A Hybrid Approach Combining Fully Kinetic Ions and Gyrokinetic Electrons for Modelling Space and Astrophysical Plasmas

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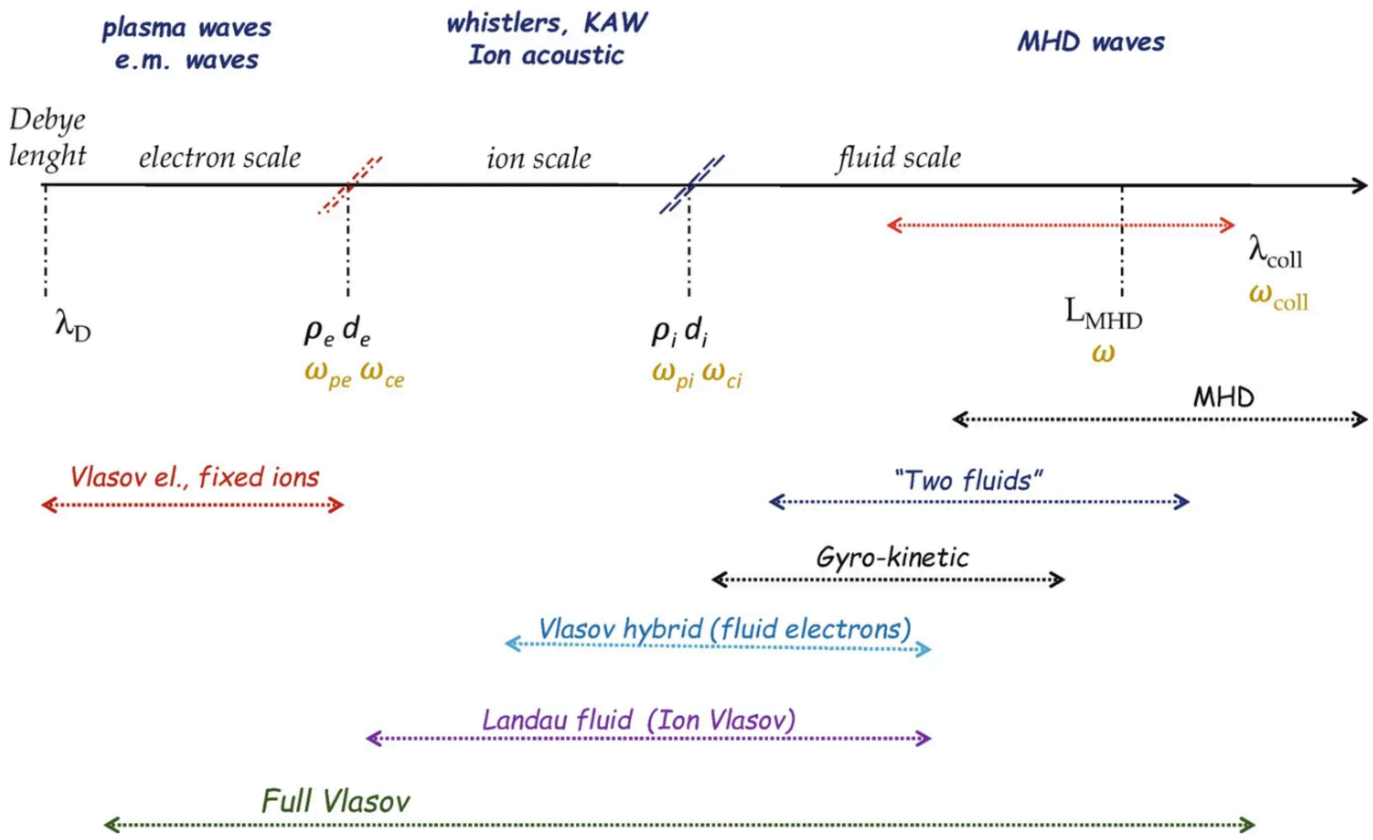
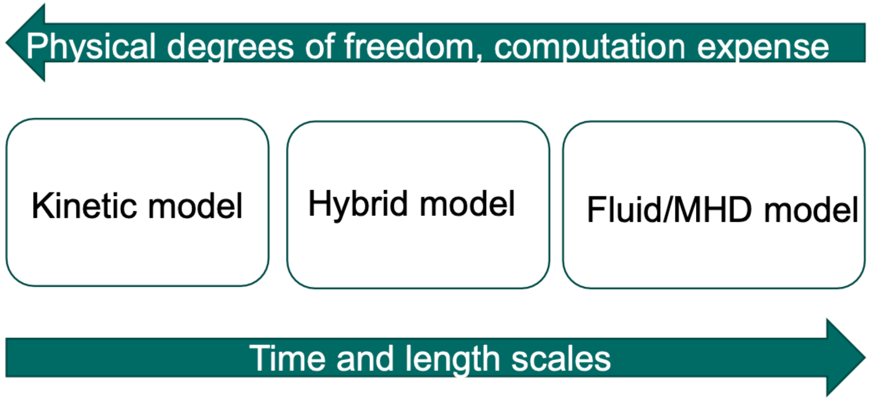
Outline

1. Background and Motivation
2. The Hybrid Model
3. Numerical Implementation and Linear Analysis
4. Nonlinear Analysis

Background and Motivation

- Emerged as a complement to fully kinetic and hybrid kinetic-fluid models.
- Initially applied to plasma turbulence at kinetic scales and magnetic reconnection



