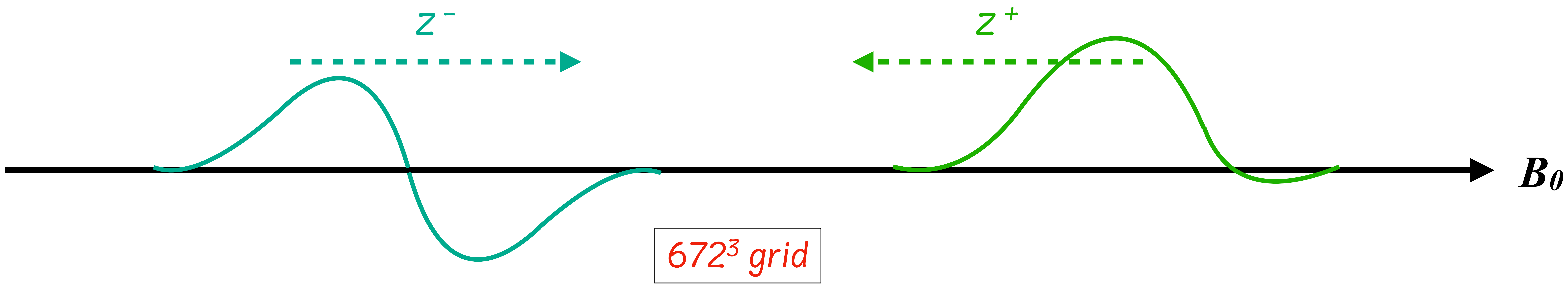
towards smaller scales

largest scales: isotropy

Simulations setup

[Cerri et al. ApJ 2022]

basic 3D setup: start from the ***building blocks of the Alfvénic cascade***!



Simulations performed with the ***Hamiltonian 2-field gyro-fluid*** model/code described in *[Passot, Sulem & Tassi, PoP (2018)]*

☞ *model retains* ***only Alfvén & kinetic-Alfvén modes*** *(assumes* ***low frequency*** $\omega << \Omega_{c,i}$ *,* ***strong anisotropy*** $k_{||} << k_\perp$ *)*

☞ *dissipation through a combination of 2nd-order Laplacian operator (with* ***resistivity*** η*) and 8th-order hyper-dissipation operator*

☞ *employed at* ***MHD scales*** *(*$0.004 \lesssim k_\perp \rho_i \lesssim 1$*), i.e., equivalent to reduced MHD (RMHD)*

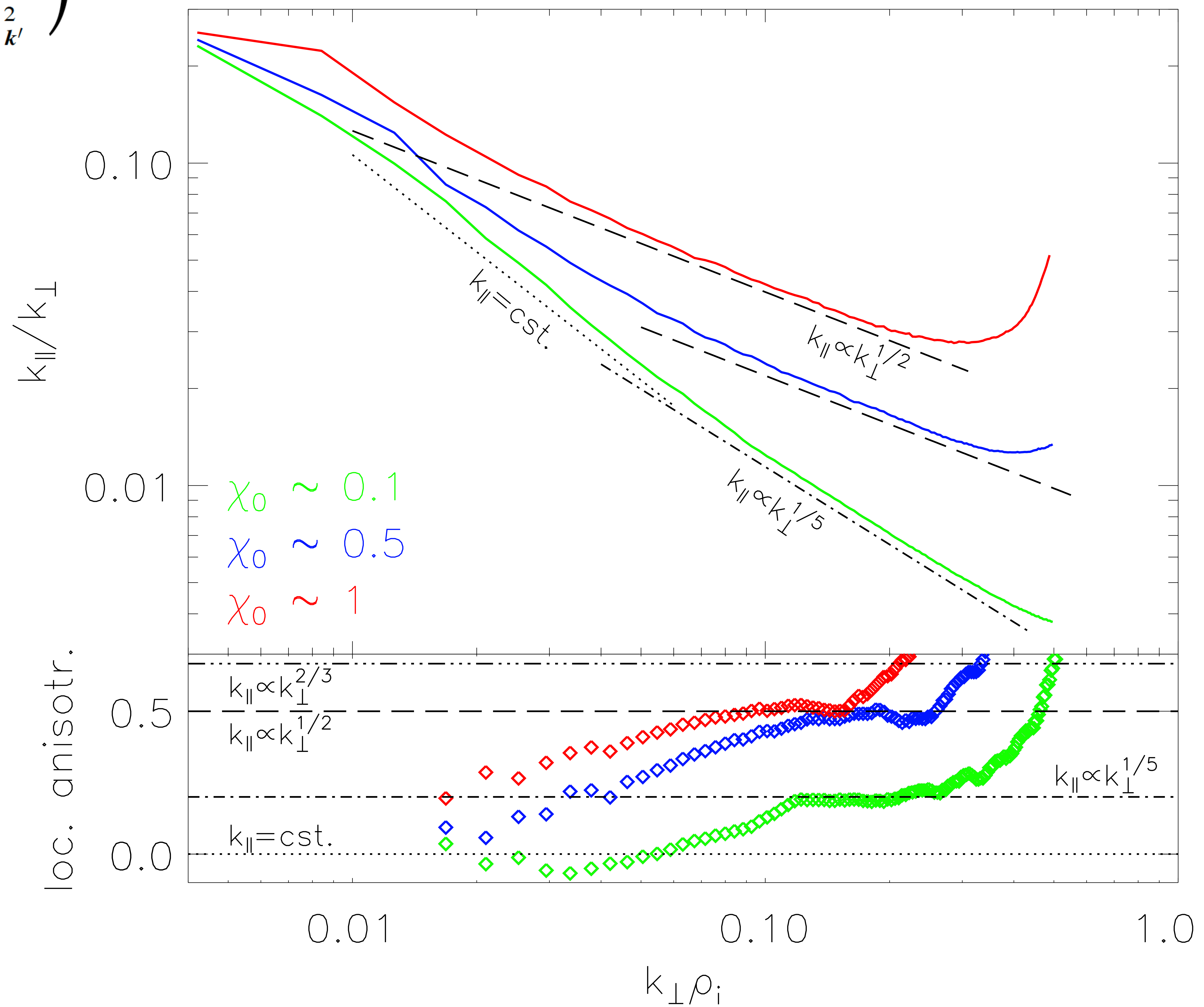
☞ *exploit* ***lower nonlinearities*** *(*$\chi < 1$*) in order to increase the turbulent-eddy lifetime at scale* λ *and "facilitate" the onset tearing instability*

3D simulations of colliding AW packets

$$k_\parallel(k_\perp) \approx \left(\frac{\sum_{k\leq|\boldsymbol{k}'|<k+1} |\widehat{\boldsymbol{B}_L \cdot \nabla \boldsymbol{b}_l}|^2_{\boldsymbol{k}'}}{B_L^2 \sum_{k\leq|\boldsymbol{k}'|<k+1} |\hat{\boldsymbol{b}}|^2_{\boldsymbol{k}'}} \right)^{1/2}$$

[Cho & Lazarian, ApJ (2004)]

[Cerri et al. ApJ 2022]



3D simulations of colliding AW packets

[Cerri et al. ApJ 2022]

