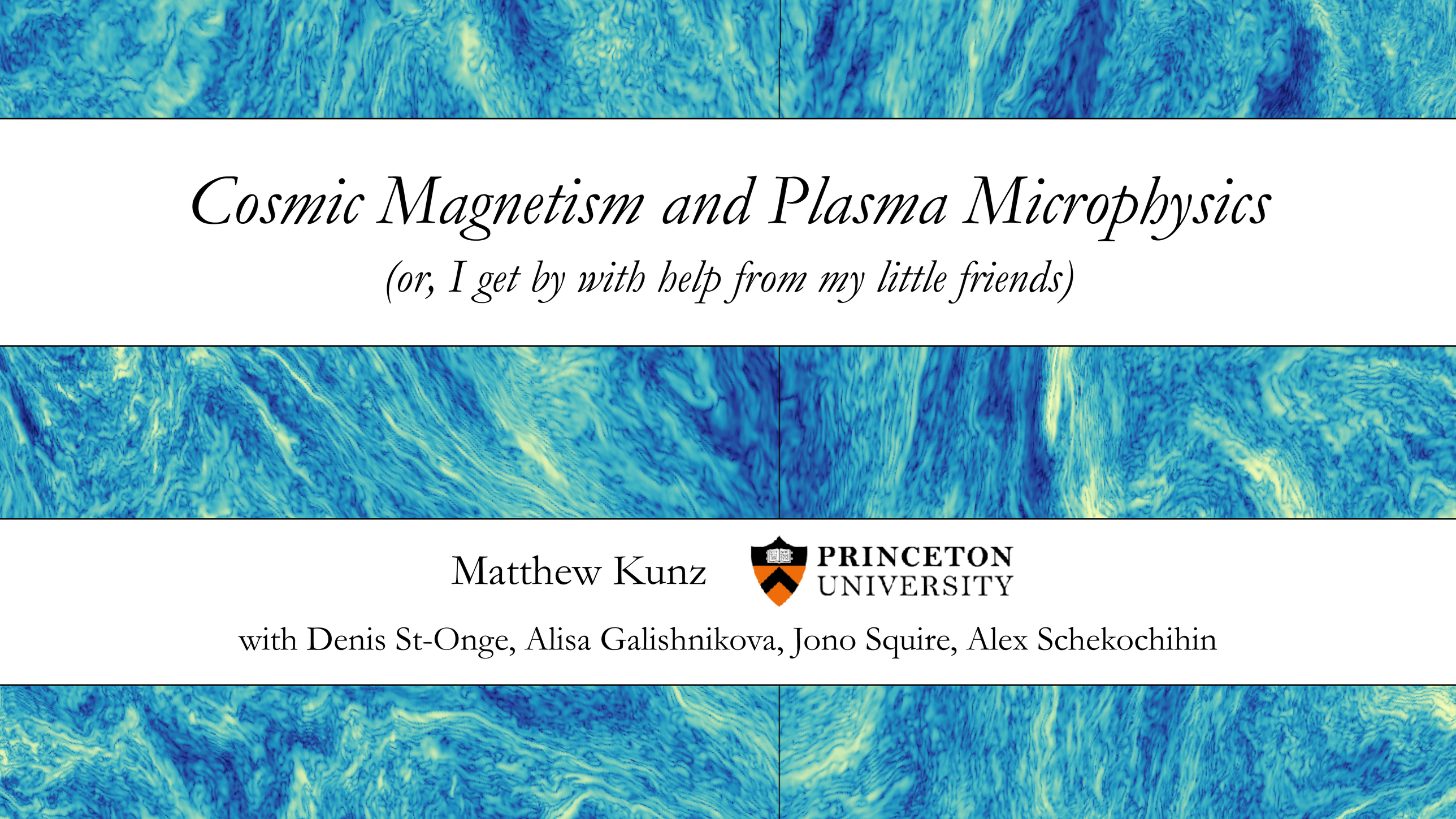


Cosmic Magnetism and Plasma Microphysics

(or, I get by with help from my little friends)

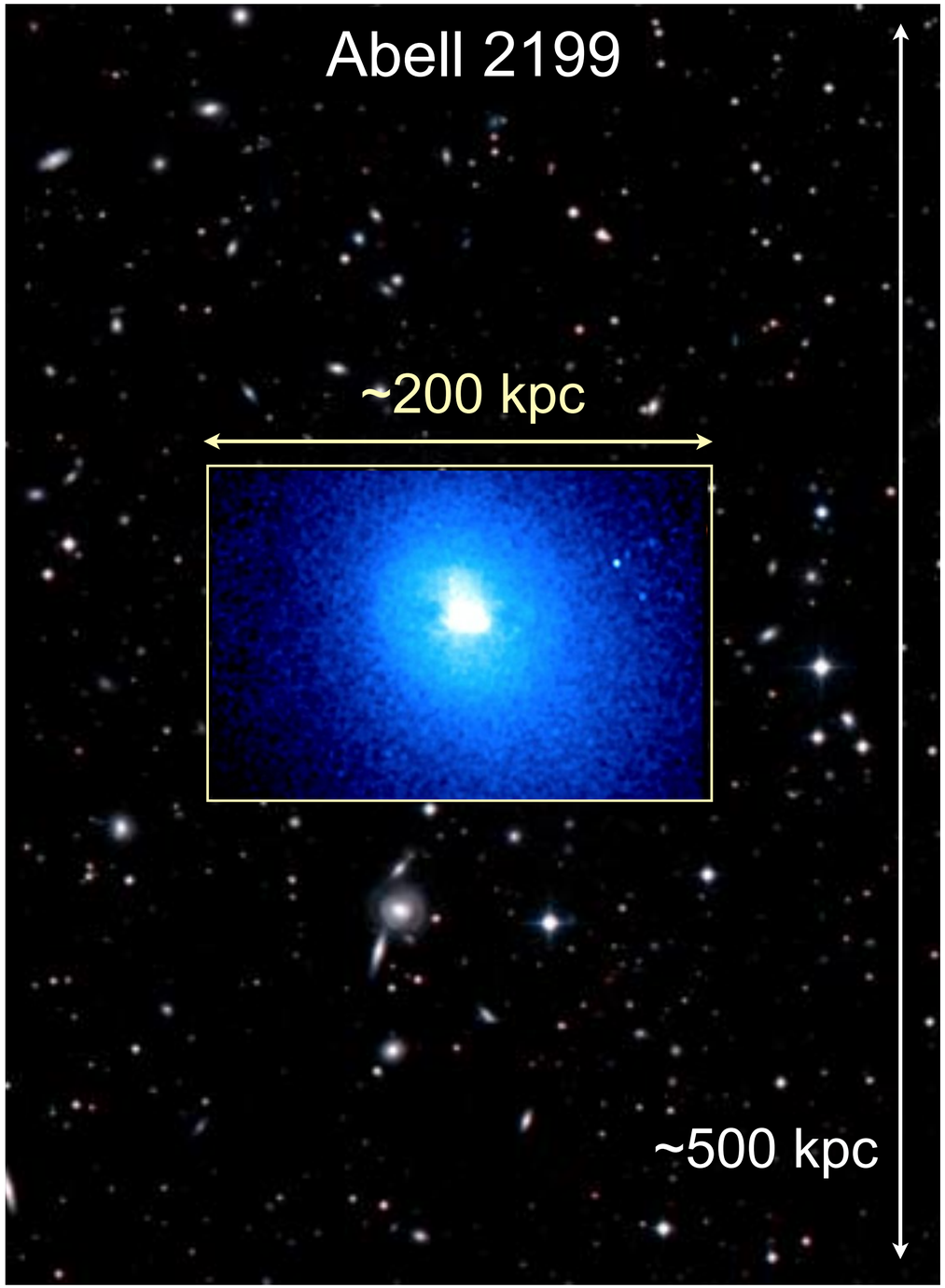
Matthew Kunz

PRINCETON UNIVERSITY

with Denis St-Onge, Alisa Galishnikova, Jono Squire, Alex Schekochihin

Outline

- Observational motivation:
  Why are intracluster magnetic fields at ~μG level and ~kpc scale?
- Fluctuation dynamo and plasma microphysics:
  Kinetic instabilities, reduced viscosity, and a fast dynamo
- Selected results from program of theory and simulations
  Featuring work with St-Onge, Galishnikova, Squire & Schekochihin
- (if time remains…) The broader challenge:
  Material properties of weakly collisional, high-beta plasmas



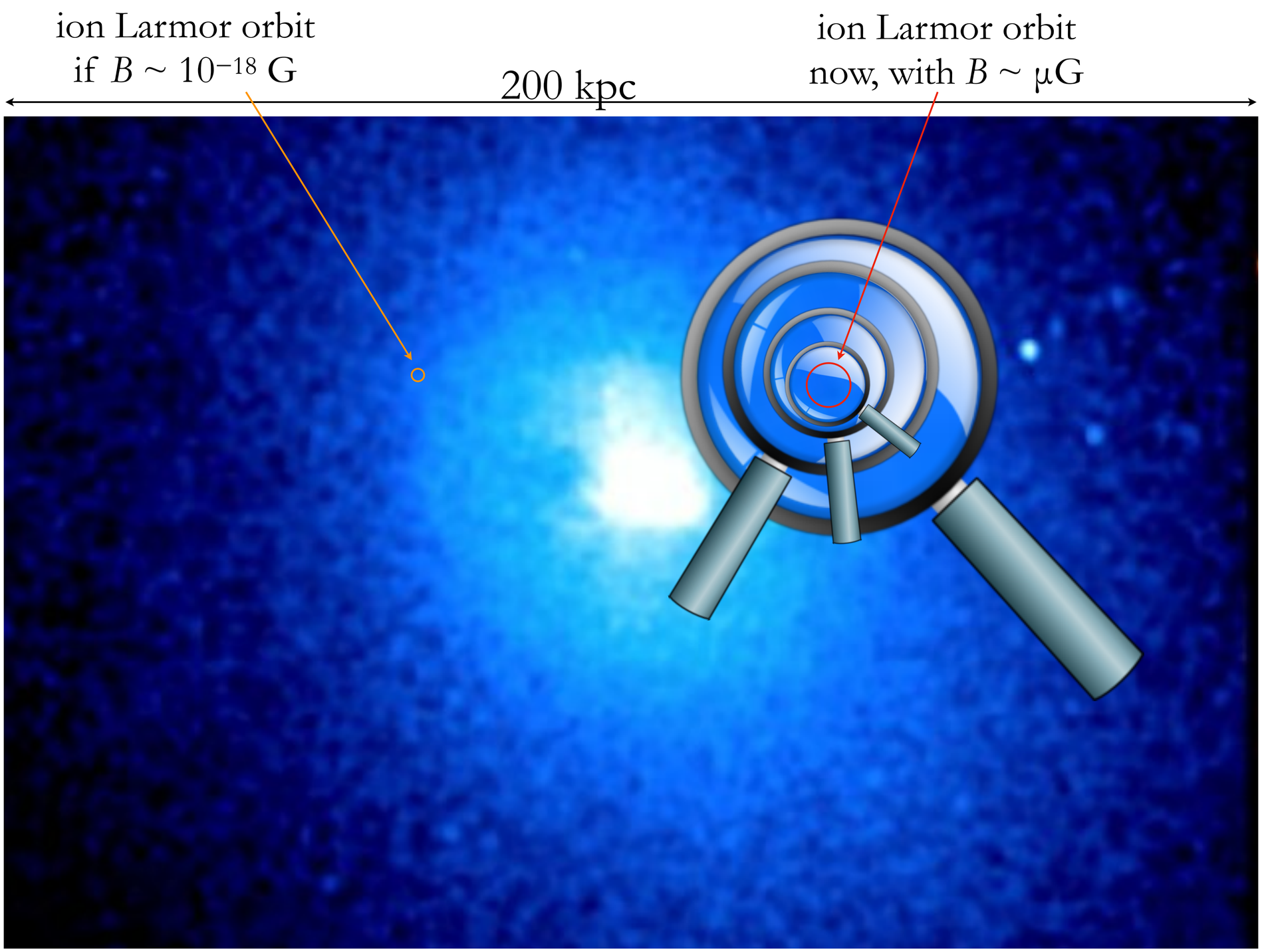
Galaxy Clusters: $\sim 10^{14-15}\ M_\odot$ in ~ 1 Mpc

intracluster medium (ICM)