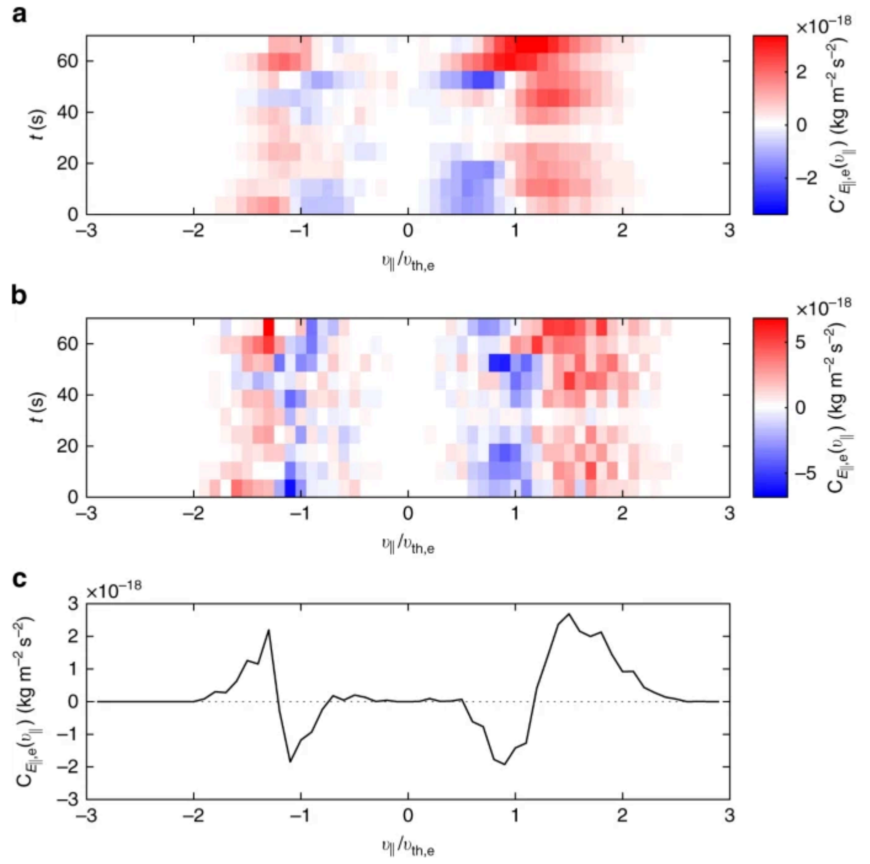


- Typical amplitudes of the turbulent fluctuations (in the densities, temperatures, etc.) tend to be small compared to the respective mean values.
- Typical frequencies of the turbulent fluctuations can cover a wide range of scales but are generally small compared to the ion and electron gyrofrequencies.
- Typical perpendicular correlation lengths of the turbulent fluctuations tend to be small compared to various macroscopic scale lengths but may be comparable to the ion and electron Larmor radii.
- Typical parallel correlation lengths of the turbulent fluctuations tend to be comparable to various macroscopic scale lengths.

Background and Motivation

- Traditional kinetic descriptions involve solving for a time-dependent, single-particle distribution function in phase space.

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s}\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$



- Importance of kinetic phenomena: Landau damping, finite Larmor radius effects, ion/electron cyclotron resonances.
- Challenges with fully kinetic models due to computational limits.

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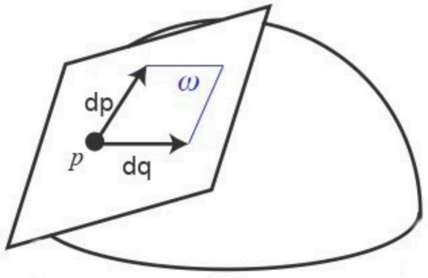
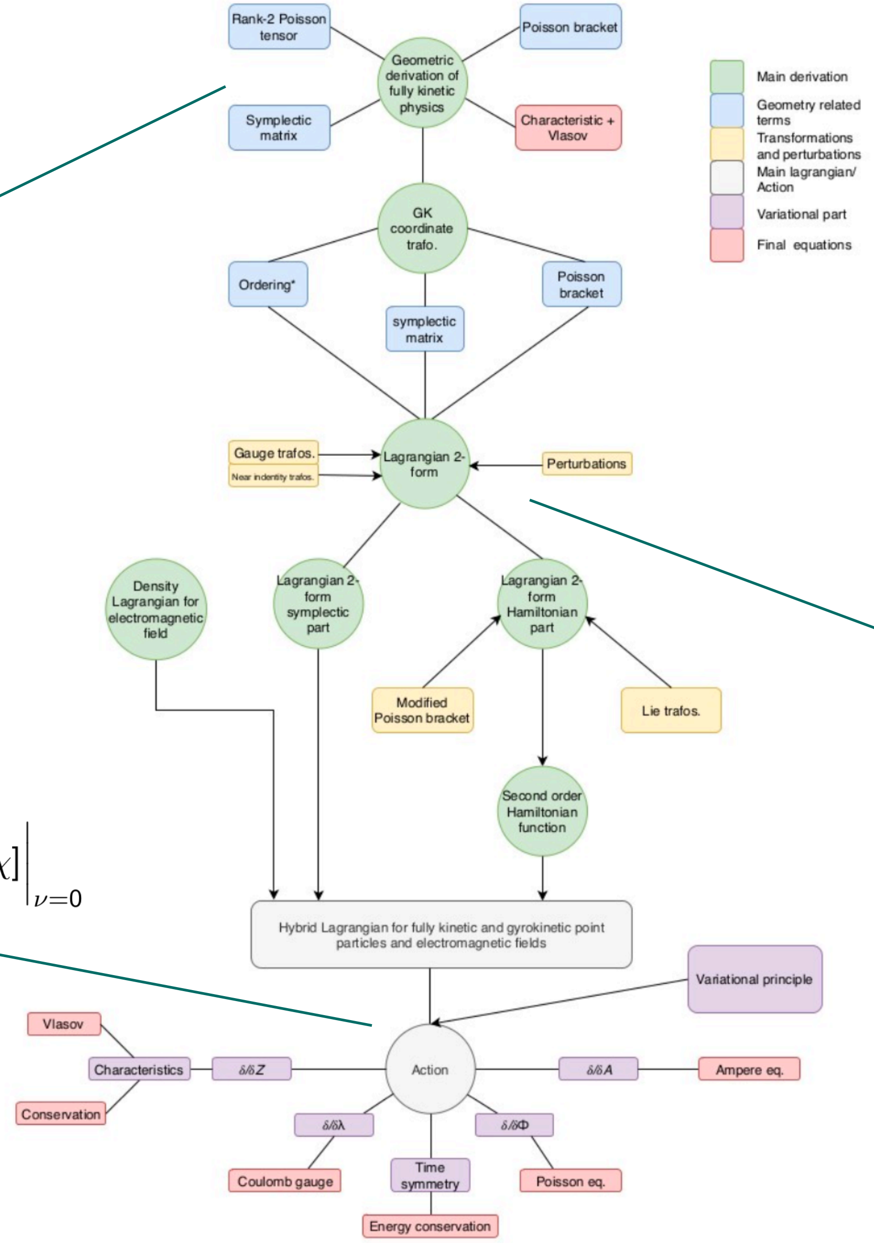
The Hybrid Model

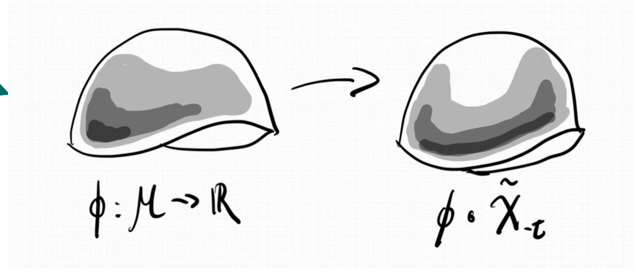
Symplectic Geometry



$$\left\langle \frac{\delta \mathcal{L}}{\delta \phi} \mid \chi \right\rangle \equiv \int_{\mathbb{X}} \frac{\delta \mathcal{L}}{\delta \phi}(X_{gy}) \chi(X_{gy}) d^m X_{gy} = \frac{d}{d\nu} \mathcal{L}[\phi + \nu\chi] \Big|_{\nu=0}$$