Two-field gyro-fluid (2fGF) Hamiltonian model

☞ *At scales $k_\perp d_e << 1$, the original 2fGF equations from* [Passot, Sulem & Tassi, PoP 2018] *reduce to ($\boldsymbol{B}_0$ along z):*

$$\frac{\partial N_e}{\partial t} + [\varphi, N_e] - [B_z, N_e] + \frac{2}{\beta_e}\nabla_\| \Delta_\perp A_\| = 0\,,$$

$$\frac{\partial A_\|}{\partial t} + \nabla_\| (\varphi - N_e - B_z) = 0\,,$$

- N_e = *number density of electron gyro-centers*
- $A_\|$ = *field-parallel component of magnetic potential*
- $[F, G] = (\partial_x F)(\partial_y F) - (\partial_y F)(\partial_x G)$ = *Poisson brackets of two fields F and G*
- electrostatic potential φ and parallel magnetic-field fluctuations B_z are given by: $B_z = M_1\varphi$ and $N_e = -M_2\varphi$

  M are operators; in Fourier space they read: $\widehat{M_1} \doteq L_1^{-1} L_2$ $\quad \widehat{M_2} \doteq L_3 + L_4 L_1^{-1} L_2$

$$L_1 \doteq 2/\beta_e + (1+2\tau)(\Gamma_0 - \Gamma_1) \qquad L_2 \doteq 1 + (1-\Gamma_0)/\tau - \Gamma_0 + \Gamma_1 \qquad L_3 \doteq (1-\Gamma_0)/\tau \qquad L_4 \doteq 1 - \Gamma_0 + \Gamma_1$$

$$\beta_e = 8\pi n_0 T_{e0}/B_0^2 \qquad \tau = T_{i0}/T_{e0} \qquad \Gamma_n(b) \doteq \mathrm{I}_n(b)\exp(-b) \qquad b \doteq k_\perp^2 \rho_i^2/2$$

Phenomenology of Alfvénic Turbulence

weak Alfvénic turbulence: a quick phenomenological derivation of the spectrum

☞ *for a formal derivation, see, e.g.,*

[Ng & Bhattacharjee, PoP 1996]
[Galtier, Nazarenko, Newell, Pouquet, JPP 2000]
[Schekochihin, JPP 2022]

Phenomenology of Alfvénic Turbulence

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$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$$
$$\omega(k_{\|,1}) + \omega(k_{\|,2}) = \omega(k_{\|,3})$$

$\Rightarrow$ *no parallel cascade ($k_{//}$ = cst.), only a cascade in $k_\perp$!*

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(i.e., $\Delta(\delta z)/\delta z \sim 1$)

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crossing time ~ linear propagation time: $\tau_A = (k_\| v_A)^{-1}$

distortion time ~ non-linear time: $\tau_{nl} = (k_\perp \delta z)^{-1}$

$$\Rightarrow \quad \Delta(\delta z) \sim \left(\frac{\tau_A}{\tau_{nl}}\right) \delta z = \chi\, \delta z$$

(change during one collision)

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(change during one collision)

$\Rightarrow$ assume changes accumulates as a random walk:

$$N_{\rm inter.} \sim \left(\frac{\delta z}{\Delta(\delta z)}\right)^2 \sim \frac{1}{\chi^2}$$

$$\Rightarrow \quad \tau_{\rm casc} \sim N \, \tau_A \sim \frac{\tau_{nl}^2}{\tau_A} = \frac{\tau_{nl}}{\chi}$$

CASCADE TIME

Phenomenology of Alfvénic Turbulence

weak Alfvénic turbulence: a quick phenomenological derivation of the spectrum

☞ fluctuations' scaling and energy spectrum from constant energy flux through scales:

$$\frac{\delta z^2}{\tau_{\rm casc}} \sim \varepsilon = \text{const.} \quad \Rightarrow \quad \delta z \propto k_\perp^{-1/2} \quad \Rightarrow \quad \mathcal{E}_{\delta z} \propto k_\perp^{-2}$$

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while linear frequency is constant because there is no parallel cascade:

$$\omega_{\rm nl} = k_\perp \delta z \sim k_\perp^{1/2}$$

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Phenomenology of Alfvénic Turbulence

weak Alfvénic turbulence: a quick phenomenological derivation of the spectrum

☞ fluctuations' scaling and energy spectum

from constant energy flux through scales: $\frac{\delta z^2}{\tau_{\rm casc}} \sim \varepsilon = \text{const.} \quad \Rightarrow \quad \delta z \propto k_\perp^{-1/2} \quad \Rightarrow \quad \mathcal{E}_{\delta z} \propto k_\perp^{-2}$

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$$\Rightarrow \quad \chi \sim k_\perp^{1/2} \quad \Rightarrow \quad \frac{\lambda_\perp^{\rm CB}}{\ell_{\|,0}} \sim \left(\frac{\varepsilon\,\ell_{\|,0}}{v_{\rm A}^3}\right)^{1/2} \sim \left(\frac{\delta z_0}{v_{\rm A}}\right)^{3/2} \quad (\ll 1)$$

transition to critical balance ($\chi \sim 1$)

Phenomenology of Alfvénic Turbulence

critically balanced (strong) Alfvénic turbulence: a quick phenomenological derivation

☞ *for furhter details, see, e.g.,*

[Goldreich & Sridhar, ApJ 1995]
[Oughton & Matthaeus, ApJ 2020]
[Schekochihin, JPP 2022]

Phenomenology of Alfvénic Turbulence

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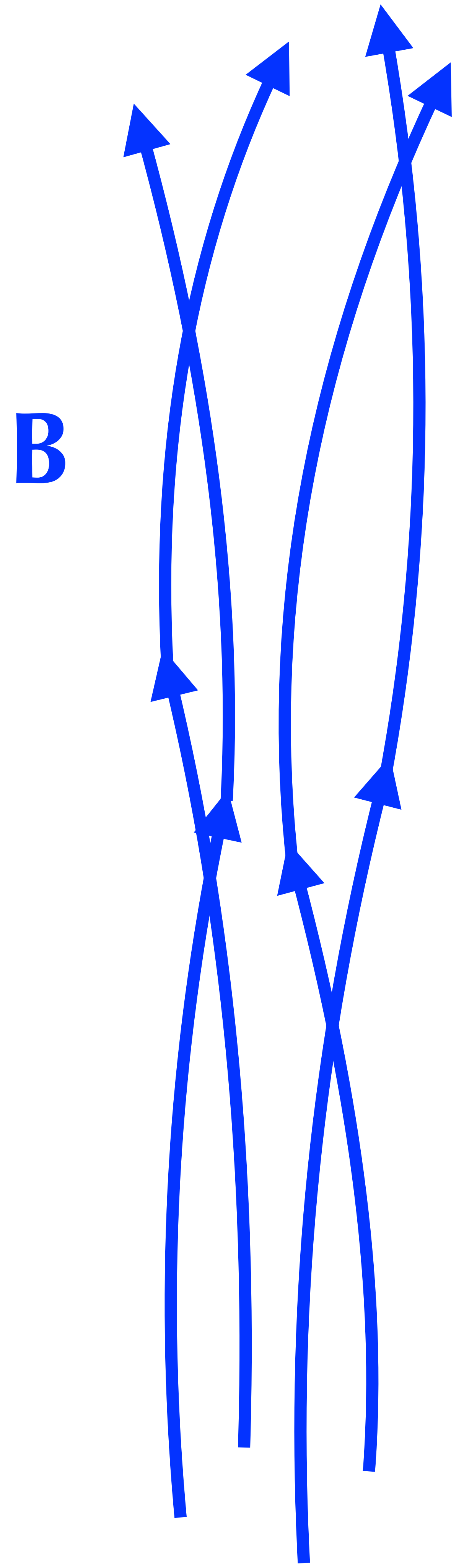