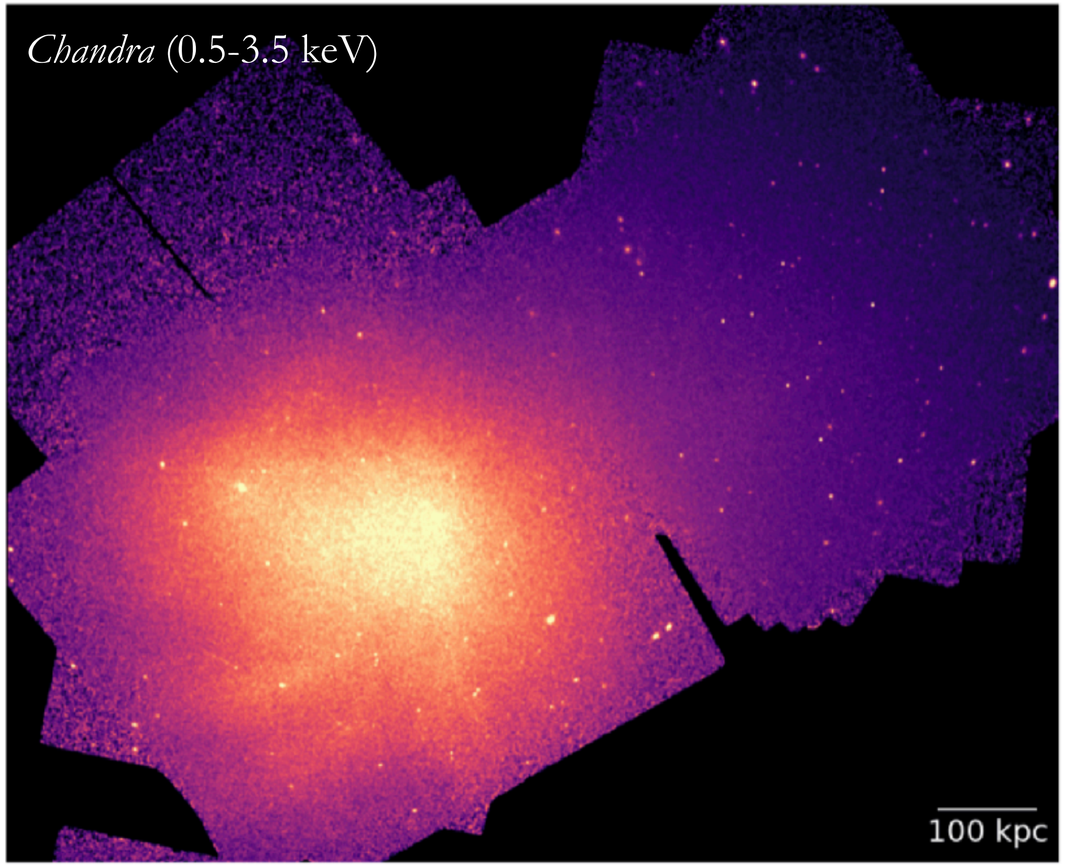
14 % thermal plasma

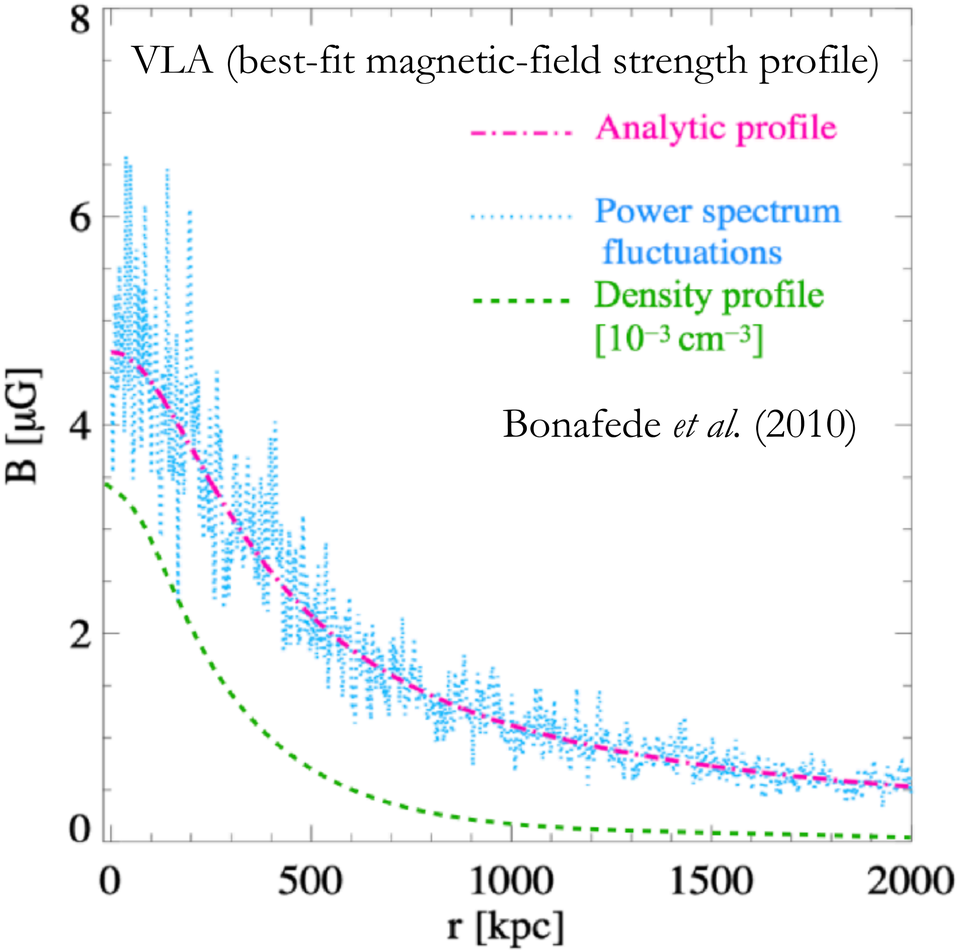
$T \sim 1 - 10$ keV

$n \sim 10^{-4} - 10^{-1}\ \mathrm{cm}^{-3}$


ion Larmor orbit
if $B \sim 10^{-18}$ G
200 kpc
ion Larmor orbit
now, with $B \sim \mu$G

Example (Coma cluster) using Faraday rotation measurements:





Similar strengths (~1 – 10 μG) on similar scales (~1 – 10 kpc) inferred from RMs in other clusters

(e.g., Murgia *et al.* 2004; Vogt & Enßlin 2005; Govoni *et al.* 2017)

How to make ~μG intracluster magnetic fields?

$$\rho_i \sim \left(\frac{T}{1\ \mathrm{keV}}\right)^{1/2} \left(\frac{B}{10^{-18}\ \mathrm{G}}\right)^{-1} \mathrm{kpc} \qquad \beta = \frac{nT}{B^2/8\pi} \sim 10^{26}$$

⇩ ?

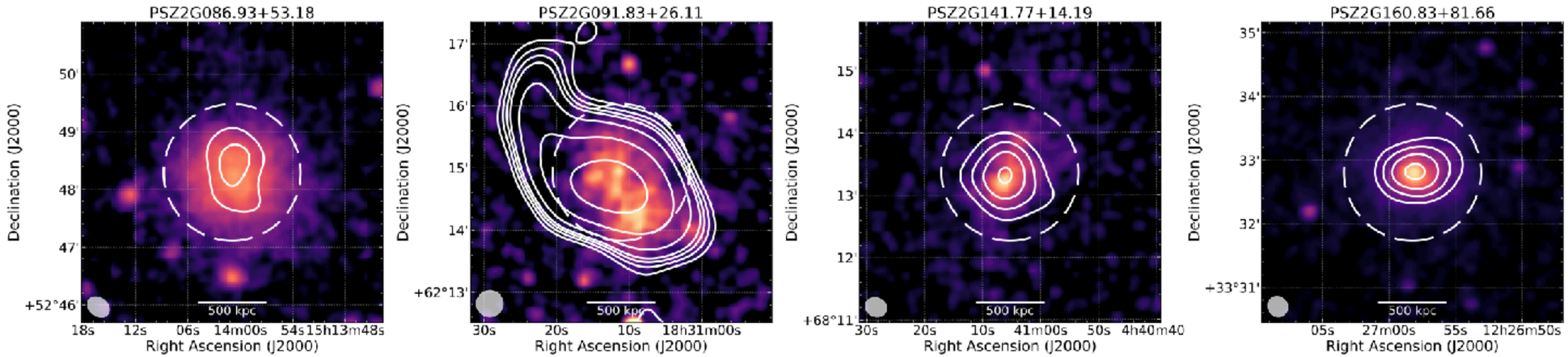
$$\rho_i \sim \left(\frac{T}{1\ \mathrm{keV}}\right)^{1/2} \left(\frac{B}{10^{-6}\ \mathrm{G}}\right)^{-1} \mathrm{npc} \qquad \beta = \frac{nT}{B^2/8\pi} \sim 10^{2}$$

(~ size of Jupiter)

…and in a cosmologically short time?

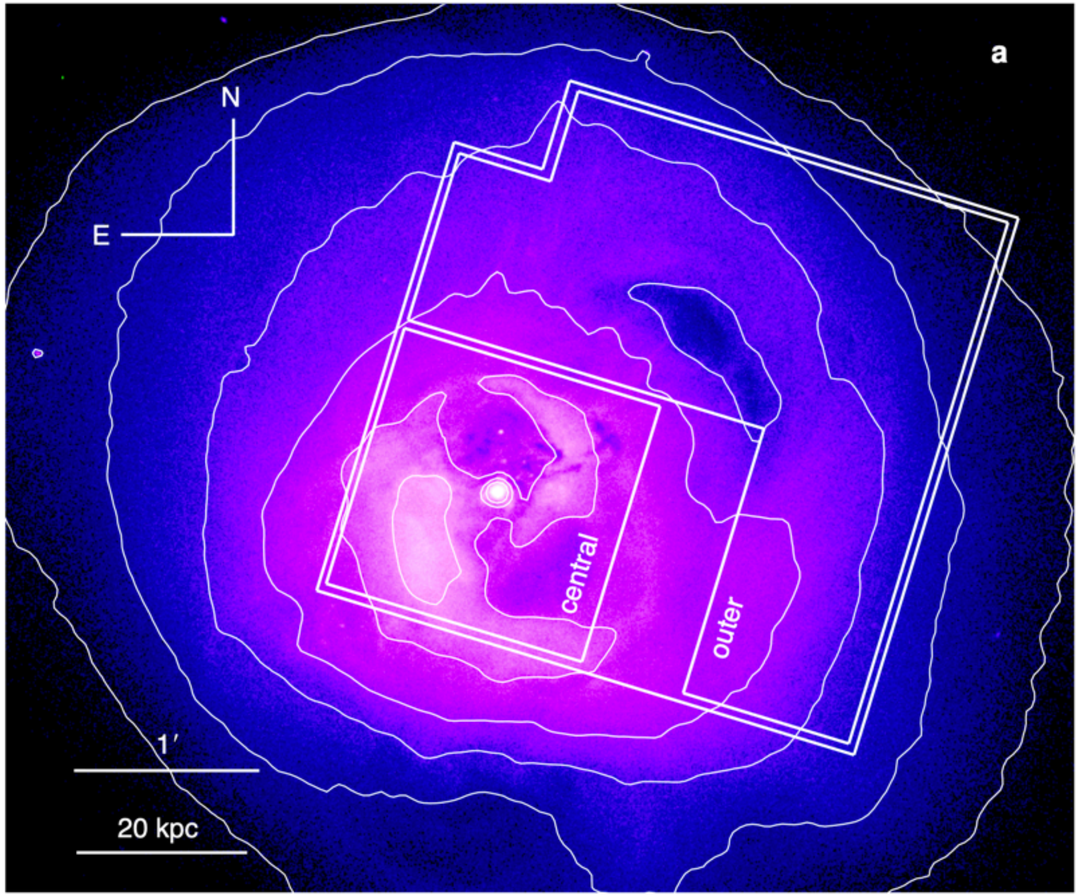
Di Gennaro *et al.* (2020, *Nature*):

"The high radio luminosities indicate that these clusters [detected by LOFAR via diffuse radio emission at $z \sim 0.7$] have similar magnetic field strengths to those in nearby clusters, and suggest that magnetic field amplification is fast during the first phases of cluster formation."



Hints from $z = 0$:

Hitomi, before its death (2016):
$u = 164 \pm 10$ km/s
in Perseus at ~50 kpc
($M \lesssim 0.2$)



typical from
RM maps

$$v_A = 77 \left(\frac{B}{5\ \mu\text{G}}\right)\left(\frac{n}{0.02\ \text{cm}^{-3}}\right)^{-1/2}\ \text{km s}^{-1}$$

likely not a coincidence that $M_A \doteq u/v_A \sim$ few
($B \propto n^{1/2}$ inferred in Coma: Bonafede *et al.* 2010)

it is then natural to attribute intracluster magnetic field to

the **fluctuation ("turbulent") dynamo**

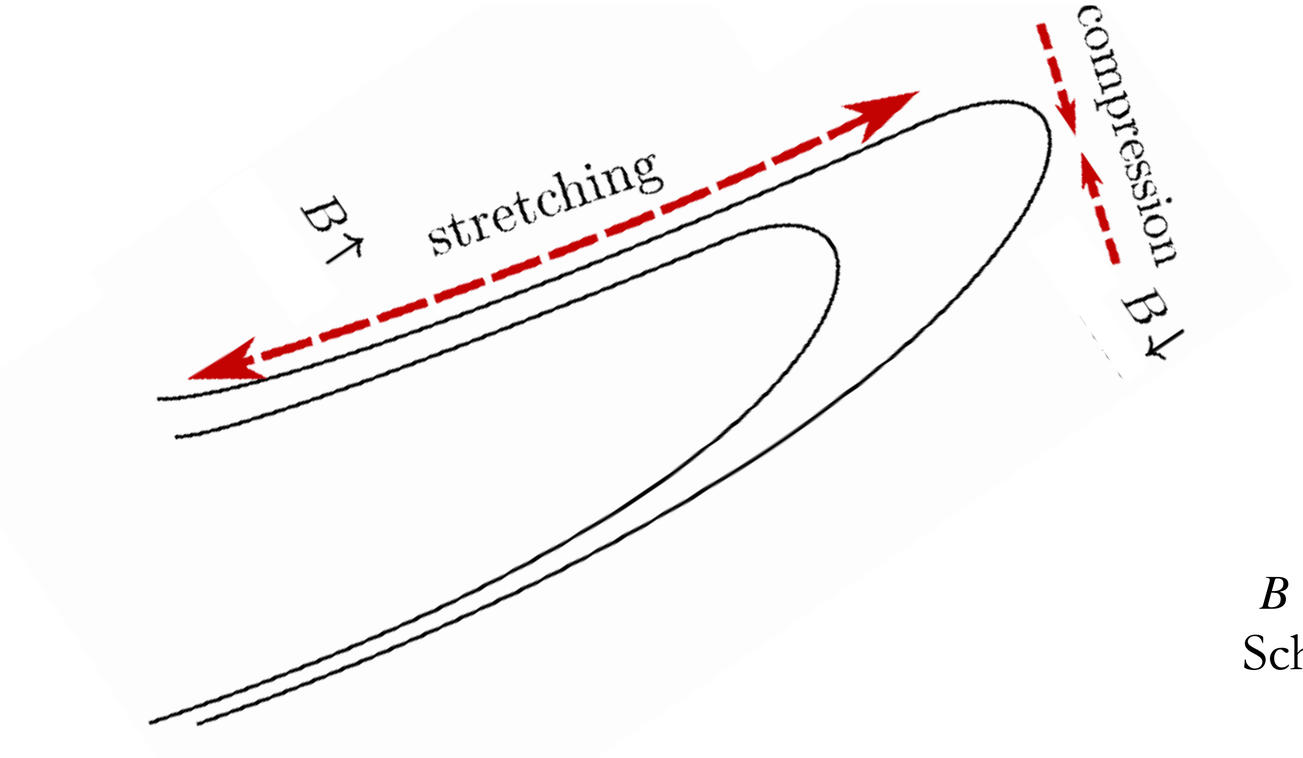
(Batchelor 1950; Kazantsev 1967; Zel'dovich et al. 1984; Childress & Gilbert 1995),