Coordinate Transformations



The Hybrid Model - FK Symplectic Geometry

Considering the following tautological one form

$$\Gamma = \mathcal{L}dt = (\Lambda_a d\Theta^a - \bar{H}dt)$$

we construct a symplectic form

$$\omega = d\Gamma = \epsilon_{ijk}B_k dx^i \wedge dx^j - m\delta_{ij}dx^i \wedge dv^j + m\delta_{ij}dv^i \wedge dx^j$$

Which can then be converted into a rank-2 Poisson tensor

$$\Pi^{ij} = \omega_{ij}^{-1} = \begin{pmatrix} 0 & \frac{1}{m}\delta^{ij} \\ -\frac{1}{m}\delta^{ij} & \frac{1}{m^2}\epsilon^{ijk}B_k \end{pmatrix}$$

and a Poisson Bracket

$$\{f,g\} = \sum_{ij} \frac{\partial f}{\partial \Theta^i}\Pi^{ij}\frac{\partial g}{\partial \Theta^j} = \frac{1}{m}\left(\nabla f \frac{\partial g}{\partial v} - \frac{\partial f}{\partial v}\nabla f\right) + B\left(\frac{\partial f}{\partial v} \times \frac{\partial g}{\partial v}\right)$$

The Hybrid Model - FK Symplectic Geometry

We use Liouville's theorem to evolve the distribution function in phase space, and identify the terms of the equation with the characteristics of the Poisson Bracket

$$\frac{\partial F}{\partial t} + \{x, H\} \nabla F + \{v, H\} \partial_v F = 0$$

The fully kinetic Vlasov equation becomes then

$$\frac{\partial F}{\partial t} + v^I \nabla F + \left(\frac{\partial \phi}{\partial x^i} + \epsilon^{ijk} v_j B_k \right) \partial_v F = 0$$

The Hybrid Model - GK Symplectic Geometry

A series of guiding centre transformation will change our Lagrangian

$$\mathcal{L}_{gc} = \left[\frac{e}{c}A(X_{gc}) + mV_\parallel \hat{b}(X_{gc}) - \frac{mc}{e}\mu R\right] \cdot dX_{gc} + \frac{mc}{e}\mu d\Theta - H_{gc}dt$$

Which means that our symplectic form becomes

$$\omega_{gc} = d\Gamma_{gc} = \frac{e}{c}\epsilon_{ijk}B^k dX^i_{gc} \wedge dX^j_{gc} + m\hat{b}(X_{gc})dV_\parallel \wedge dX^i_{gc} + \frac{mc}{e}d\mu \wedge d\theta$$

And therefore, the Poisson bracket is also modified

$$\{f,g\}_{gc} = \frac{e}{mc}\frac{1}{\epsilon_B}\left(\frac{\partial f}{\partial \theta}\frac{\partial g}{\partial \mu} - \frac{\partial f}{\partial \theta}\frac{\partial g}{\partial \mu}\right) + \frac{B^*}{mB^*_\parallel}\left(\nabla^* f \frac{\partial g}{\partial V_\parallel} - \nabla^* g \frac{\partial f}{\partial V_\parallel}\right) - \epsilon_B \frac{c\hat{b}}{eB^*_\parallel}\left(\nabla^* f \times \nabla^* g\right)$$

The Hybrid Model - GK Symplectic Geometry

The theta dependence left in the Hamiltonian can be worked out with a Lie transformation

$$H_{gc}(Z_{gc}) = e^{-\mathcal{L}_{S_{gc}}} H(Z)$$

Our gyrokinetic Lagrangian is now

$$\Gamma_{gy} = \left(\frac{e}{\varepsilon_\delta c} \mathbf{A}_1 + m \mathbf{v}_{gy,\parallel} \hat{\mathbf{b}}(X_{gy}) \right) \cdot \dot{\mathbf{X}}_{gy} + \varepsilon_\delta \frac{mc}{e} \mu_{gy} \dot{\theta}_{gy} -$$
$$\frac{1}{2} m v_{gy,\parallel}^2 - \mu_{gy} B(\mathbf{X}_{gy}) - \varepsilon_\delta e \langle \psi_1 \rangle - \varepsilon_\delta^2 e^2 \left(\frac{1}{2mc^2} \langle |\mathbf{A}_1|^2 \rangle - \frac{1}{2B(\mathbf{X}_{gy})} \partial_{\mu_{gy}} \langle \psi_1^2 \rangle \right)$$

And our total action is written as

$$\mathcal{S} = \mathcal{S}_{gy}^p + \sum_i \mathcal{S}_{fk,i}^p + \mathcal{S}^f$$

The Hybrid Model - GK Symplectic Geometry

Similarly to the fully kinetic case, we use Liouville's theorem to evolve the distribution function in phase space, in the present case, using the modified Poisson Bracket. The gyrokinetic Vlasov equation becomes then

$$
\frac{\partial F}{\partial t} + \frac{\mathbf{B}^*}{m\mathbf{B}^*_\parallel} \cdot \left(m v_{gy,\parallel} - \frac{e}{c} \langle A_{1\parallel} \rangle \right) \nabla_{gy} F
$$

$$
+ \frac{c\hat{\mathbf{b}}}{e\mathbf{B}^*_\parallel} \times \left(\mu_{gy} \nabla_{gy} B(\mathbf{X}_{gy}) + \varepsilon_\delta e \nabla \langle \phi_1 \rangle - \varepsilon_\delta \frac{e}{c} v_{gy,\parallel} \langle \nabla \mathbf{A}_{1\parallel} \rangle \right) \nabla_{gy} F
$$

$$
- \frac{\mathbf{B}^*}{m\mathbf{B}^*_\parallel} \cdot \left(\mu_{gy} \nabla_{gy} B(\mathbf{X}_{gy}) + \varepsilon_\delta e \nabla \langle \phi_1 \rangle - \varepsilon_\delta \frac{e}{c} v_{gy,\parallel} \langle \nabla \mathbf{A}_{1\parallel} \rangle \right) \partial_{v_{gy,\parallel}} F = 0,
$$

The Hybrid Model - Field Equations

We make use of variational derivatives to perform the derivation of the field equations

$$\frac{\delta \mathcal{S}[\chi(\Omega)]}{\delta \chi(\Omega)} \circ \hat{\chi}(\Omega) = 0$$