☞ At this point, linear, non-linear, and cascade timescales match each other:

$$\tau_{\rm nl} \sim \tau_{\rm A} \quad \Rightarrow \quad \tau_{\rm casc} \sim \tau_{\rm nl}$$

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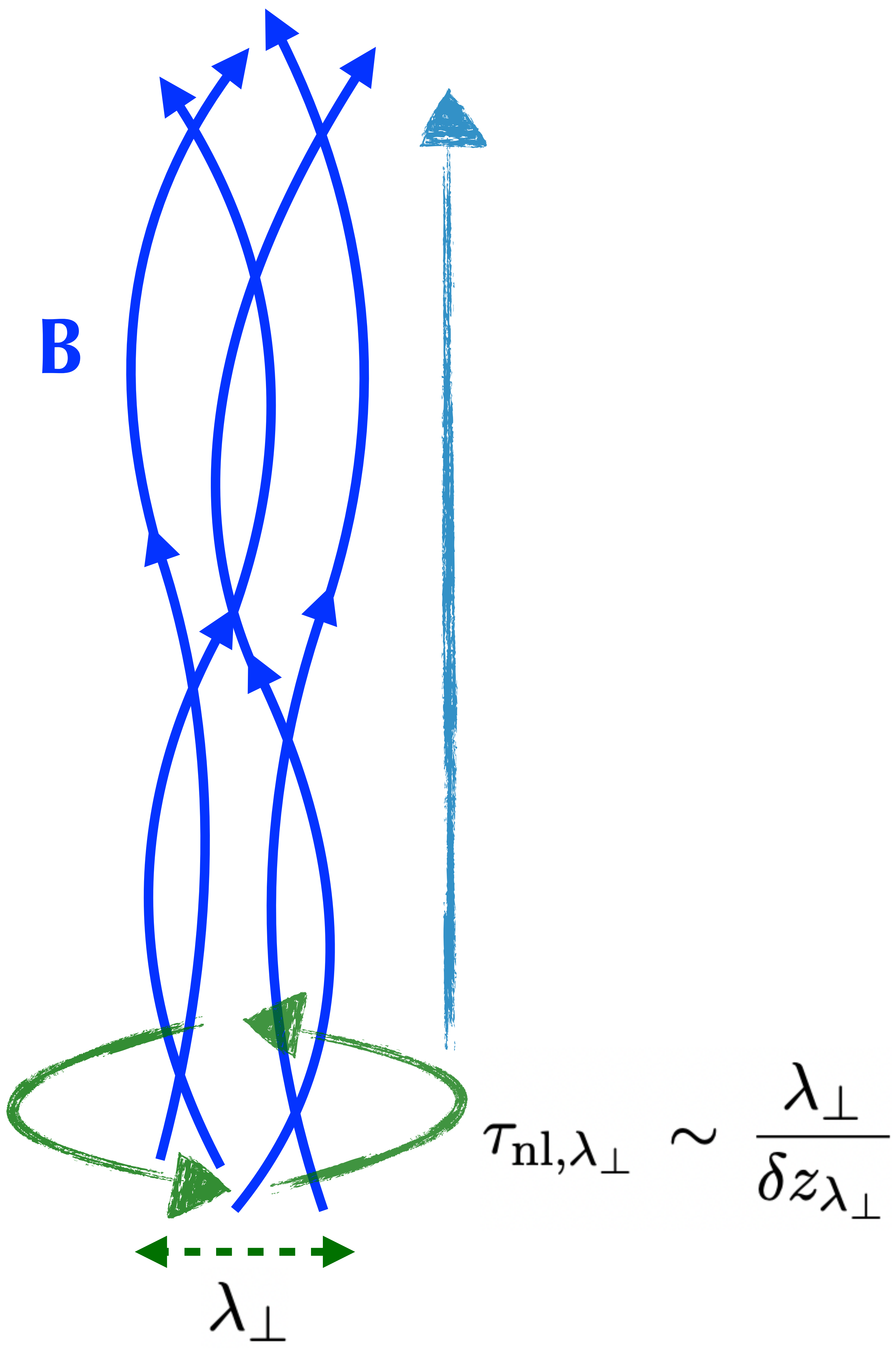

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you can see the *``critical-balance condition" as the result of causality*:

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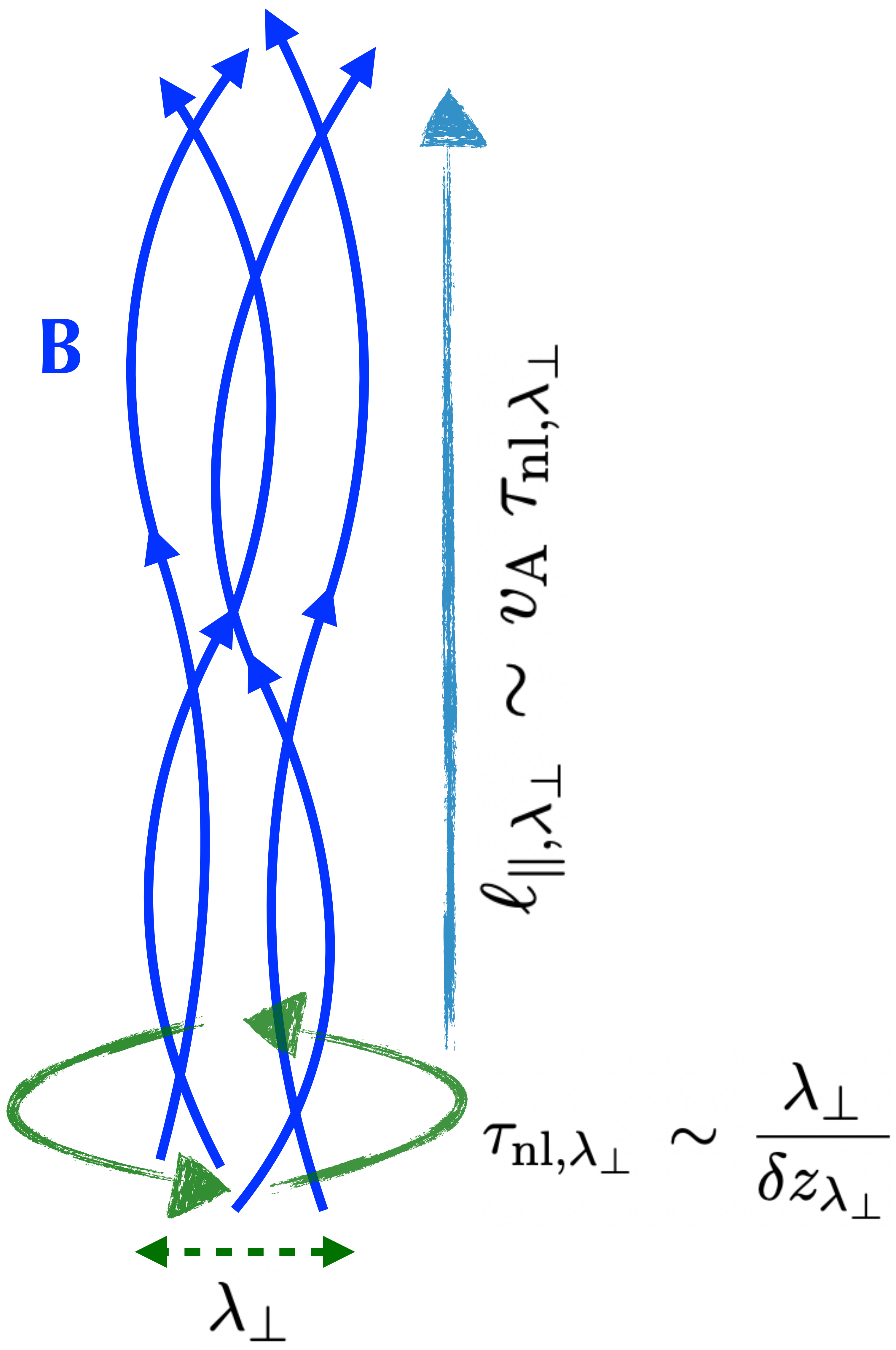