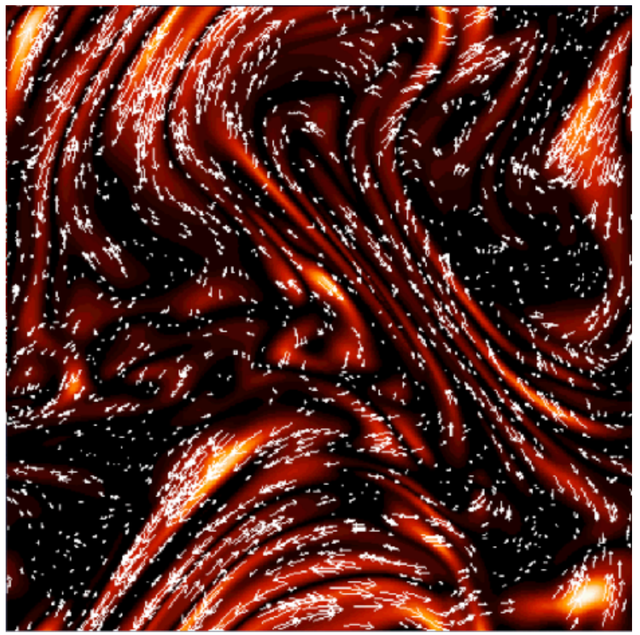
whereby a succession of random velocity shears stretches the field

and leads on the average to its growth to dynamical strengths.

$$\frac{\mathrm{d}\ln B}{\mathrm{d}t} = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}{:}\boldsymbol{\nabla}\boldsymbol{u}$$

magnetic energy grows in a 3D, smooth, chaotic velocity field





B from Re ~ 1, Pm ≫ 1
Schekochihin *et al.* (2004)

material properties of the plasma matter

$$\mathrm{Re} \doteq \frac{UL}{\nu} \approx 2 \left(\frac{U}{200\ \mathrm{km\ s^{-1}}} \right) \left(\frac{L}{100\ \mathrm{kpc}} \right) \left(\frac{T}{10^8\ \mathrm{K}} \right)^{-5/2} \left(\frac{n}{10^{-3}\ \mathrm{cm^{-3}}} \right)$$

$$\mathrm{Pm} \doteq \frac{\nu}{\eta} \approx 2 \times 10^{29} \left(\frac{T}{10^8\ \mathrm{K}} \right)^{4} \left(\frac{n}{10^{-3}\ \mathrm{cm^{-3}}} \right)^{-1}$$

viscous scale, where $\mathbf{u} \cdot \nabla \mathbf{u} \sim \nu \nabla^2 \mathbf{u}$: $\ell_\nu \sim L\,\mathrm{Re}^{-3/4}$

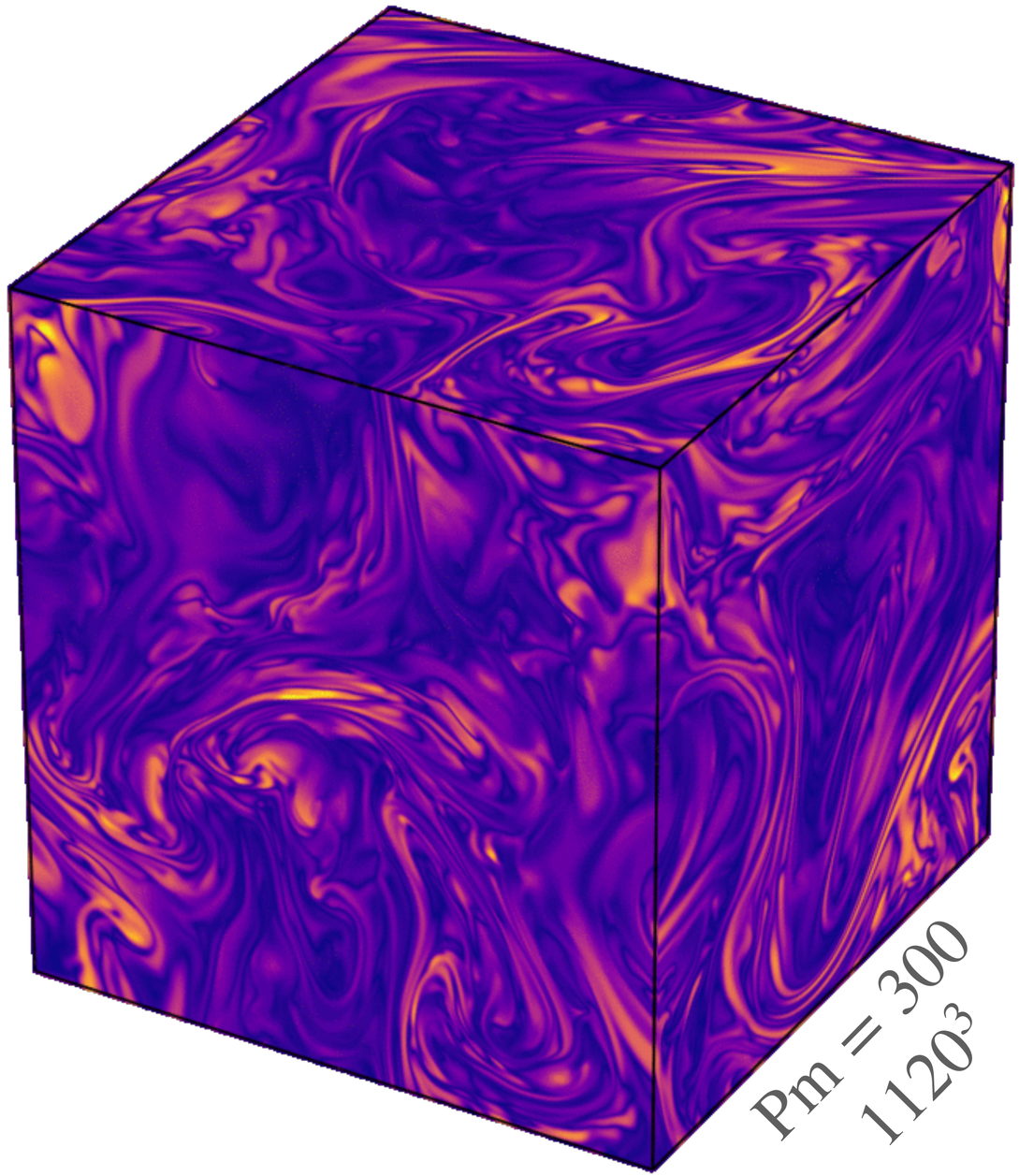
resistive scale, where $\mathbf{B} \cdot \nabla \mathbf{u} \sim \eta \nabla^2 \mathbf{B}$: $\ell_\eta \sim \ell_\nu\,\mathrm{Pm}^{-1/2} \ll \ell_\nu$

magnetic field arranged in folds

% A. Galishnikova

if magnetic field dynamically weak, then

$$\frac{\mathrm{d}\ln B}{\mathrm{d}t} \sim \frac{U}{L}\,\mathrm{Re}^{1/2}$$



Seeds?

1. Primordial or pre-galactic
   (e.g., Kulsrud *et al.* 1997: Biermann + fluctuation dynamo;
   see also Zhou, Zhdankin, Kunz, Loureiro, Uzdensky (2022, *PRX*) on Weibel instability → fluctuation dynamo)

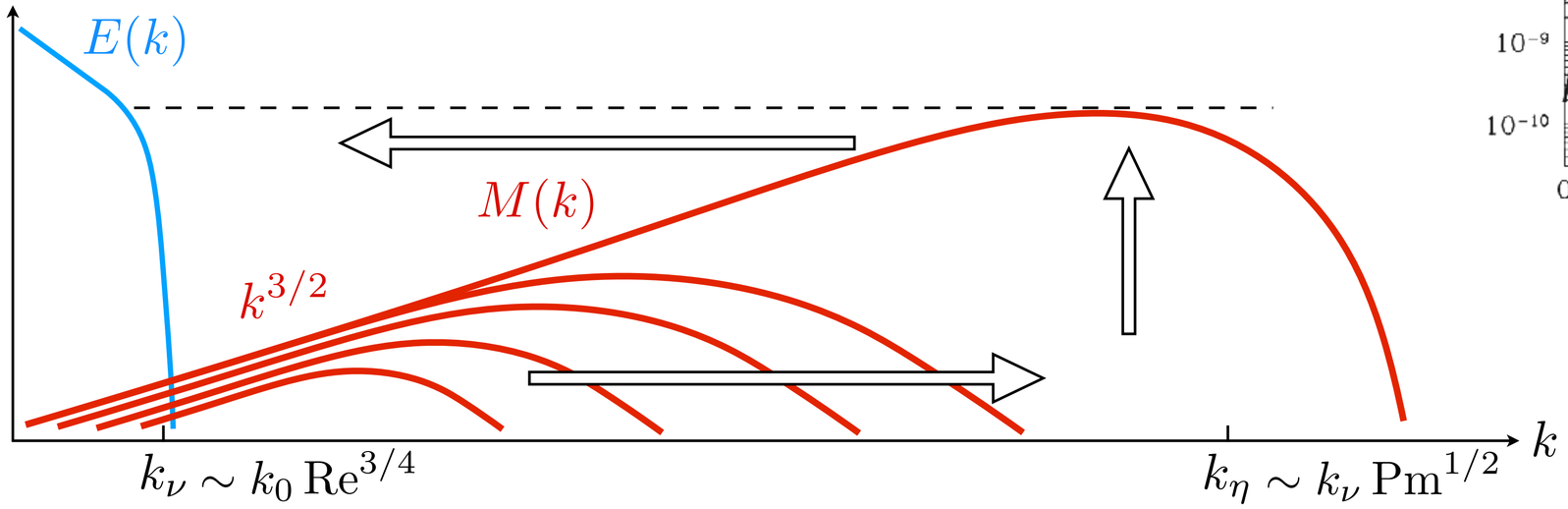
2. Galactic + dispersion & dilution throughout cluster volume
   (e.g., Rees & Setti 1968; Rees 1987; Rephaeli 1988; Furlanetto & Loeb 2001)

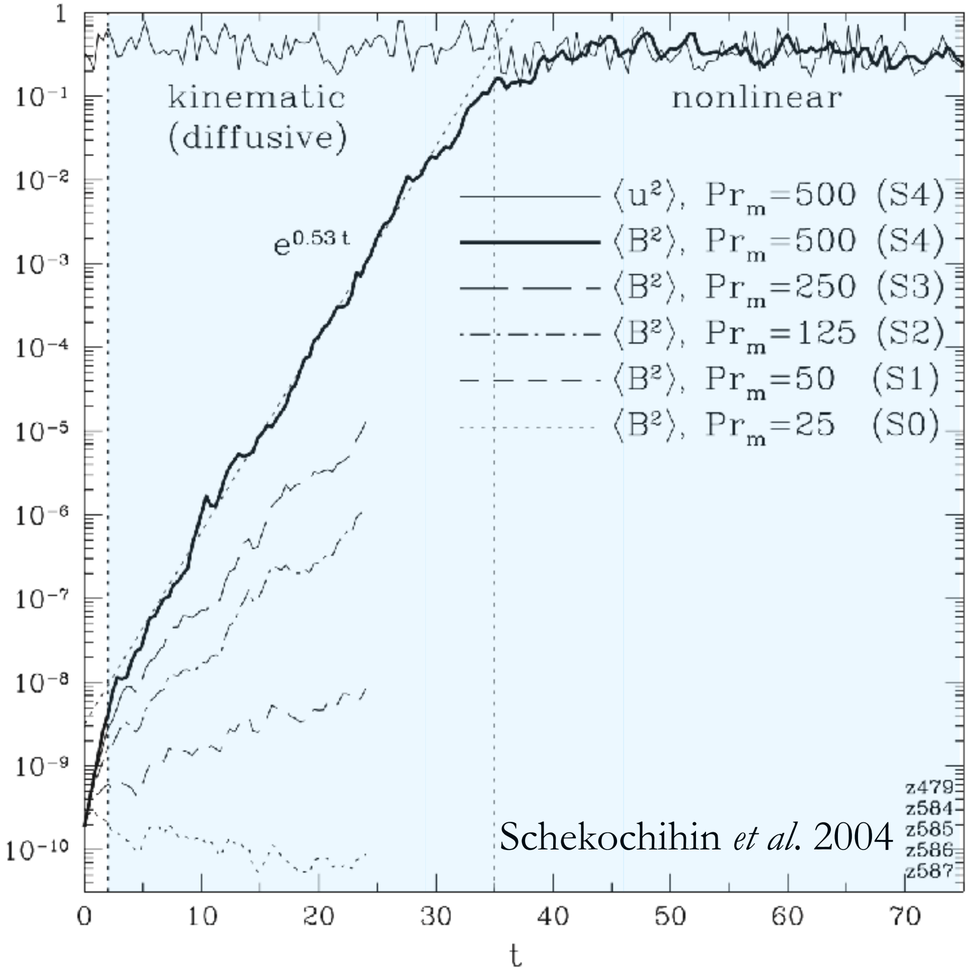
   supported indirectly by observed early metal enrichment of clusters
   (e.g., Mantz *et al.* 2020: $M \sim 2 \times 10^{14}$ $M_\odot$ at $z \simeq 1.7$ with $\sim 1/3$ Solar metallicity)