Effectively, in the case of the perturbed electric potential, that looks like

$$\frac{\delta \phi_1(X_{gy} + \rho)}{\delta \phi_1(\mathbf{x})} \circ \hat{\chi}(\mathbf{x}) = \frac{\delta}{\delta \phi_1(\mathbf{x})} \left(\phi_1(X_{gy} + \rho)\right) \circ \hat{\chi}(\mathbf{x})$$

$$= \frac{\delta}{\delta \phi_1(\mathbf{x})} \left[\int d^3x \phi_1(\mathbf{x}) \delta^3(X_{gy} + \rho - x) \right] \circ \hat{\chi}(\mathbf{x})$$

$$= \frac{d}{d\nu} \left[\int \int d\nu d^3x \left(\phi_1(\mathbf{x}) + \nu \hat{\chi}(\mathbf{x})\right) \delta^3(X_{gy} + \rho - x) \right] |_{\nu=0}$$

$$= \frac{d}{d\nu} \left[\phi_1(X_{gy} + \rho) + \nu \hat{\chi}(X_{gy} + \rho)\right] |_{\nu=0} = \hat{\chi}(X_{gy} + \rho).$$

The Hybrid Model - Field Equations

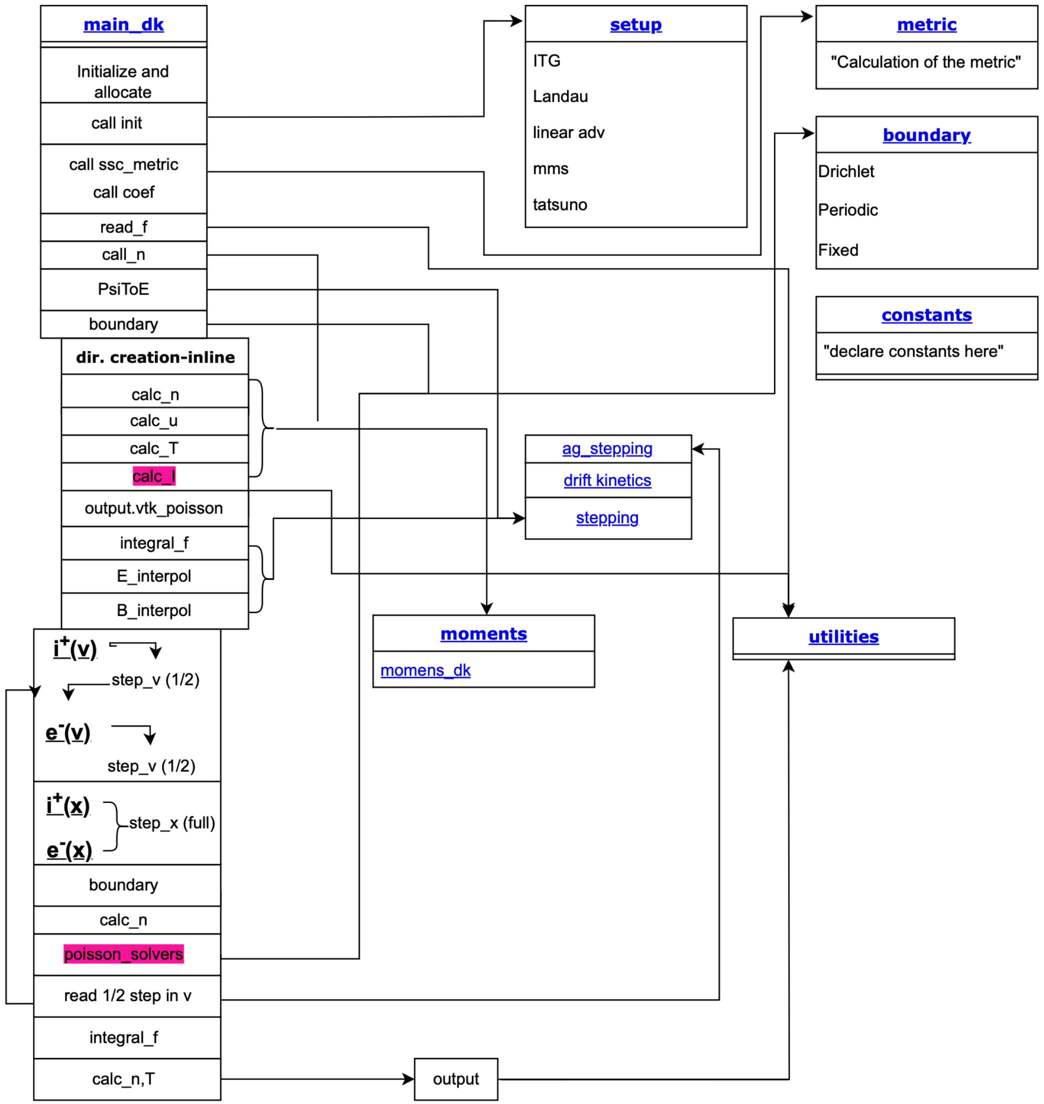
Our Poisson equations become then

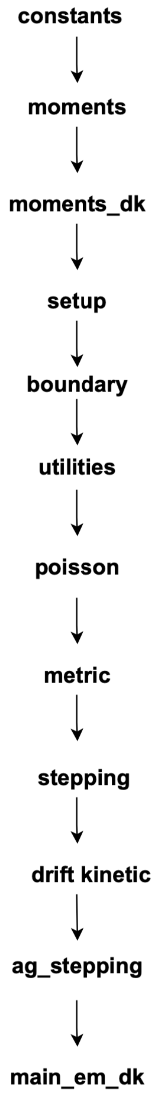
$$\frac{1}{4\pi}\nabla_\perp^2\phi_1\left(4\pi\frac{\rho_{th}^2}{\lambda_D^2}-1\right)+u_{e\parallel}\frac{\rho_{th}^2}{\lambda_D^2}\nabla_\perp^2 A_{1\parallel}(\mathbf{x})=\sum_i q_i n_i(\mathbf{x})-en_e(\mathbf{x})$$

and the parallel Ampere equation is written as

$$\frac{c}{4\pi}\nabla_\perp^2 A_{1\parallel}\left(1+\frac{\beta_e}{2}\right)+u_e\frac{\rho_e^2}{\lambda_D^2}\nabla_\perp^2\phi_1(\mathbf{x})=\frac{e^2}{m_e c}n_e A_{1\parallel}(\mathbf{x})-I_e+\sum_i I_{i\parallel}$$

Numerical Implementation - The Semi Lagrange Method





- The **semi-Lagrangian scheme uses Eulerian framework but the discrete equations come from the Lagrangian perspective**.