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the information about Alfvénic fluctuations decorrelating in the perpendicular plane over an eddy turn-over time τ_{nl} can only propagate along the field for a length $\ell_{||}$ at maximum speed v_A.

"So… CB is essentially AWs trying to keep up with the turbulent eddies…"

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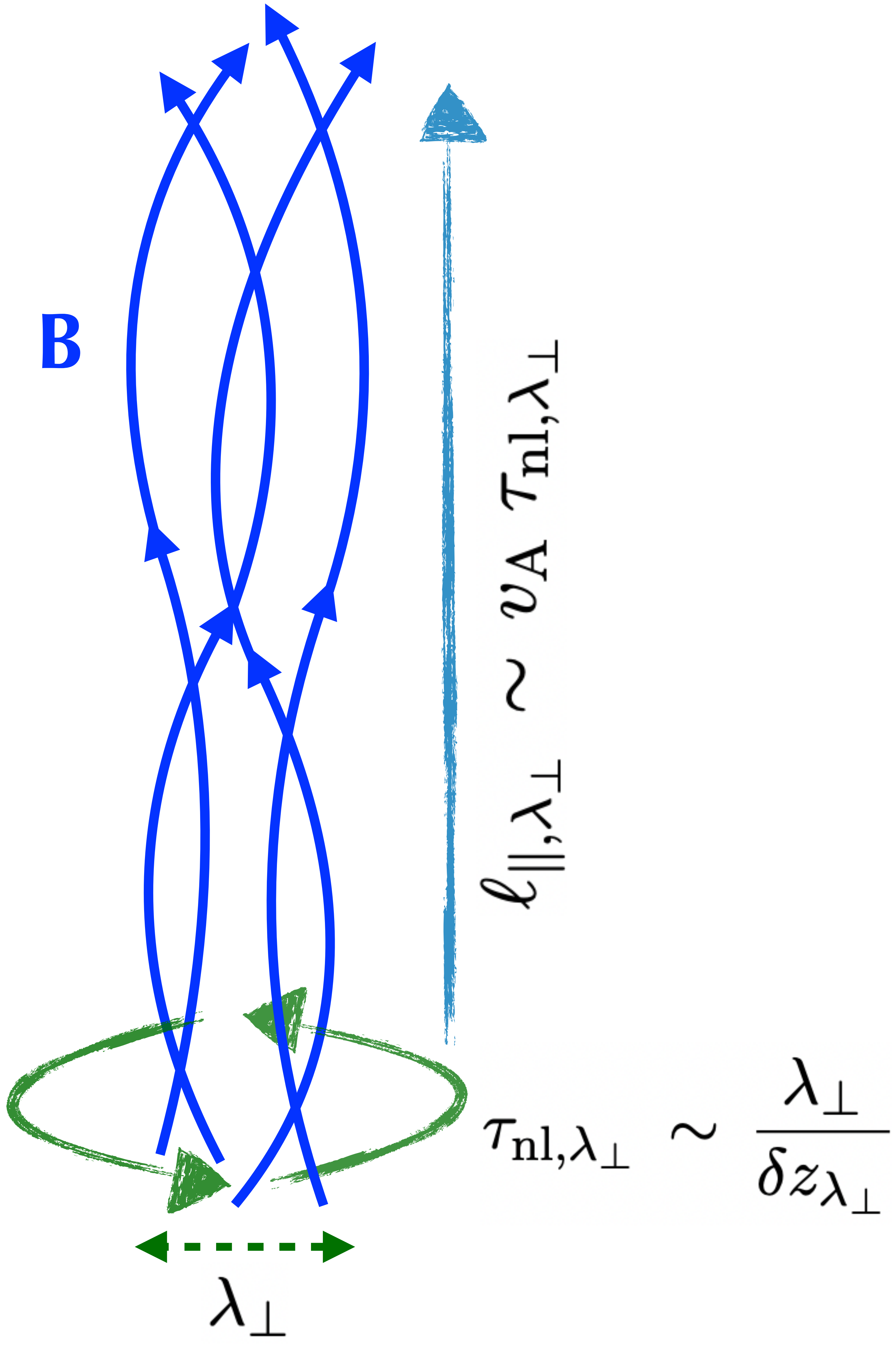

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"So… CB is essentially AWs trying to keep up with the turbulent eddies…"

Therefore, once $\tau_{nl} \sim \tau_A$ is reached, the balance is mantained.

(In principle, this could be done by continuing the cascade with τ_{nl} = const., or by generating smaller $\ell_{||}$ such that $\tau_A \sim \ell_{||}/v_A \sim \tau_{nl}$ keeps holding… it is the latter)