in either case, turbulent amplification
and maintenance of fields needed

MHD fluctuation dynamo evolution at Pm ≫ 1

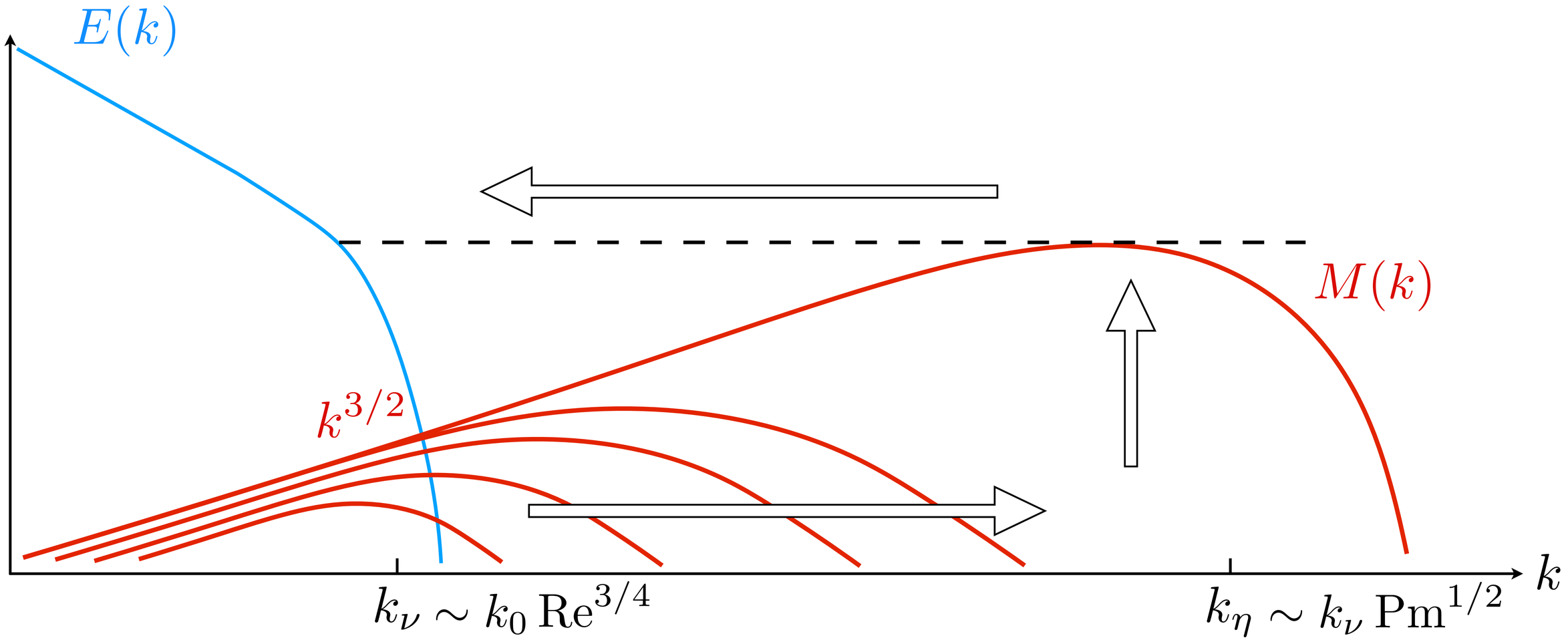
1) kinematic
   no feedback from B on u; exponential growth
   Kazantsev $k^{3/2}$ spectrum, peaking at k_η
   development of folded structure
2) nonlinear
   tension affects viscous-scale eddies:
   $\boldsymbol{B}\cdot\boldsymbol{\nabla}\boldsymbol{B} \sim \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} \sim u_\nu^2/\ell_\nu$
   slower, larger-scale eddies take over stretching
   secular growth $\langle B^2\rangle$ (Sch02&04, Cho+09, Beresnyak12)
3) saturation at $\langle B^2\rangle \sim \langle u^2\rangle$
   not scale-by-scale! suppression of $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}{:}\boldsymbol{\nabla}\boldsymbol{u}$





Pm ≐ viscosity/resistivity = Rm/Re

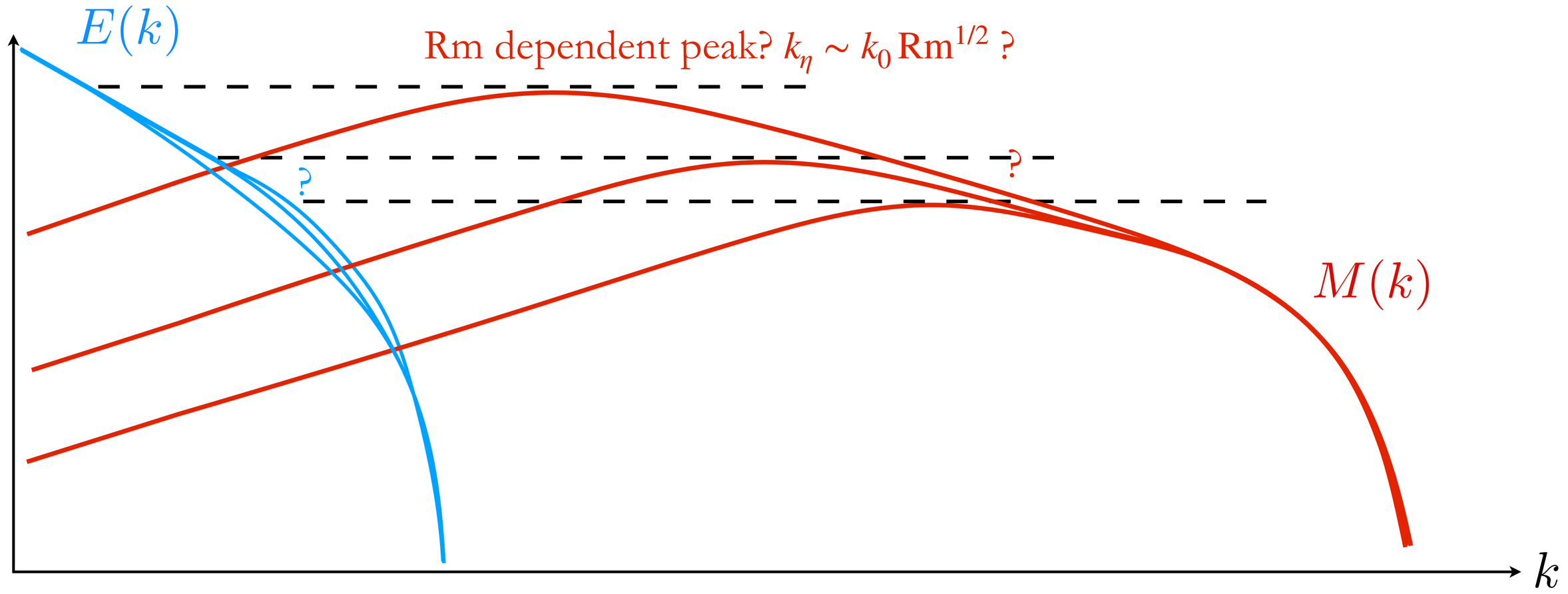
stir incompressible, subsonic turbulence at box scale…



$$\mathrm{Re} = \frac{UL}{\nu}$$

$\mathrm{Pm} \doteq \text{viscosity/resistivity} = \mathrm{Rm}/\mathrm{Re}$

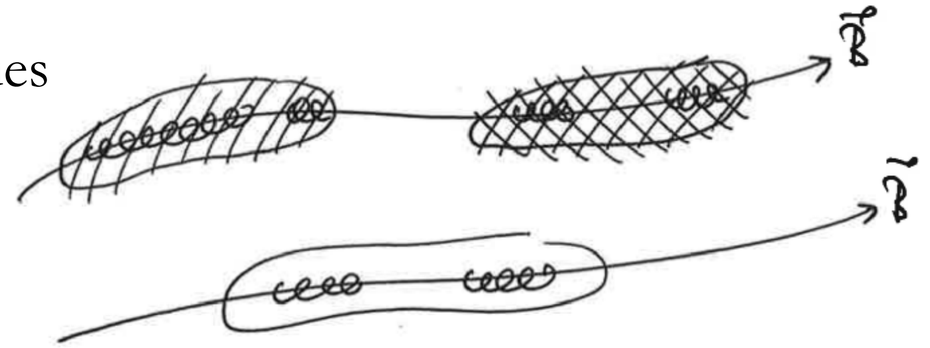
stir incompressible, subsonic turbulence at box scale…



complicating issues…

- ICM is **well magnetized**, even at $\beta \sim 10^{22}$; implies that viscous transport is anisotropic:

$$\frac{\lambda_{\rm mfp}}{\rho_{\rm i}} \sim 0.1 \left(\frac{\rm Pm}{\beta_{\rm i}}\right)^{1/2}$$ using "Spitzer" (collisional) values



$$\Downarrow$$

(M is Mach number; $\mathrm{Pm} \equiv \nu/\eta$)

$$\mathrm{Re}_{\parallel} \doteq M\frac{L}{\lambda_{\rm mfp}} \ll \mathrm{Re}_{\perp} \sim M\frac{L}{\rho_{\rm i}}$$ "Braginskii-MHD"

- ICM is **weakly collisional**, i.e., not rigorously a fluid on all but the largest scales
  (e.g., $\mathrm{Re}_{\parallel} \sim 1 - 10$ from Coulomb collisions in ICM)

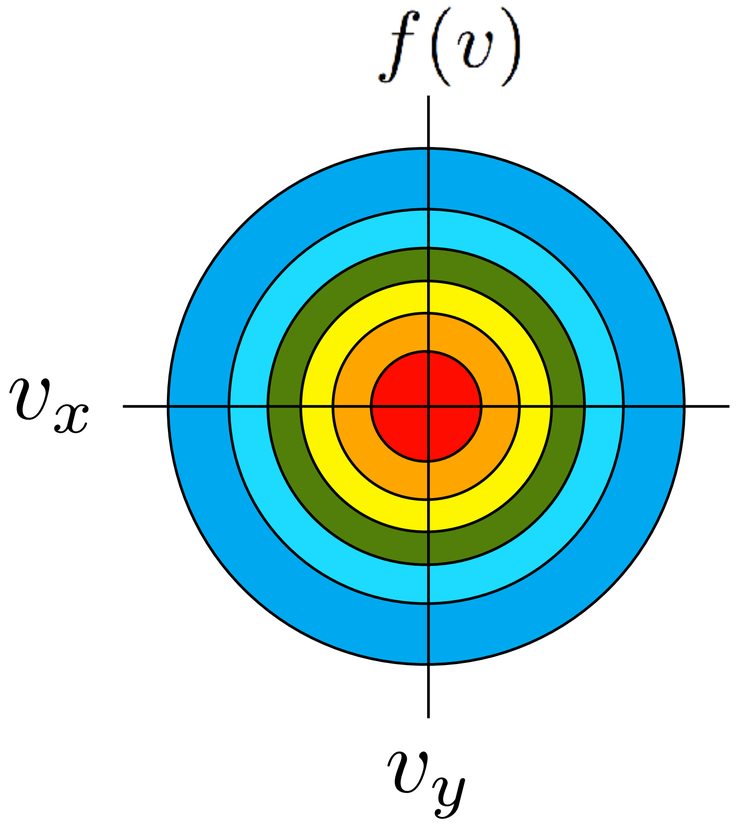
  Departures from LTE: $|p_{\perp}/p_{\parallel} - 1| \sim M^2\,\mathrm{Re}_{\parallel}^{-1/2} \sim 0.01 - 0.1$ (will return to this shortly…)

- Dynamo is slow if such departures aren't regulated (e.g., $|\mathrm{d}\ln B/\mathrm{d}t|^{-1} \sim 0.5$ Gyr in ICM)
  (and it's impossible in a plasma that conserves adiabatic invariants; Helander *et al.* 2016)

let’s pause here for a
plasma physics primer…

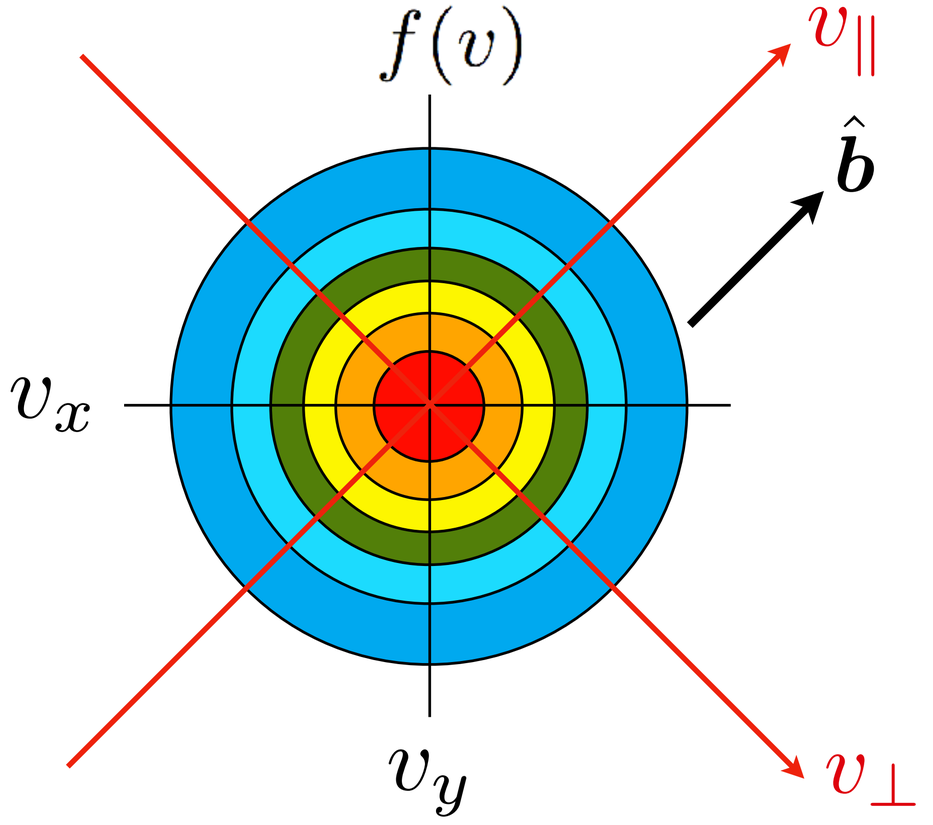
Suppose I have an isotropic, Maxwellian velocity distribution function of the plasma particles:

$$f(v) \propto \exp\left(-\frac{v^2}{v_{\rm th}^2}\right)$$

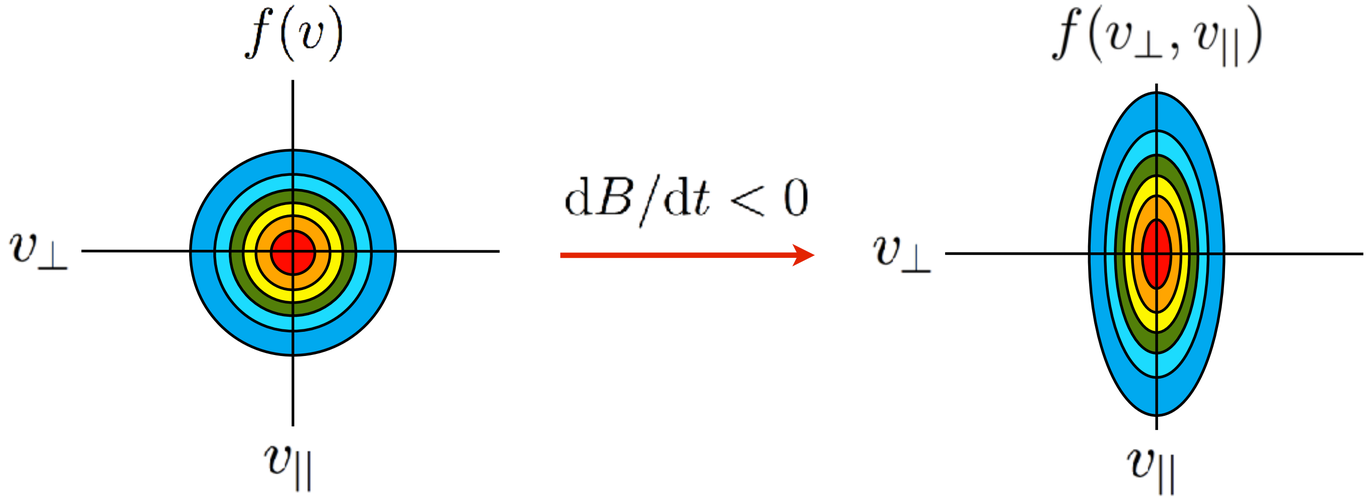


now thread the plasma with a magnetic field:

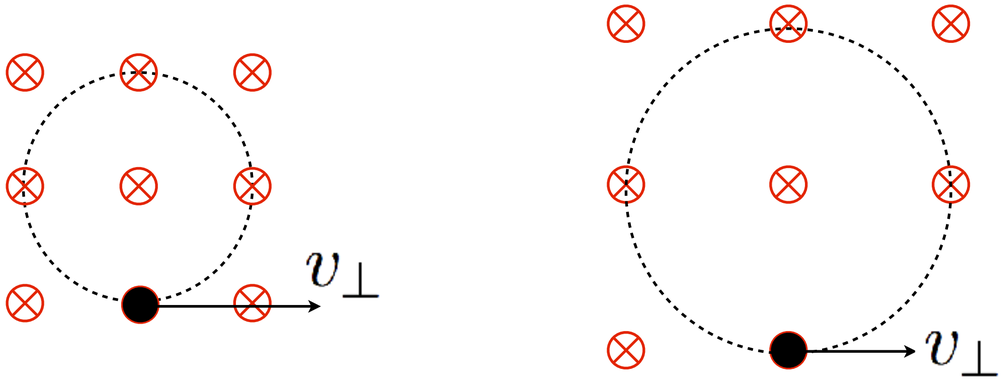
$$f(v) \propto \exp\left(-\frac{v_\parallel^2}{v_{\rm th}^2}\right) \exp\left(-\frac{v_\perp^2}{v_{\rm th}^2}\right)$$

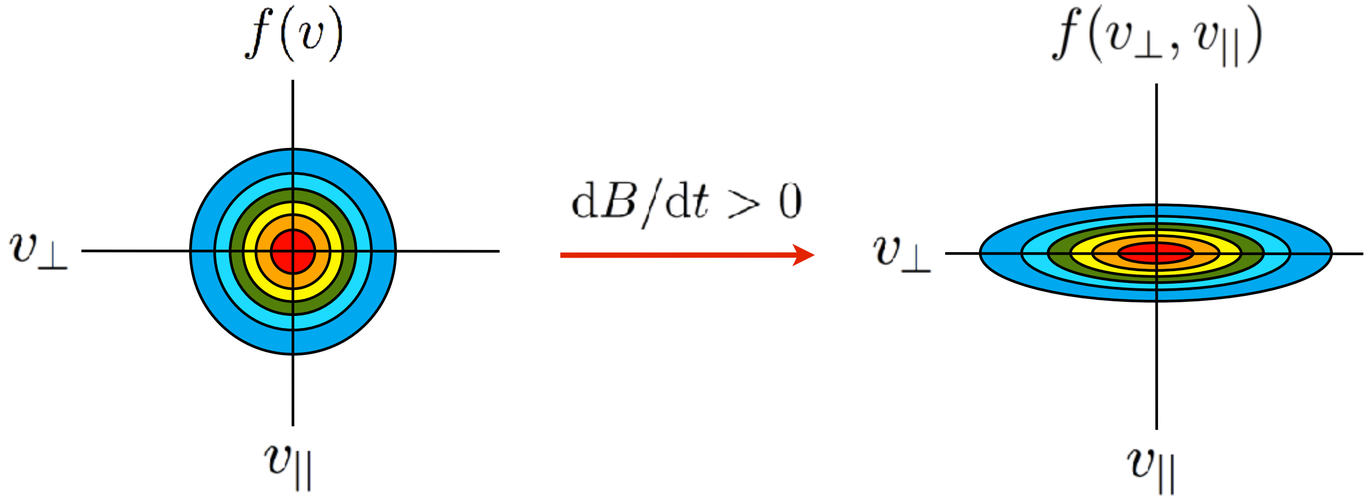


adiabatic evolution distorts the particle distribution function in this field-aligned frame…

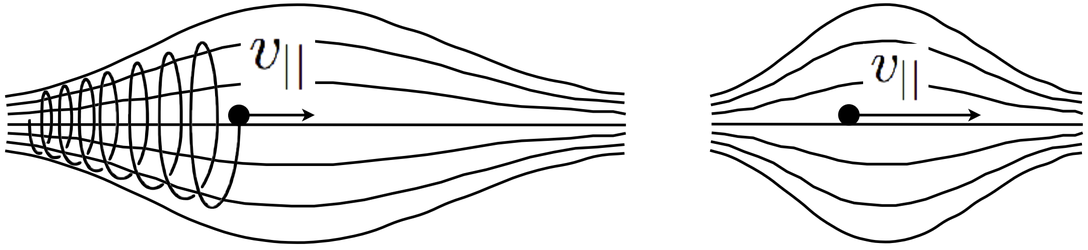


μ conservation:



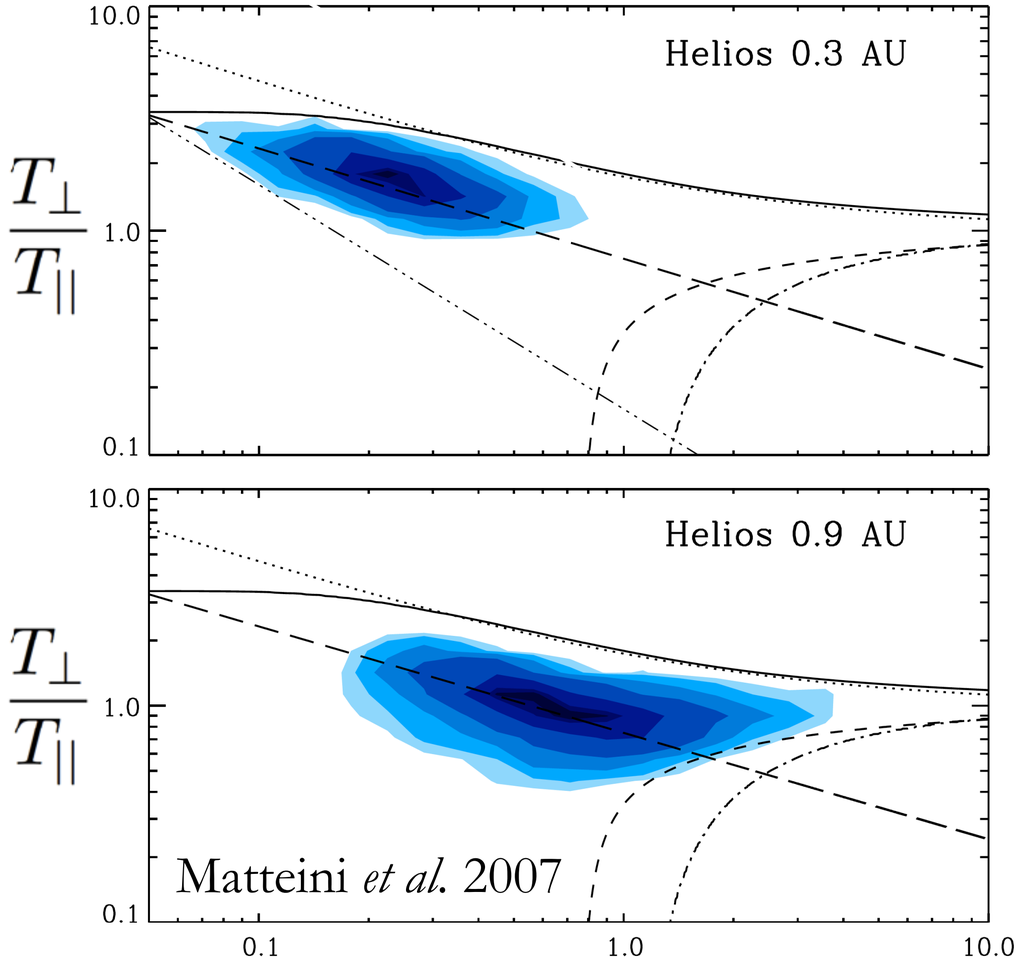


J conservation:

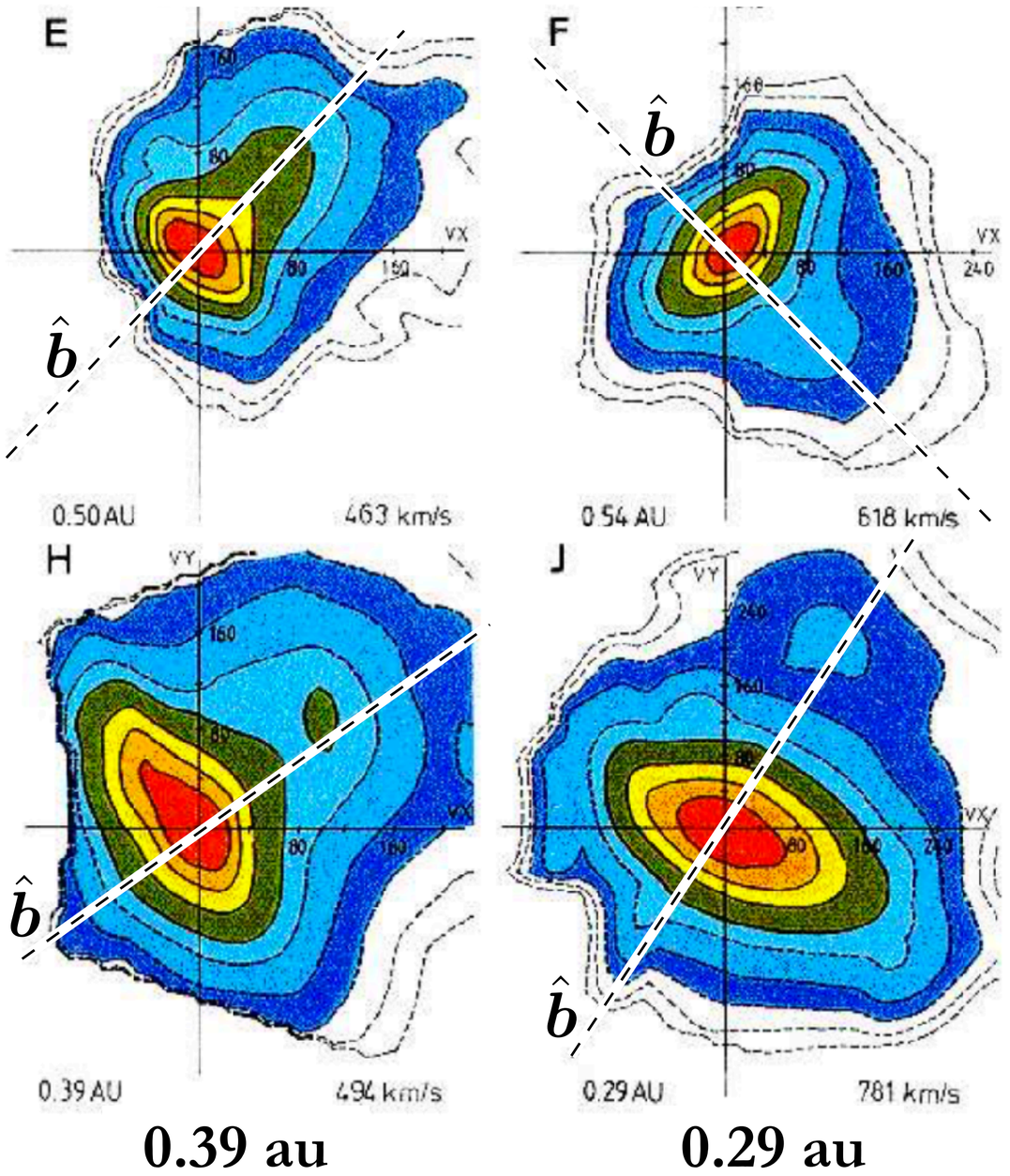


$$f(v_\perp, v_\parallel) \propto \exp\left(-\frac{v_\parallel^2}{v_{\mathrm{th}\parallel}^2}\right) \exp\left(-\frac{v_\perp^2}{v_{\mathrm{th}\perp}^2}\right)$$

These field-biased distortions are measured directly in the solar wind:

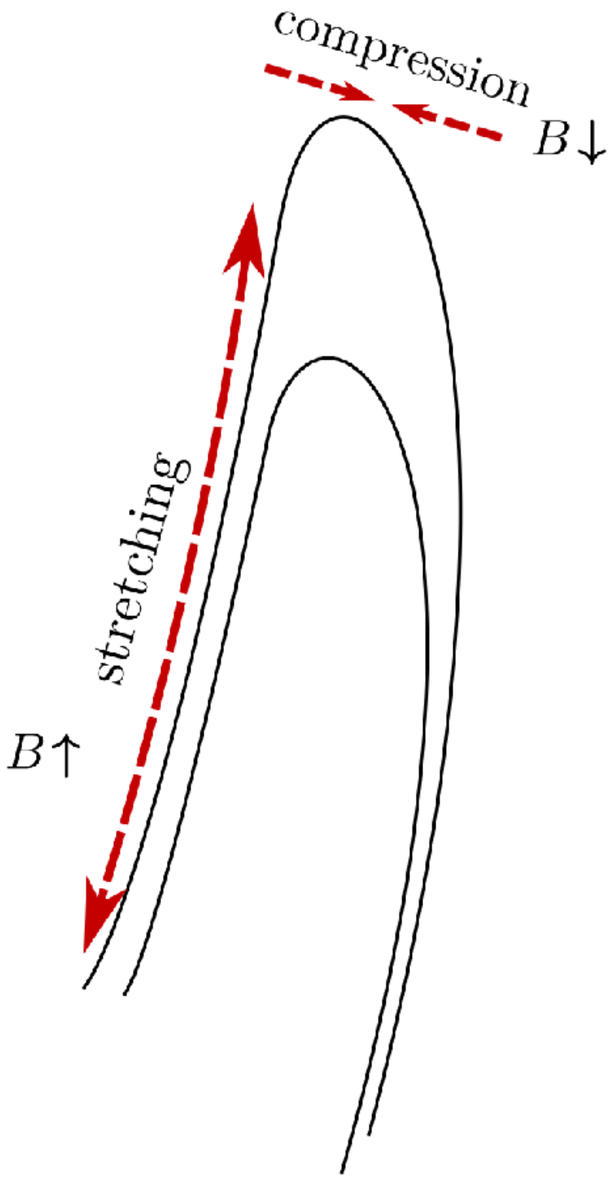


$$\beta_{||} = \frac{8\pi n T_{||}}{B^2}$$

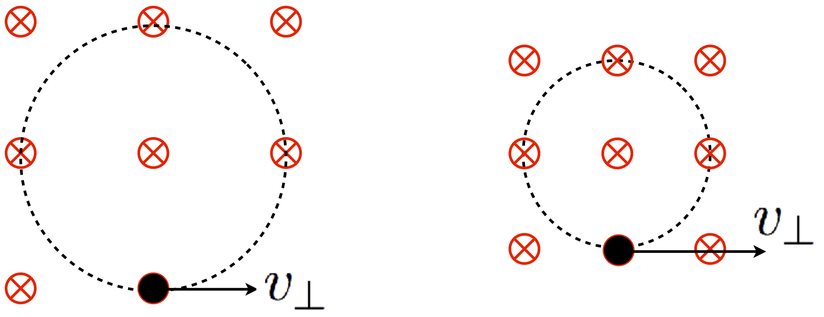


Marsch (2006)

dynamo context: can't move a plasma differentially without stretching/compressing B



μ-conservation implies pressure anisotropy:



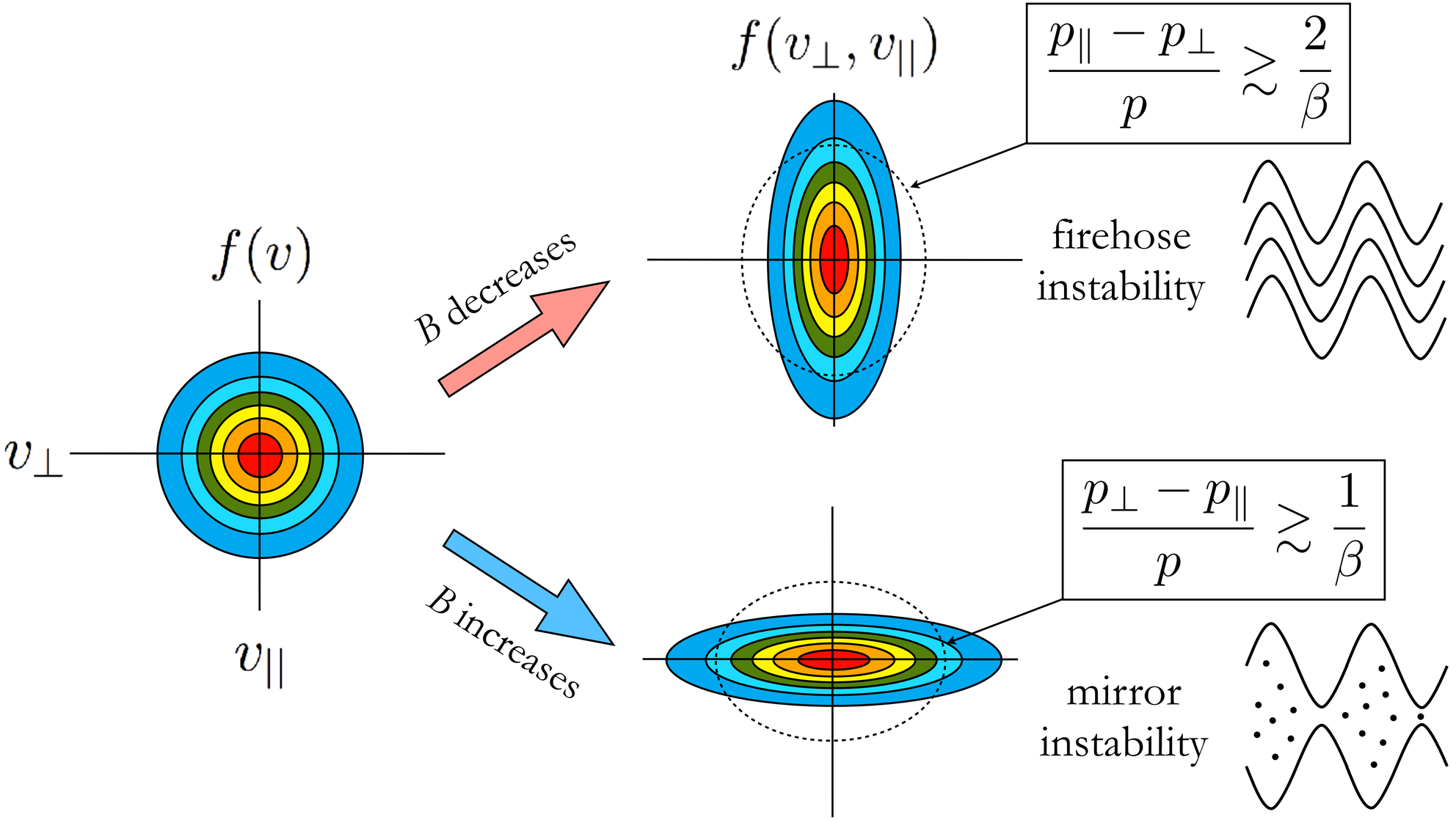
$$\frac{p_\perp}{p} \propto \frac{B}{B_0}$$

appreciable dynamo growth is *impossible* if μ is conserved; there's not enough free energy (Helander *et al.* 2016)

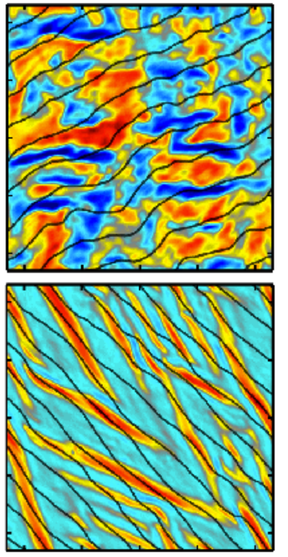
implies (at least) two things:

1) μ must be broken, e.g., by **kinetic instabilities** that feed off $p_\perp \neq p_\parallel$
2) no "kinematic" phase… ***B*, no matter how weak, influences the flow**

if $\beta \gg 1$, then even small departures from LTE affect **microscale** dynamics:

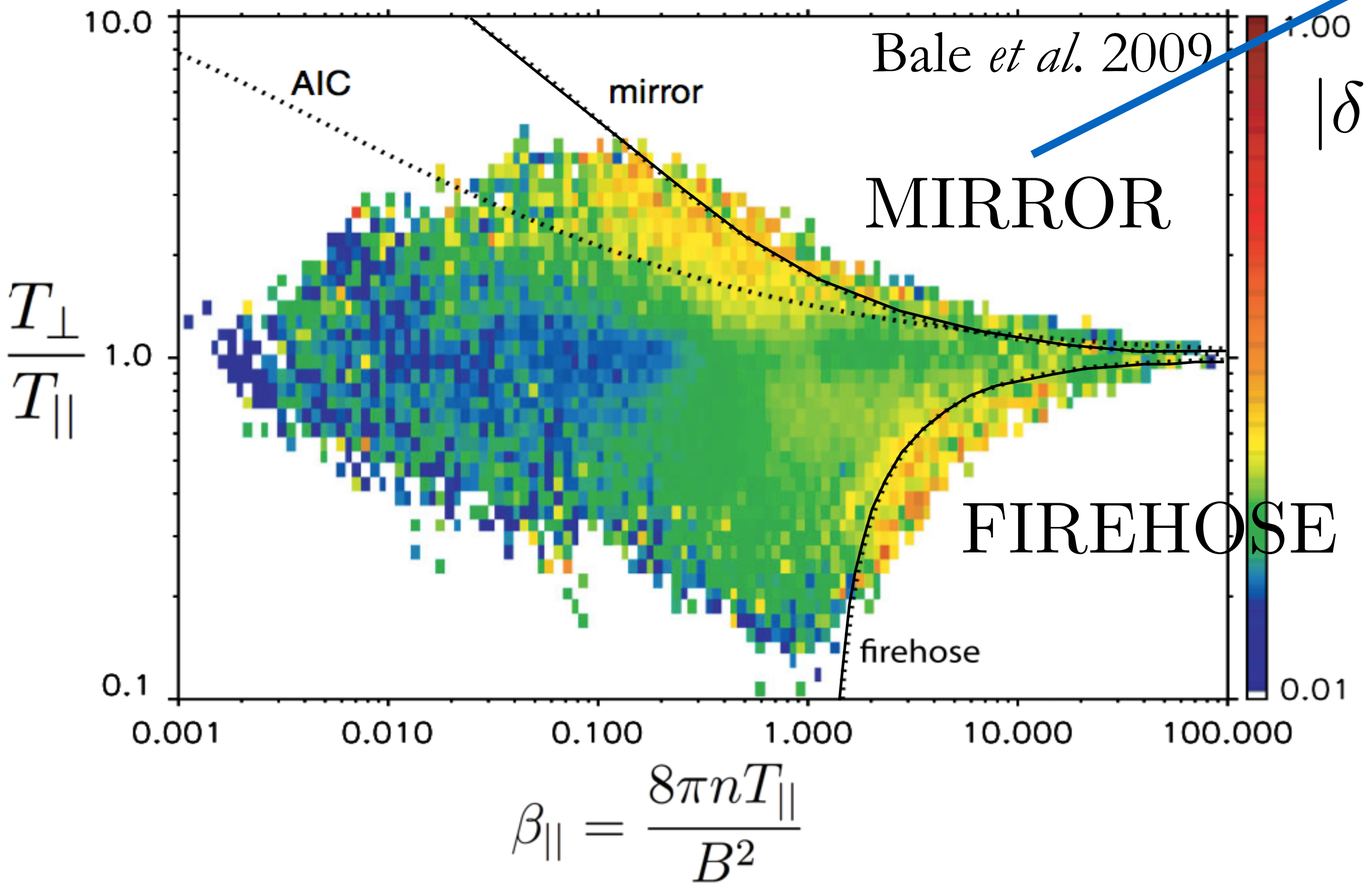


rapidly produce ion-Larmor-scale bends in field lines, which scatter and trap particles and thereby regulate departures from LTE ($p_\perp \neq p_{||}$)

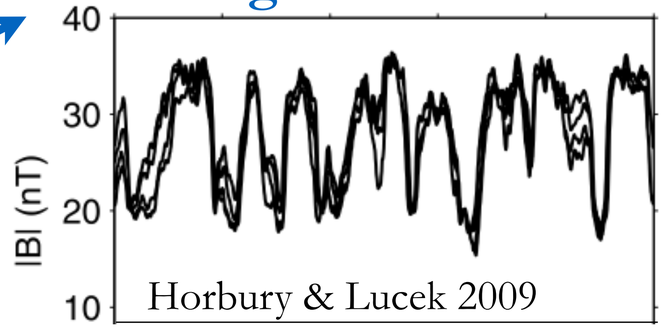


Kunz, Stone & Schekochihin 2014, *PRL*

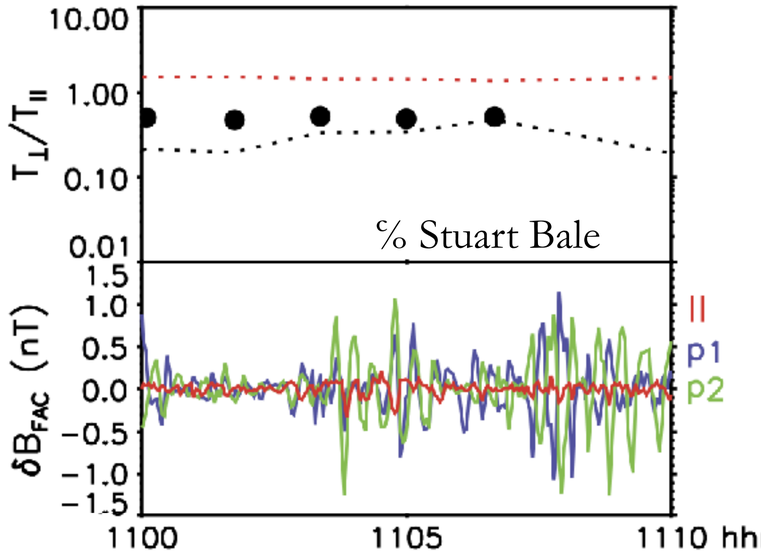
evidence from near-Earth solar wind:



magnetic mirrors



firehose fluctuations



can estimate deviations from LTE in ICM: $\Delta \doteq \dfrac{P_\perp - P_\parallel}{P} \sim M\dfrac{\lambda_{\rm mfp}}{L} \sim 0.01 - 0.1 \gtrsim \dfrac{1}{\beta}$

implication: scattering by kinetic Larmor-scale instabilities $\gtrsim$ Coulomb scattering