- In the Gyrokinetic context, and in the collisionless limit, **the values of the gyrocenter distribution functions on phase-space grid points at the next time step are computed by tracing back the respective characteristics to their positions at the present time step and using interpolations to obtain the required values.**

Numerical Implementation - Vlasov and Field Equations

$$\left(\frac{1}{4\pi}+\frac{\rho_{th,e}^2}{\lambda_{D,e}^2}\right)\Delta_\perp\varphi_1+u_{e\parallel}\frac{\rho_{th,e}^2}{\lambda_{D,e}^2}\Delta_\perp A_{1\parallel}=\sum_i q_i n_i-en_e$$

$$\frac{c}{4\pi}\left(\frac{\beta_e}{2}+1\right)\Delta_\perp A_{1\parallel}+u_{e\parallel}\frac{\rho_{th,e}^2}{\lambda_{D,e}^2}\Delta_\perp\varphi_1=\frac{e^2}{m_e c}n_e A_{1\parallel}+\sum_i I_{i\parallel}-I_{e\parallel}$$

Advection in real space

$$\frac{\partial f_i}{\partial t}+\boldsymbol{v}\cdot\boldsymbol{\nabla}f_i+\frac{q_i}{m_i}(-\nabla\varphi_1+\boldsymbol{v}\times\boldsymbol{B})\cdot\boldsymbol{\nabla}_{\boldsymbol{v}}f_i=0$$

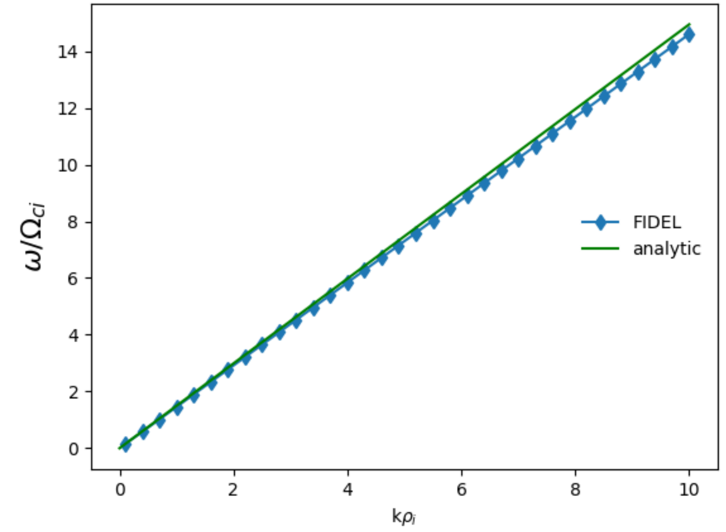
Advection in velocity space + Lorentz

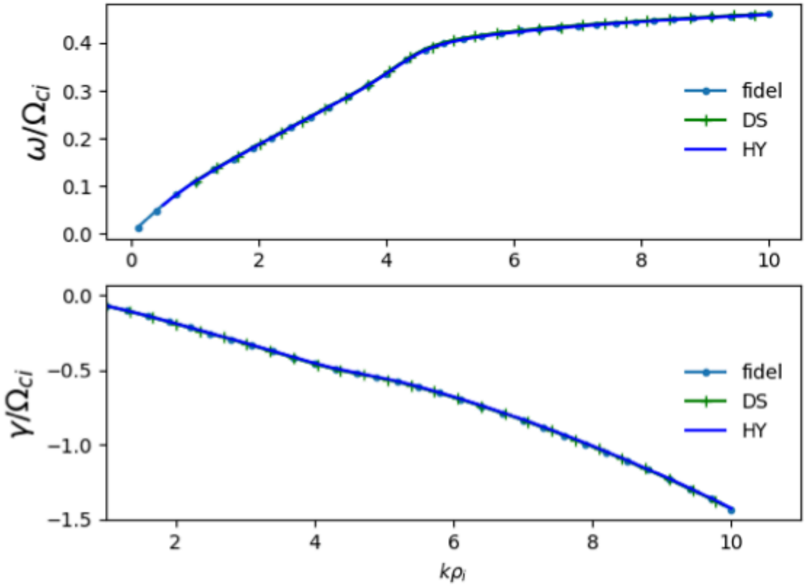
Time dependent — Parallel Advection — Perpendicular drift

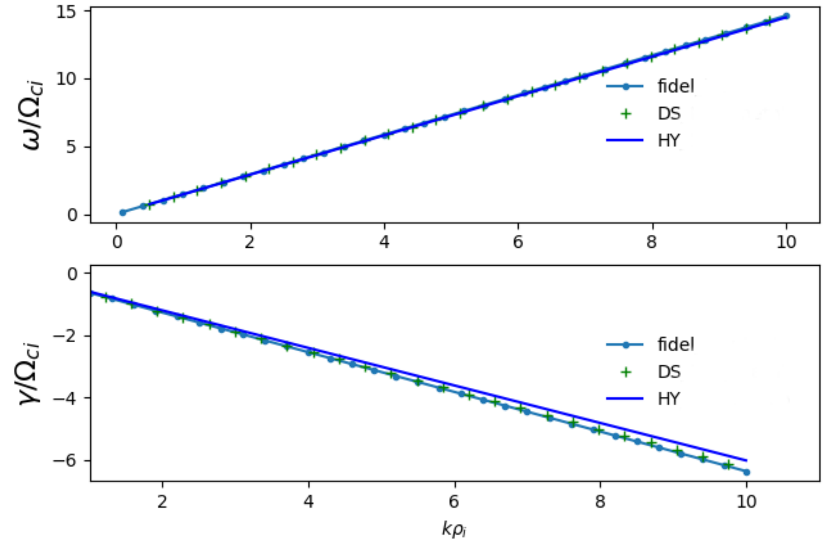
$$\frac{\partial f_e}{\partial t}+\frac{\mathbf{B}^*}{m\mathbf{B}_\parallel^*}\cdot\left(mv_{gy,\parallel}+\frac{e}{c}\langle\mathbf{A}_{1\parallel}\rangle\right)\nabla_{gy}f_e+\frac{c\hat{b}}{e\mathbf{B}_\parallel^*}\times\left(\mu_{gy}\nabla_{gy}B(\mathbf{X}_{gy})+e\nabla\langle\varphi_1\rangle-\frac{e}{c}v_{gy,\parallel}\langle\nabla\mathbf{A}_{1\parallel}\rangle\right)\nabla_{gy}f_e$$
$$-\frac{\mathbf{B}^*}{m\mathbf{B}_\parallel^*}\cdot\left(\mu_{gy}\nabla_{gy}B(\mathbf{X}_{gy})+e\nabla\langle\varphi_1\rangle-\frac{e}{c}v_{gy,\parallel}\langle\nabla\mathbf{A}_{1\parallel}\rangle\right)\frac{\partial f_e}{\partial v_{gy,\parallel}}=0$$