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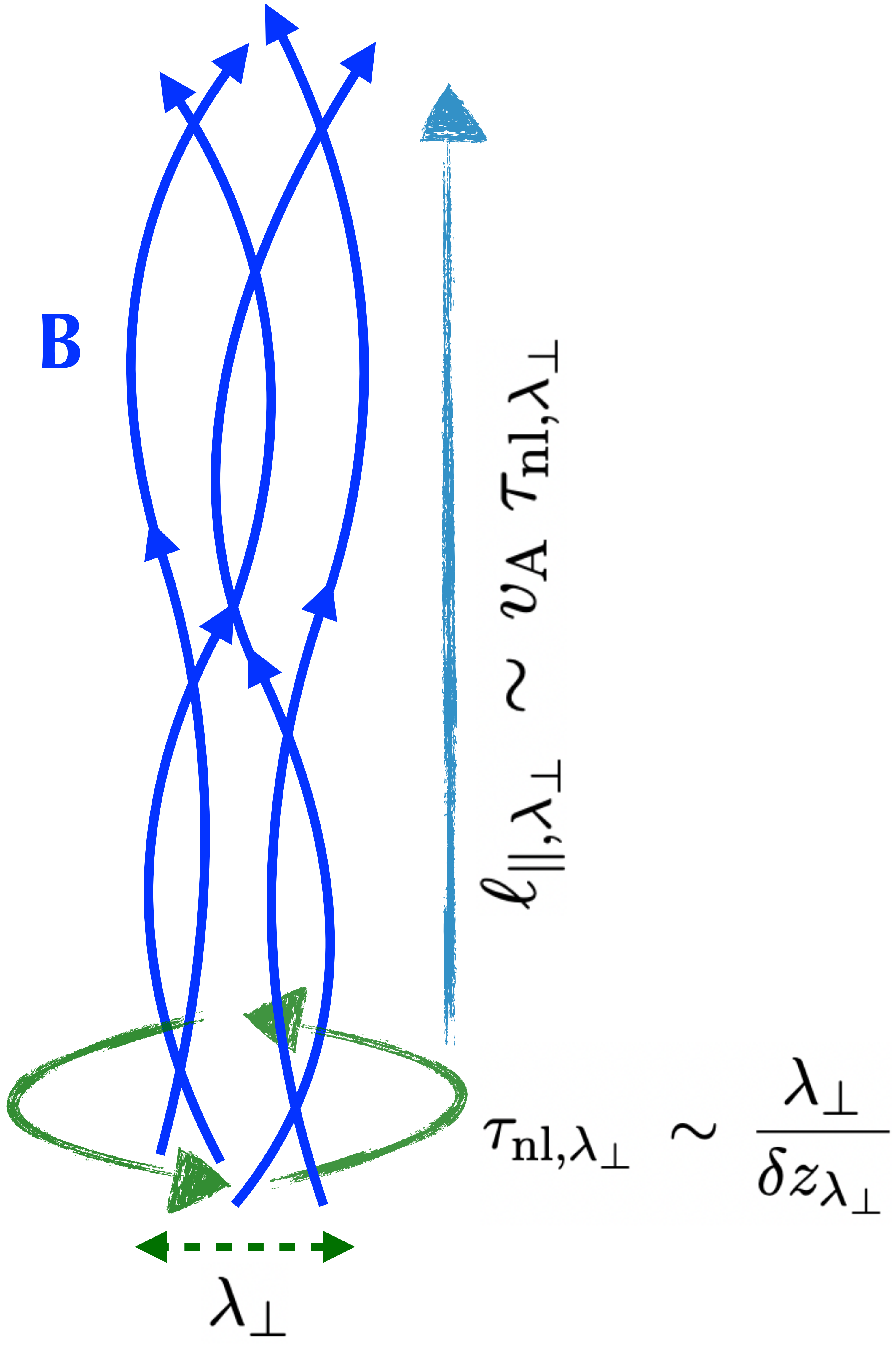

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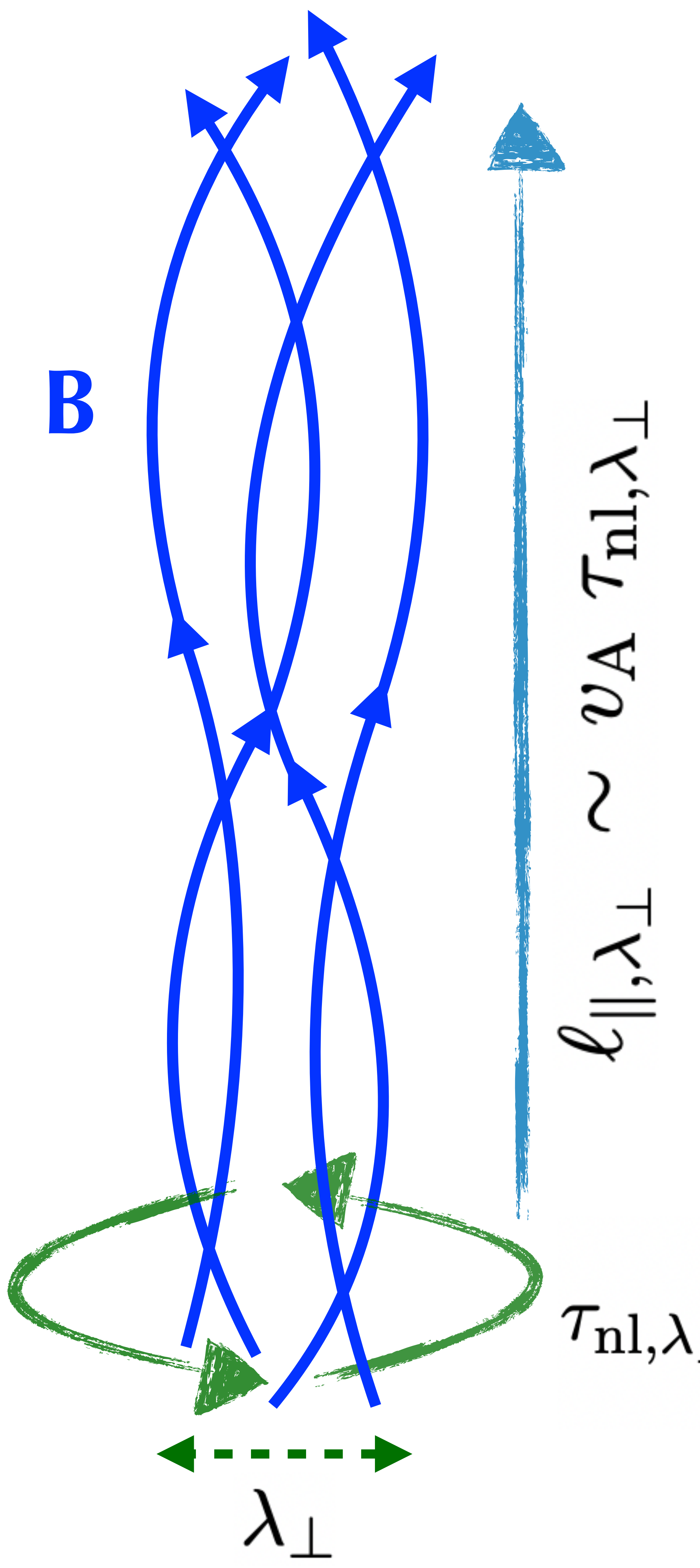

$$\tau_{\mathrm{nl},\lambda_\perp} \sim \frac{\lambda_\perp}{\delta z_{\lambda_\perp}}$$

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☞ fluctuations' scaling + spectum from **ε** = const. (*you know the drill*):

$$\frac{\delta z_{k_\perp}^2}{\tau_{\mathrm{nl},k_\perp}} \sim \varepsilon = \text{const.} \quad \Rightarrow \quad \boxed{\delta z_{k_\perp} \propto k_\perp^{-1/3} \quad \Rightarrow \quad \mathcal{E}_{\delta z}(k_\perp) \propto k_\perp^{-5/3}}$$