$\mathrm{Re}_\parallel$ likely controlled by what's going on at Larmor scale

(and increasingly so in the early stages of dynamo, when $\beta \ggg 1$)

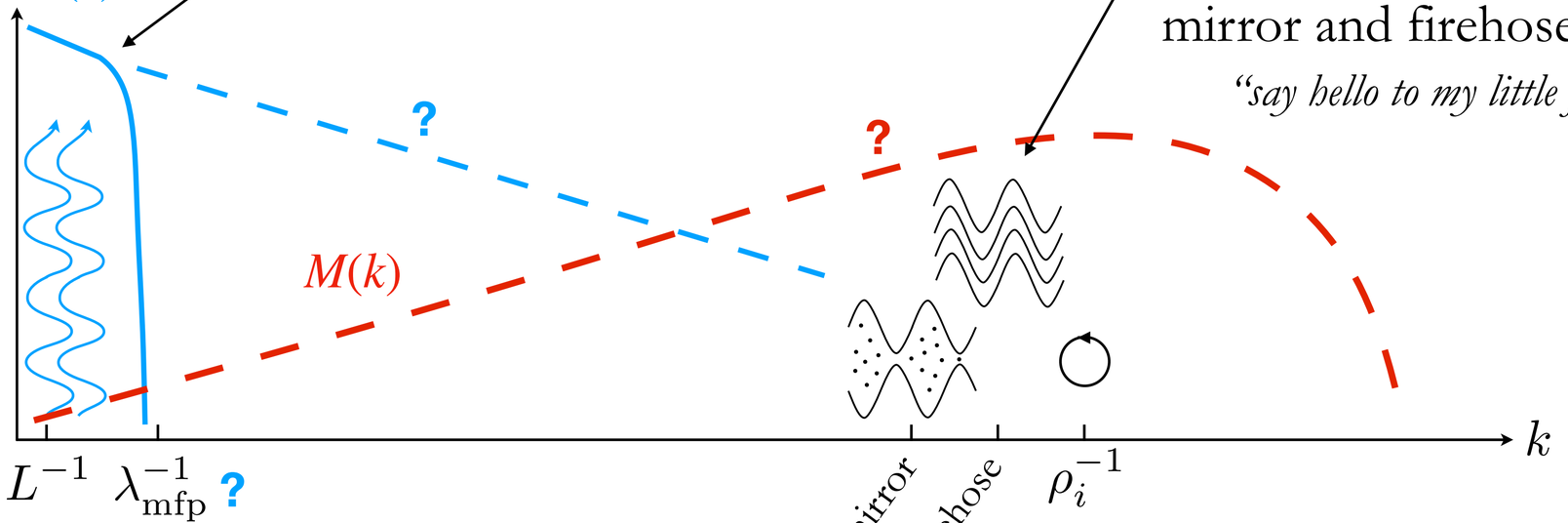
back to plasma dynamo…

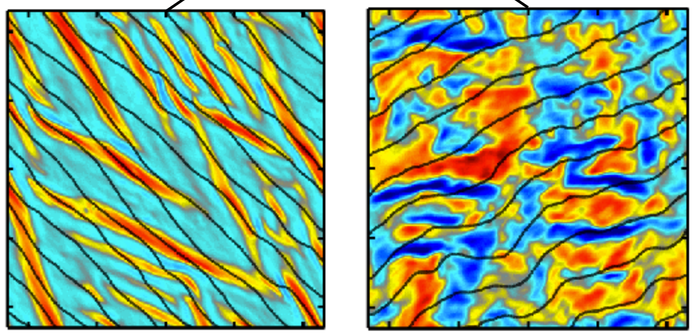
fastest stretching motions at parallel-viscous scale

(recall $\mathrm{d}\ln B/\mathrm{d}t = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\boldsymbol{\nabla u}$)

parallel-viscous scale set by effective scattering rate of kinetic instabilities: mirror and firehose

"say hello to my little friends"





these try to maintain

$$-\frac{B^2}{4\pi} \lesssim p_\perp - p_\parallel \lesssim \frac{B^2}{8\pi}$$

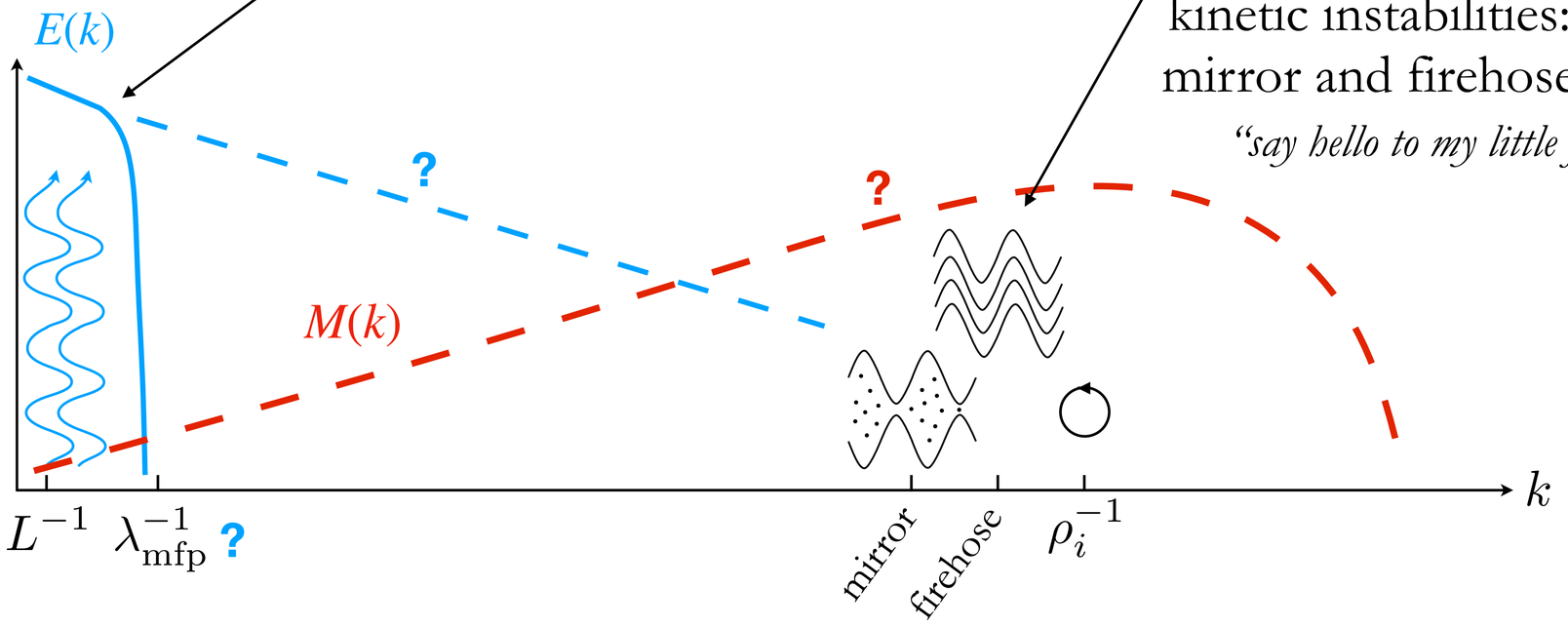
Kunz, Stone & Schekochihin 2014, PRL

fastest stretching motions at parallel-viscous scale

(recall $\mathrm{d}\ln B/\mathrm{d}t = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\boldsymbol{\nabla u}$)

parallel-viscous scale set by effective scattering rate of kinetic instabilities: mirror and firehose

"say hello to my little friends"



$$\mathrm{Re}_{\|} \sim M^2\,\nu_{\mathrm{eff}}\,\tau_{\mathrm{eddy},L} \longrightarrow \tau_{\mathrm{eddy},\ell_{\mathrm{visc}}} \sim \tau_{\mathrm{eddy},L}\,\mathrm{Re}_{\|}^{-1/2} \propto \nu_{\mathrm{eff}}^{-1/2}$$

the faster the instabilities scatter, the faster is the dynamo

One can show that, for

$$B \gtrsim 10 \left(\frac{U}{200\ \mathrm{km\ s^{-1}}}\right)^{3/5} \left(\frac{L}{100\ \mathrm{kpc}}\right)^{-1/5} \left(\frac{n}{10^{-3}\ \mathrm{cm^{-3}}}\right)^{2/5} \left(\frac{T}{10^{8}\ \mathrm{K}}\right)^{1/5} \mathrm{nG},$$

the effective collisionality induced by firehose/mirror instabilities is

$$\nu_{\mathrm{eff}} \sim \frac{U}{L}\left(\frac{U}{v_{\mathrm{A}}}\right)^{2} \beta\,.$$

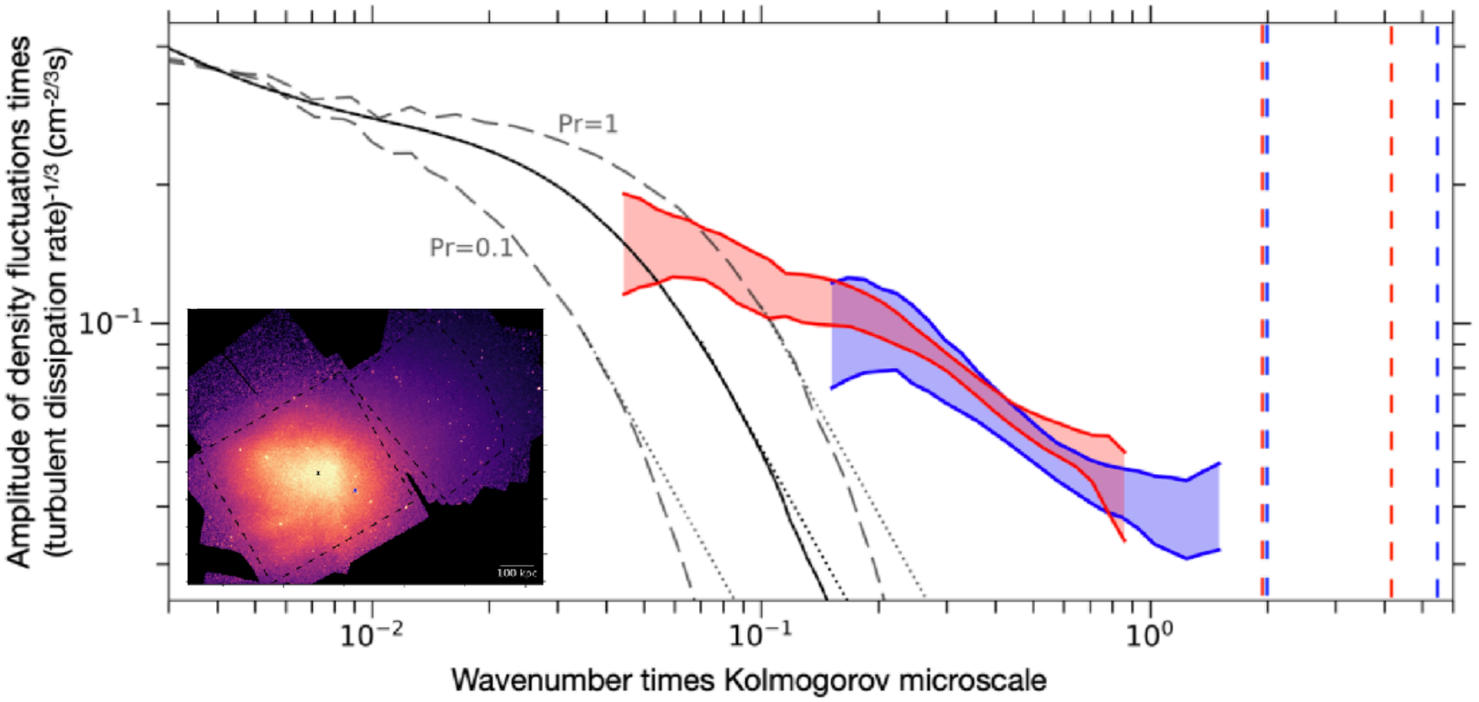
This implies

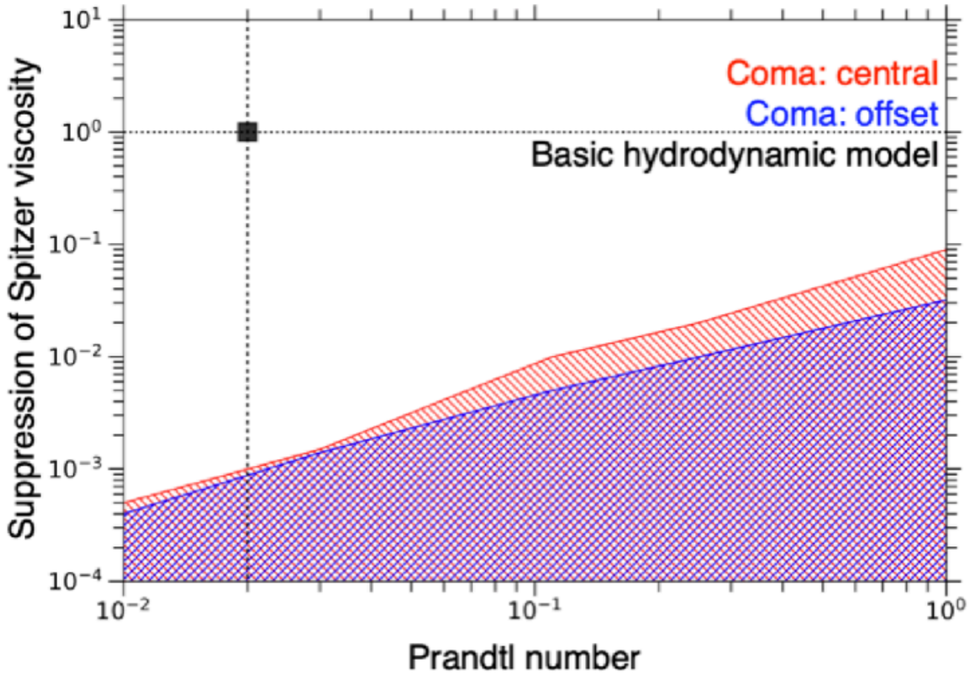
$$\lambda_{\parallel\mathrm{mfp,eff}} \sim L\left(\frac{U}{v_{\mathrm{A}}}\right)^{-3} \beta^{-1/2} \qquad \mathrm{Re}_{\parallel,\mathrm{eff}} \sim \left(\frac{U}{v_{\mathrm{A}}}\right)^{4} \qquad \ell_{\nu,\mathrm{eff}} \sim L\left(\frac{U}{v_{\mathrm{A}}}\right)^{-3}.$$