Acceleration

Linear Analysis - Electrostatic







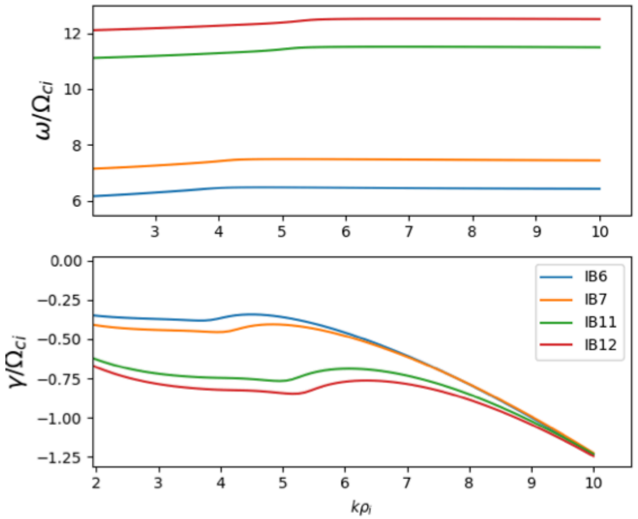
IAW is a longitudinal oscillation of charged particles similar to acoustic waves on natural gas. Such waves can be damped due to Coulomb collisions or Landau damping.

arXiv:2302.05473v1

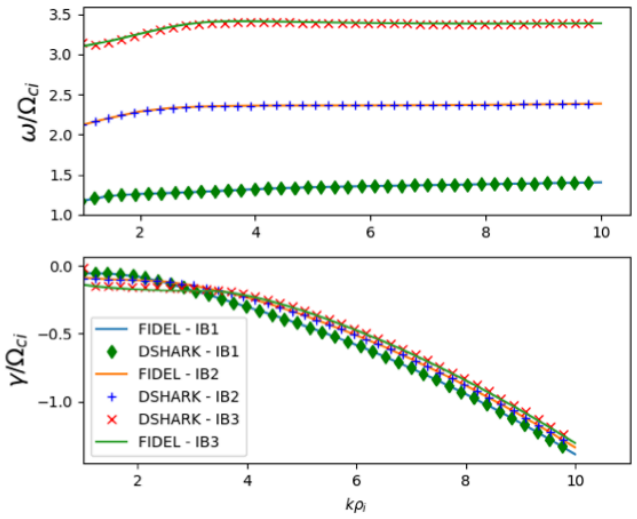
Linear Analysis - Electrostatic

IBW

Higher frequency

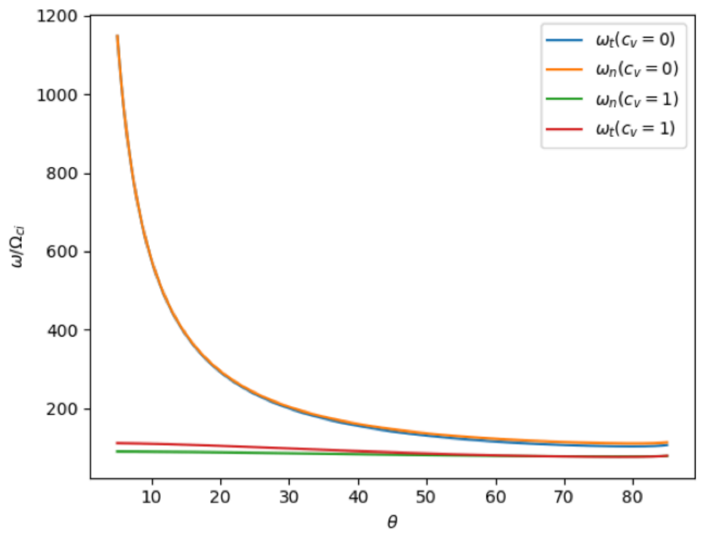


Comparison



IBW are slowly propagating, longitudinal hot plasma waves. Such waves interact with kinetic Alfven waves, and are also subject to strong localised electron Landau damping.

Linear Analysis - Electrostatic

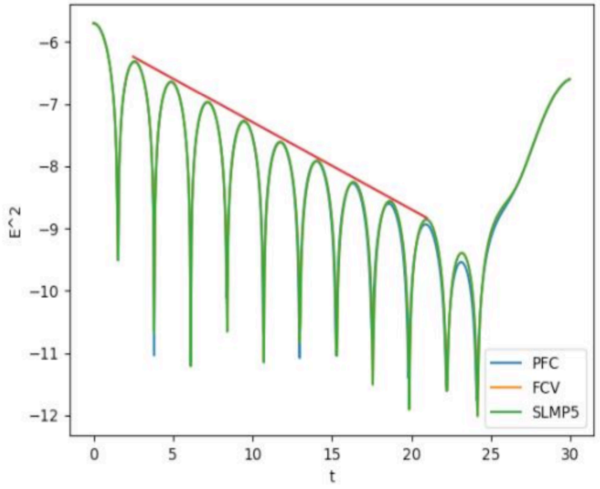


“High frequency waves” a.k.a. Lower Hybrid Drift Waves

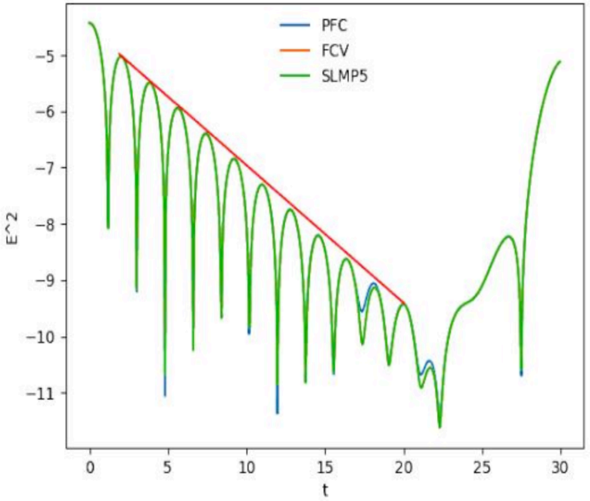
One of the main issues with the use of gyrokinetic theory on space plasma physics is its limitations regarding high frequency physics. Here we demonstrated that our model is capable of simulating high frequency waves on the range of the plasma electron frequency, which is orders of magnitude higher than the plasma ion cyclotron frequency.