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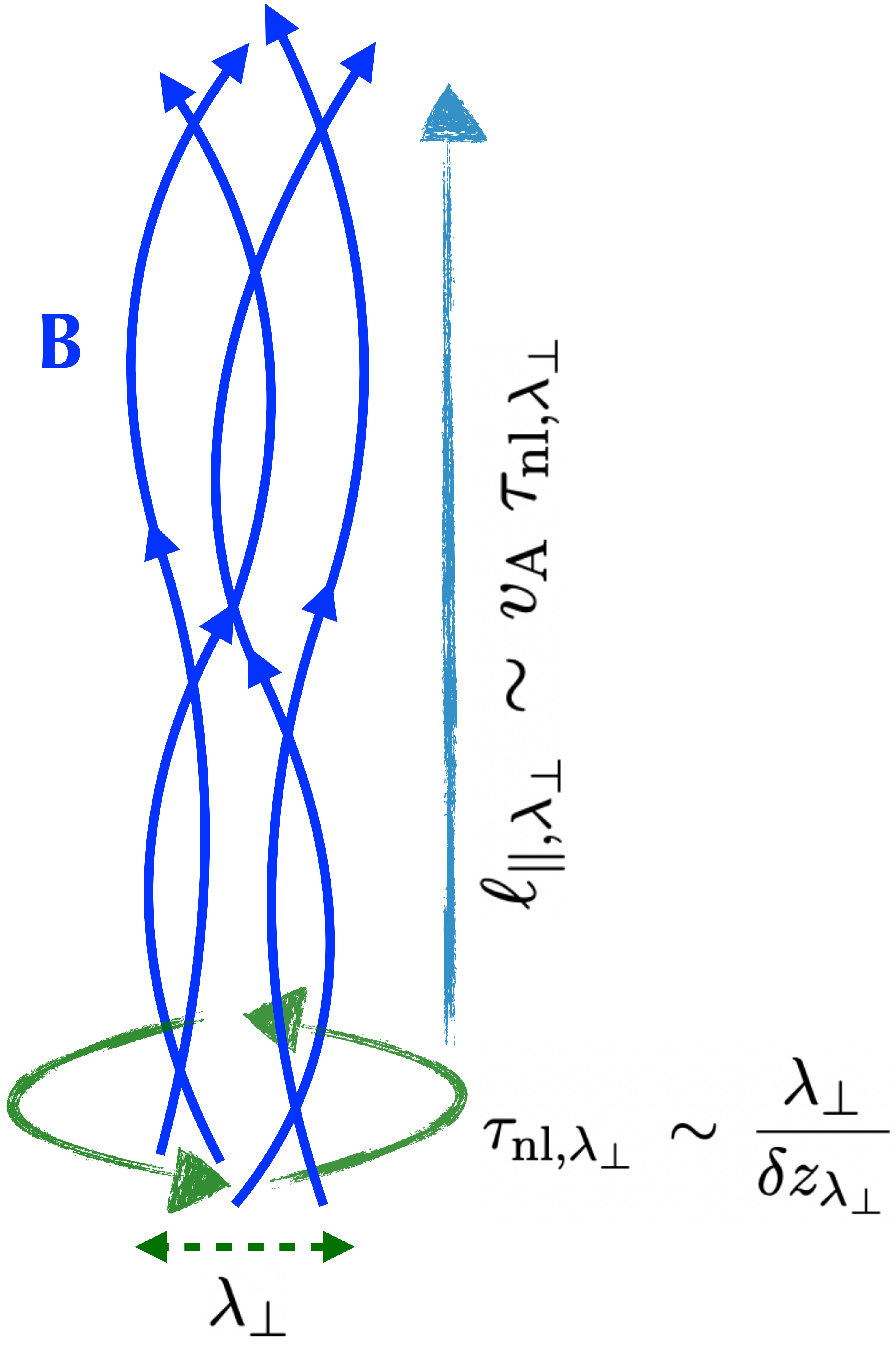

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$$\frac{\delta z^2_{k_\perp}}{\tau_{nl,k_\perp}} \sim \varepsilon = \text{const.} \quad \Rightarrow \quad \boxed{\delta z_{k_\perp} \propto k_\perp^{-1/3} \quad \Rightarrow \quad \mathcal{E}_{\delta z}(k_\perp) \propto k_\perp^{-5/3}}$$

☞ now, you can also compute the fluctuations' wavenumber anisotropy:

$$k_\perp \delta z_{k_\perp} \sim k_\| v_A \quad \Rightarrow \quad \boxed{k_\| \propto k_\perp^{2/3}} \quad \left(\Rightarrow \quad \mathcal{E}_{\delta z}(k_\|) \propto k_\|^{-2} \right)$$

Further Developments in Theoretical Models

dynamic alignment in Alfvénic turbulence: three-dimensional anisotropy

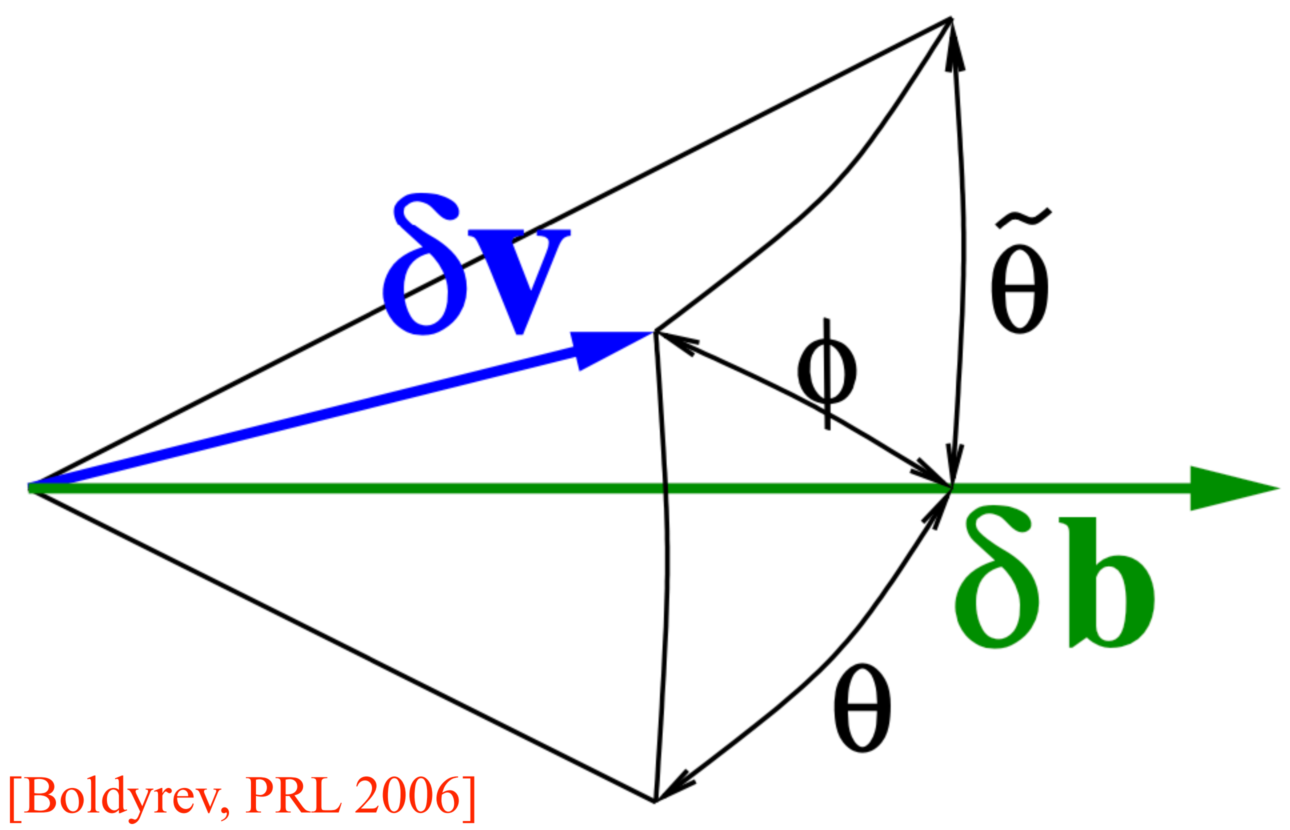
☞ *for furhter details, see, e.g.,*

[Boldyrev, PRL 2006]