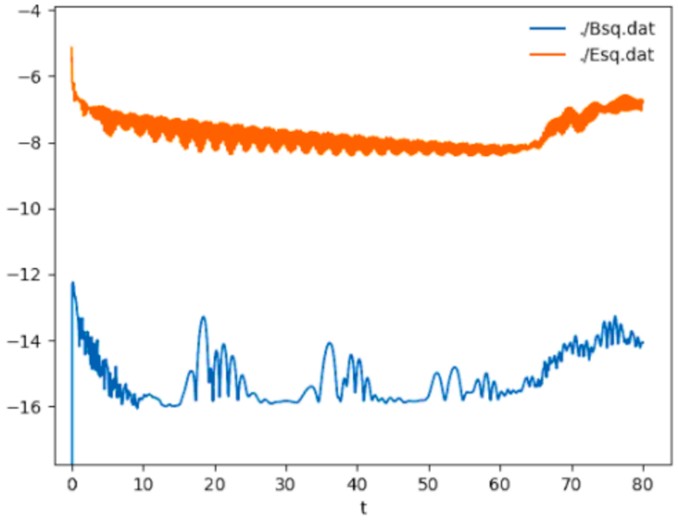
Nonlinear Analysis - Electrostatic

1D Linear landau setup:



2D Linear landau setup:





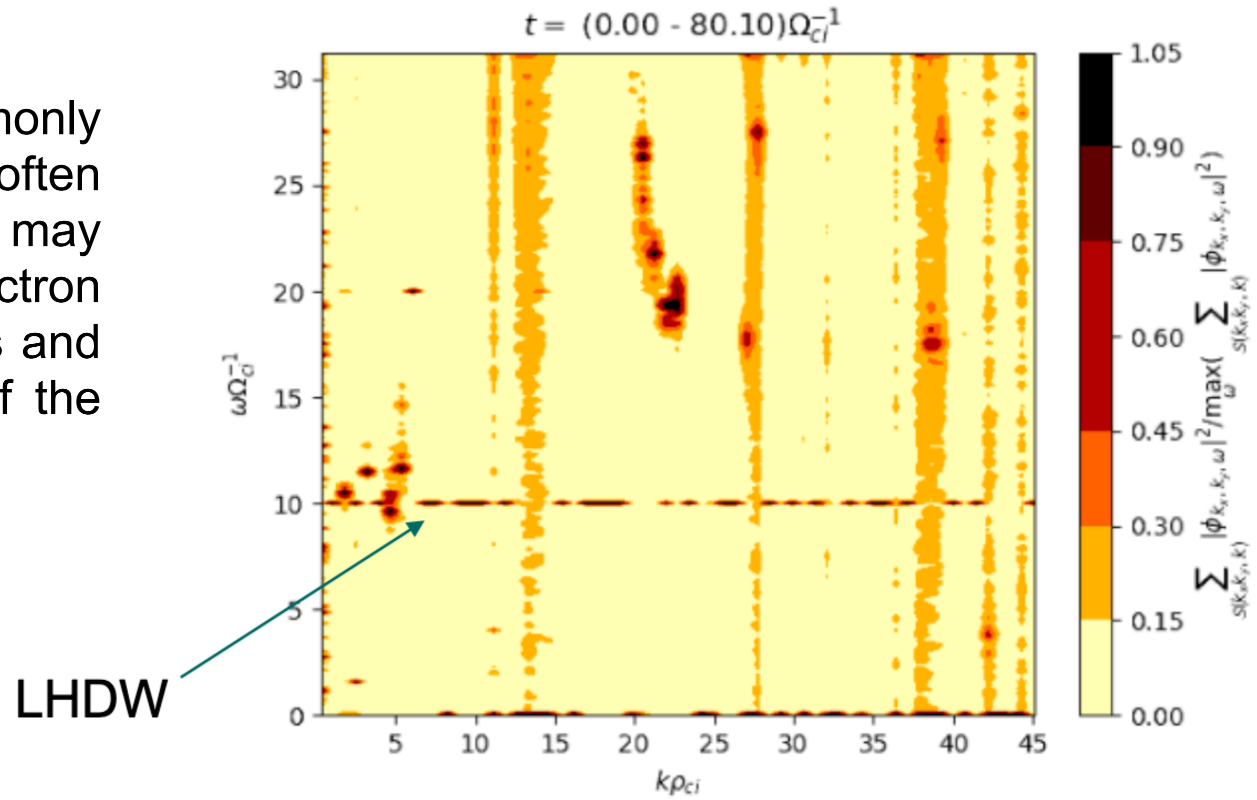
Square of amplitude of EM fields for IBW setup

Squared amplitude of field energy for Landau setup. The theoretical damping rate in both cases come from [A.Myers 2016]. EM Landau slope found to be around 0.17.

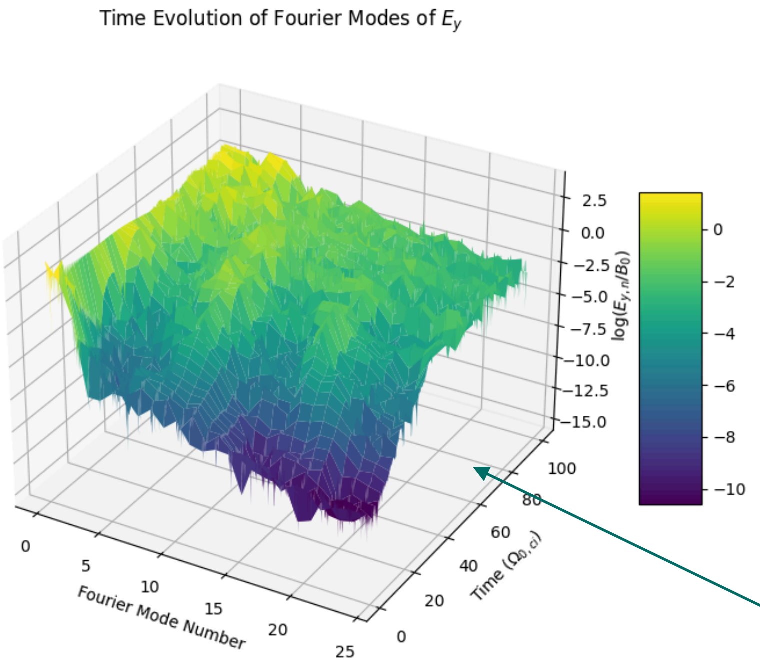
Nonlinear Analysis - Electrostatic

The lower hybrid drift waves (LHDWs) are commonly observed at plasma boundaries, where they often account for one of the strongest electric fields and may result in anomalous diffusion and resistivity and electron acceleration.The LHDWs are electron scale waves and therefore detailed experimental characterisation of the properties present a challenging task.