[Schekochihin, JPP 2022]

Further Developments in Theoretical Models

dynamic alignment in Alfvénic turbulence: three-dimensional anisotropy



[Boldyrev, PRL 2006]

☞ Observations and simulations show that $\boldsymbol{\delta v}_\lambda$ and $\boldsymbol{\delta b}_\lambda$ have a spontaneous tendency to align in the plane perpendicular to the local mean field $\langle \mathbf{B} \rangle_\lambda$, within an angle θ_λ

(e.g., Podesta et al., JGR 2009; Hnat et al., PRE 2011; Mason et al., ApJ 2011; Wicks et al., PRL 2013; Mallet et al., MNRAS 2016; …)

Further Developments in Theoretical Models

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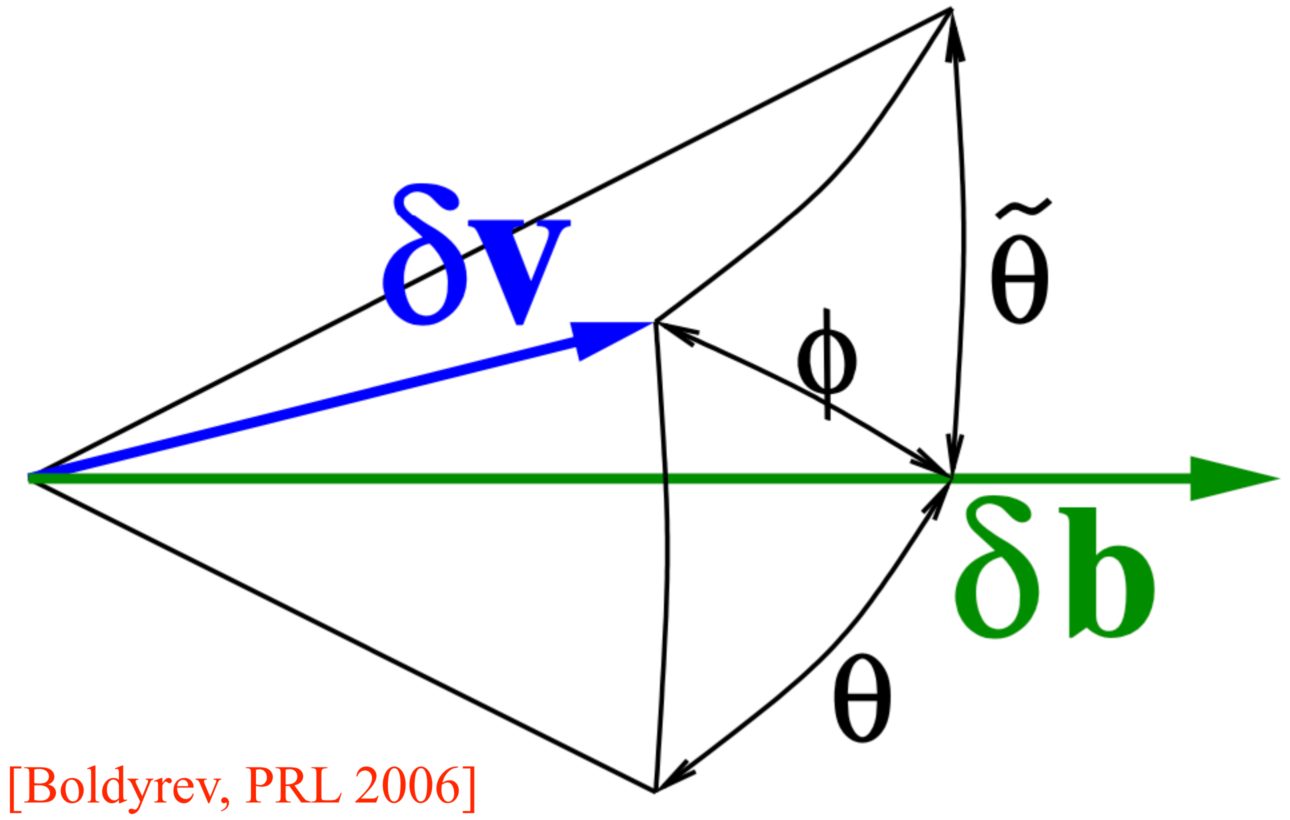

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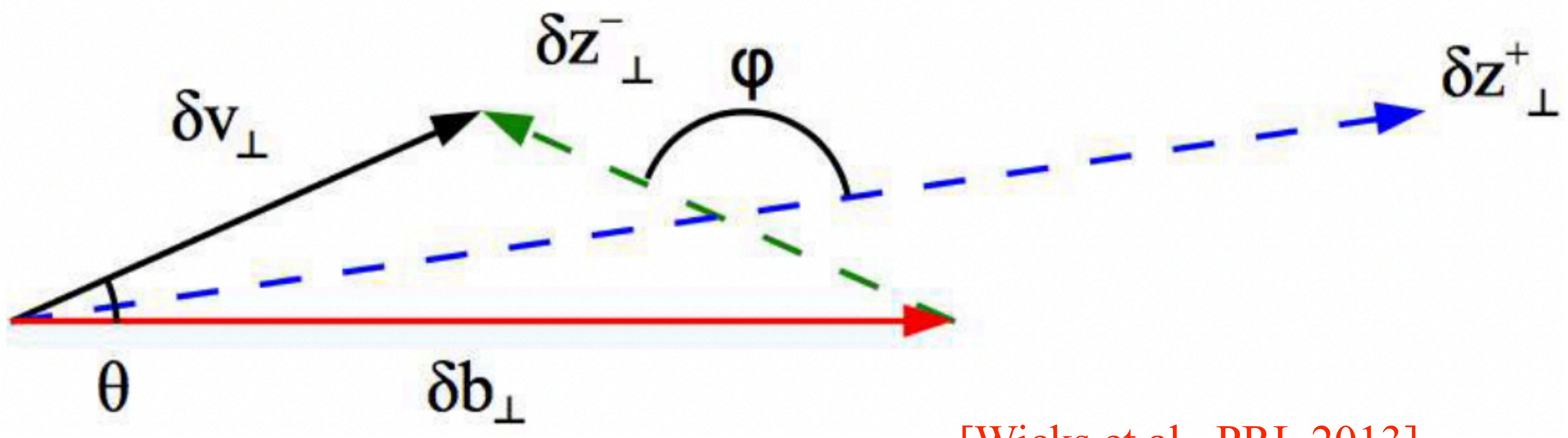
⚠️ the alignment between $\boldsymbol{\delta v}_\lambda$ and $\boldsymbol{\delta b}_\lambda$ is *not the same* as the alignment between $\boldsymbol{\delta z}^+_\lambda$ and $\boldsymbol{\delta z}^-_\lambda$!