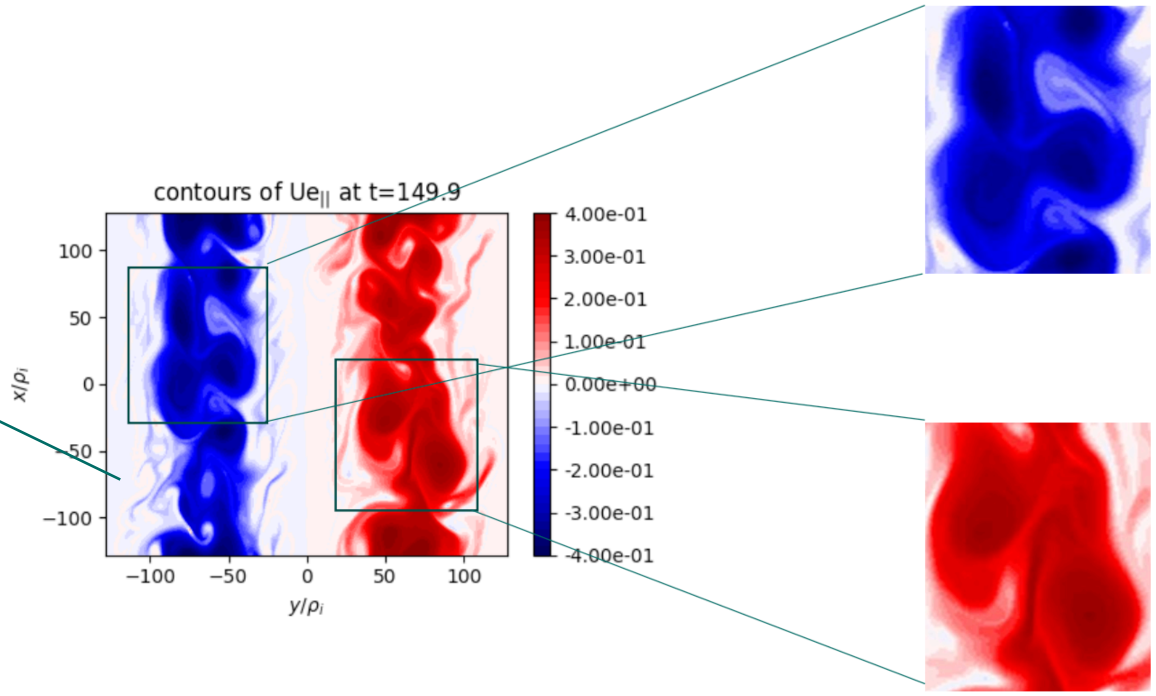
10.1103/PhysRevLett.109.055001



Nonlinear Analysis - Electromagnetic (ongoing)

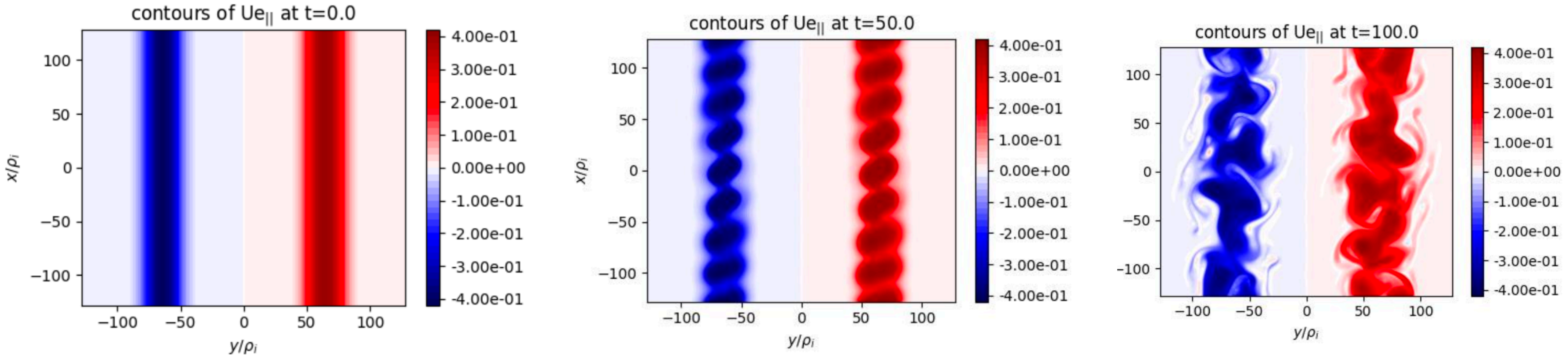


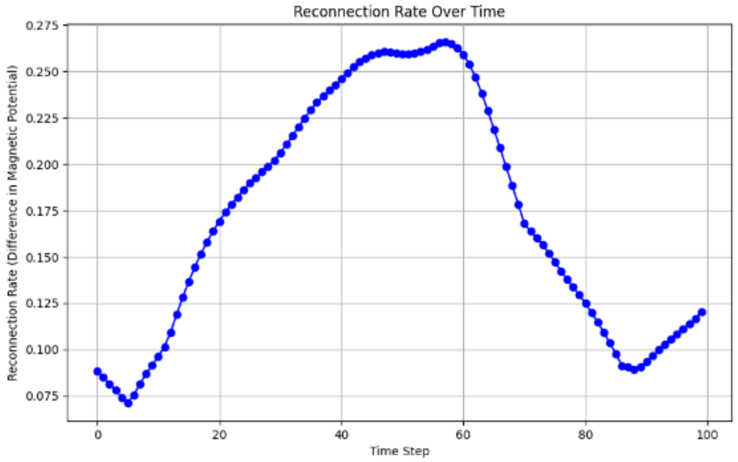
Relationship between density gradients and the onset of LHDWs, and investigation of resonant mechanism of nonlinear triggering of global instability of thin current sheets.



What is the role of mass ratio on the onset of instabilities?

Nonlinear Analysis - Electromagnetic (ongoing)





How does that value compare to the literature and theoretical values?

Summary

1. **Derivation of a hybrid model using a variational method for field equations and symplectic geometry for the fully kinetic gyro-kinetic Vlasov equations.**

2. **Numerical scheme using a semi-Lagrangian method avoids the drawbacks of a PIC implementation.**

3. **Linear analysis demonstrates the existence of waves of interest for desired frequency range.**

4. **Hybrid model could work as a viable alternative for investigating >>\Omega_{ci} phenomena, i.e. closer to the dissipation range.**

Thank you