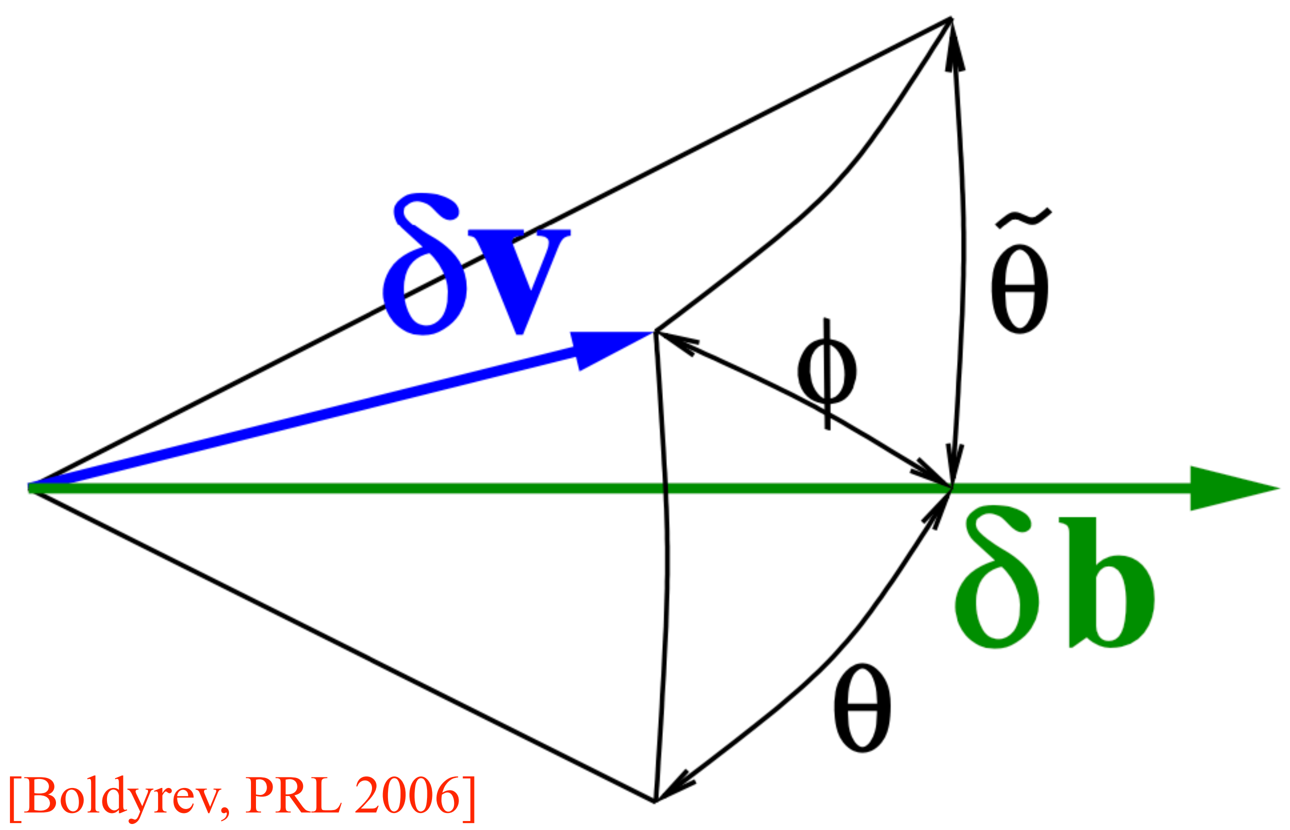
(but they are related: see Schekochihin arXiv:2010.00699)



[Wicks et al., PRL 2013]

Further Developments in Theoretical Models

dynamic alignment in Alfvénic turbulence: three-dimensional anisotropy



[Boldyrev, PRL 2006]

☞ Observations and simulations show that $\delta\mathbf{v}_\lambda$ and $\delta\mathbf{b}_\lambda$ have a spontaneous tendency to align in the plane perpendicular to the local mean field $\langle\mathbf{B}\rangle_\lambda$, within an angle θ_λ

(e.g., Podesta et al., JGR 2009; Hnat et al., PRE 2011; Mason et al., ApJ 2011; Wicks et al., PRL 2013; Mallet et al., MNRAS 2016; …)

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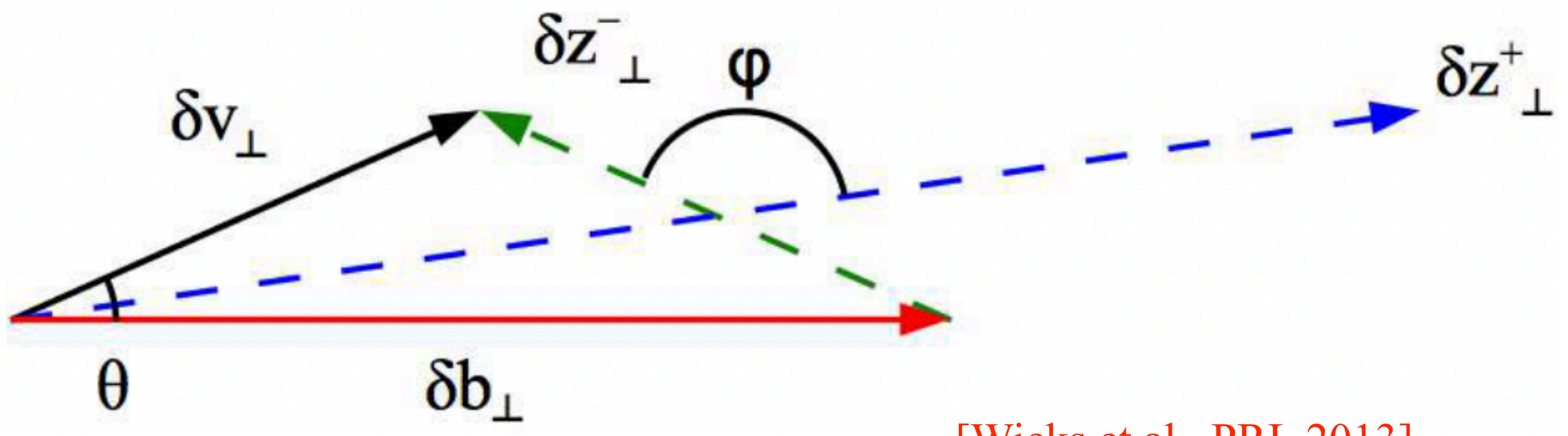

[Wicks et al., PRL 2013]

alignment* ⇒ *depletion of non-linearities: $\delta\mathbf{z}^\mp\cdot\nabla\delta\mathbf{z}^\pm \sim \sin\varphi_\lambda\frac{\delta z_\lambda^2}{\lambda} \approx \varphi_\lambda\frac{\delta z_\lambda^2}{\lambda} \longleftrightarrow \theta_\lambda\frac{\delta v_\lambda^2}{\lambda}$

⚠ but remember that *fluctuations cannot be perfectly aligned* ($\theta_\lambda = 0$) in order to have a non-linear cascade