Choosing $L = 100$ kpc and blindly promoting $\sim$ to $\approx$…

$$\frac{\lambda_{\parallel\mathrm{mfp,eff}}}{\lambda_{\mathrm{mfp,C}}} \approx 0.05 \left(\frac{U}{200\ \mathrm{km\ s^{-1}}}\right)^{-3} \left(\frac{B}{2\ \mu\mathrm{G}}\right)^{4} \left(\frac{n}{10^{-3}\ \mathrm{cm^{-3}}}\right)^{-1} \left(\frac{T}{10^{8}\ \mathrm{K}}\right)^{-5/2}.$$

such a collisionality comports with obs'd turbulence spectrum in Coma cluster





Zhuravleva *et al.* (2019) claims that viscosity is ~0.01 – 0.1 × Spitzer value; consistent with prediction

if so, then from ~10 nG levels,

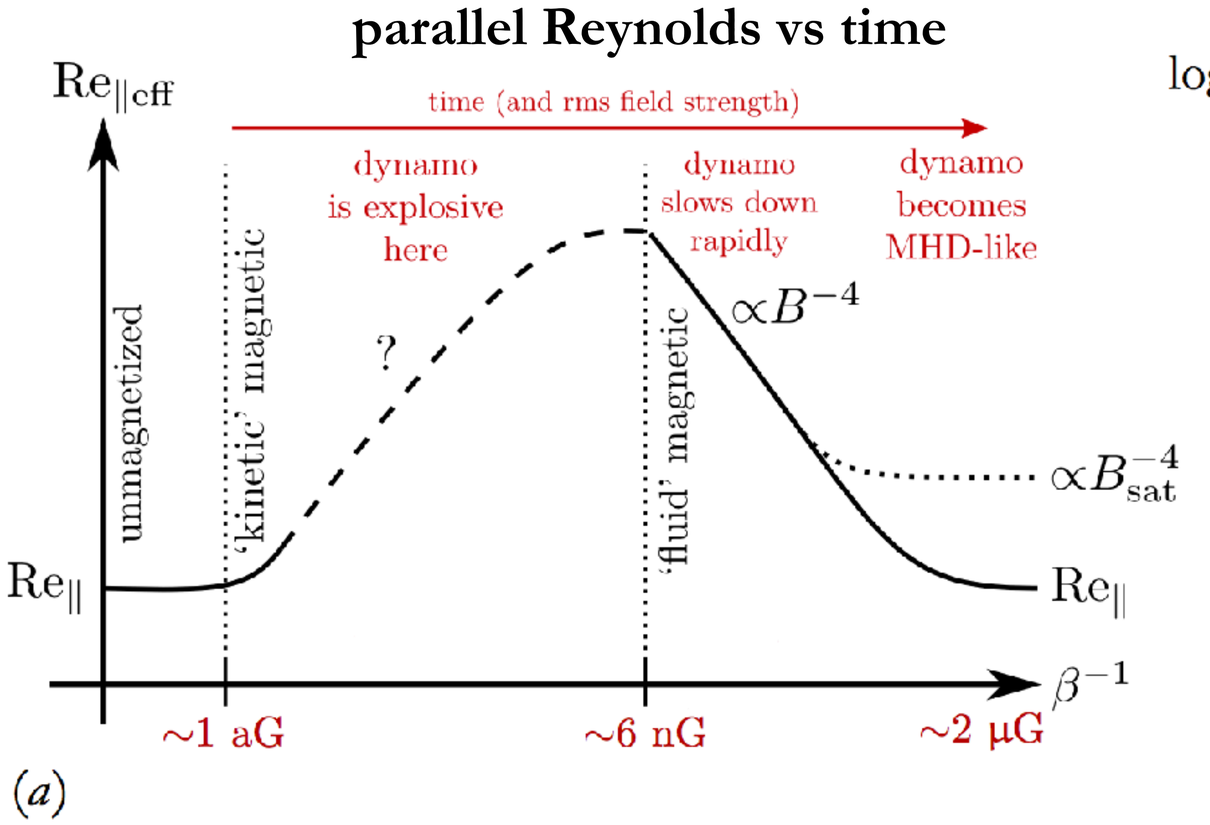
$$\frac{\mathrm{d}\ln B}{\mathrm{d}t} \sim \frac{U}{L}\mathrm{Re}_{\parallel,\mathrm{eff}}^{1/2} \sim \frac{U^3}{L}\frac{4\pi m_\mathrm{i} n}{B^2} \propto B^{-2}$$

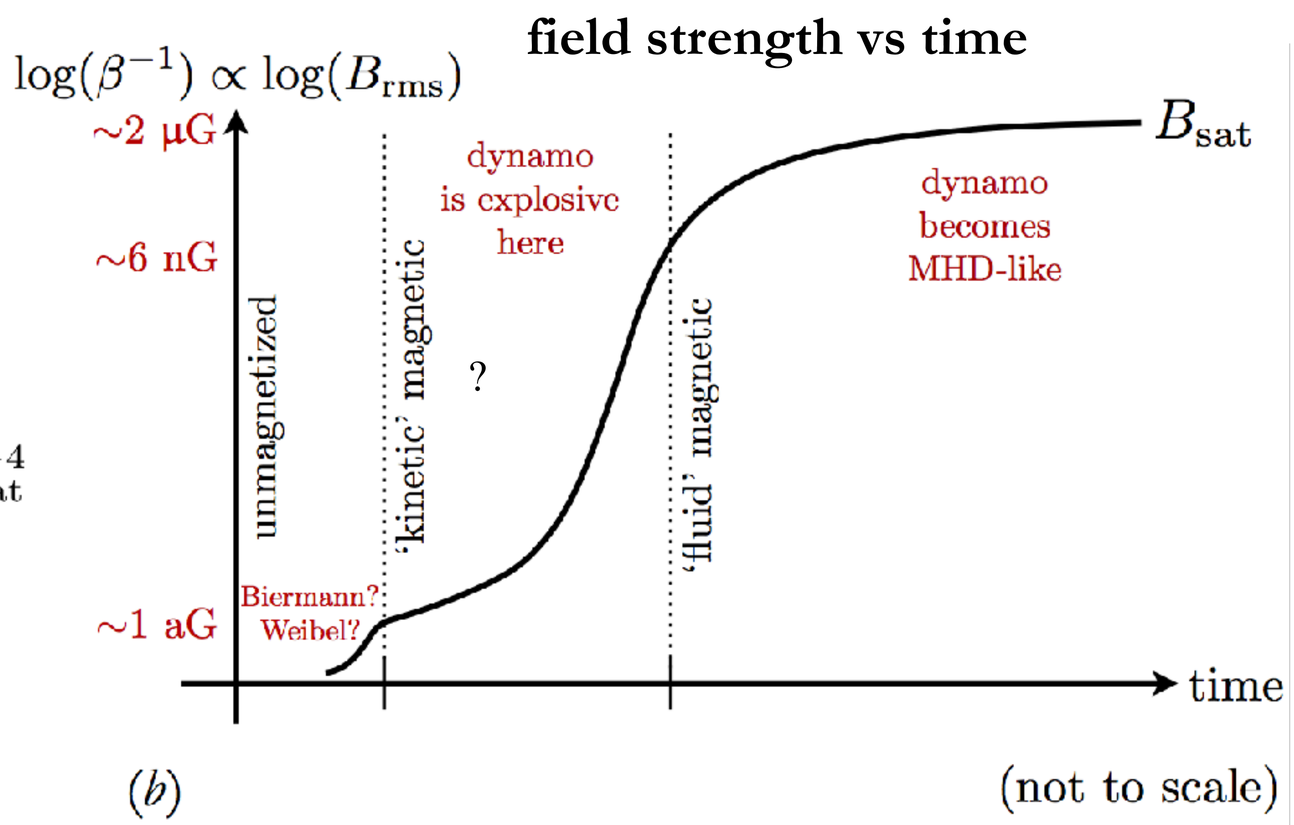
integrating and using typical values implies
~μG fields within ~30 Myr . *Fast!!!*

What happens before ~10 nG? not sure…

- Weibel growth up to ~0.1 nG… but on skin-depth scales (Zhou *et al.* 2022)
- Galactic/stellar pollution of early intergalactic medium… dilution? (Rees 1987, …)
- Explosive growth from ~10^{-16} G?
  (Schekochihin & Cowley 2006; Mogavero & Schekochihin 2014; Melville, Schekochihin & Kunz 2016)

motivates the following idea of 3 dynamo regimes:





explosive growth? predicts ~nG fields in cosmologically short time.

prior theories of explosive growth: Schekochihin & Cowley (2006); Mogavero & Schekochihin (2014); Melville, Schekochihin & Kunz (2016)

We have begun a study of fluctuation dynamo
in collisionless and weakly collisional plasmas
using hybrid-kinetic particle-in-cell simulations,
Braginskii-MHD simulations, and
analytic modeling.

hybrid-kinetic particle-in-cell simulations using Pegasus

$$\frac{\partial f_\mathrm{i}}{\partial t} + \boldsymbol{v}\cdot\nabla f_\mathrm{i} + \left[\frac{Z_\mathrm{i} e}{m_\mathrm{i}}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right) + \frac{\boldsymbol{F}}{m_\mathrm{i}}\right]\cdot\frac{\partial f_\mathrm{i}}{\partial \boldsymbol{v}} = 0$$

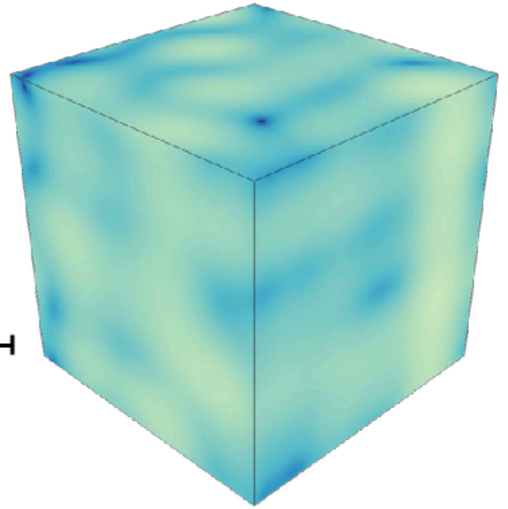
+ fluid equation for massless, isothermal electrons

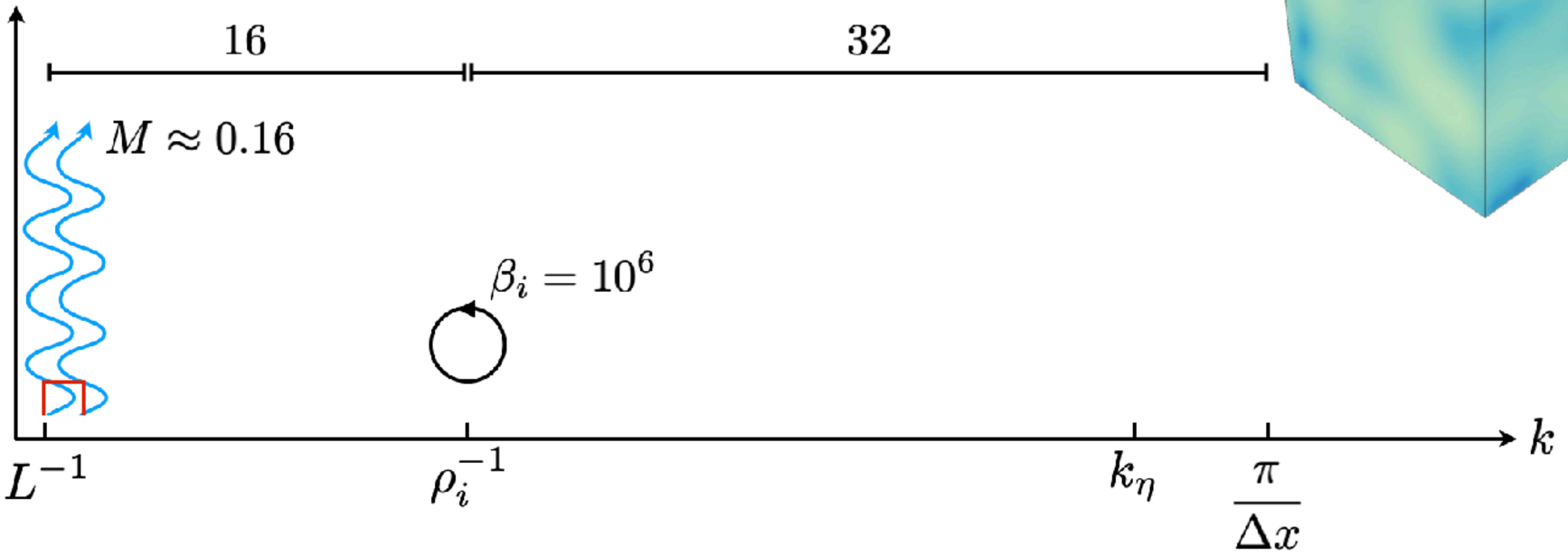
$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\boldsymbol{\nabla}\times\boldsymbol{E}$$

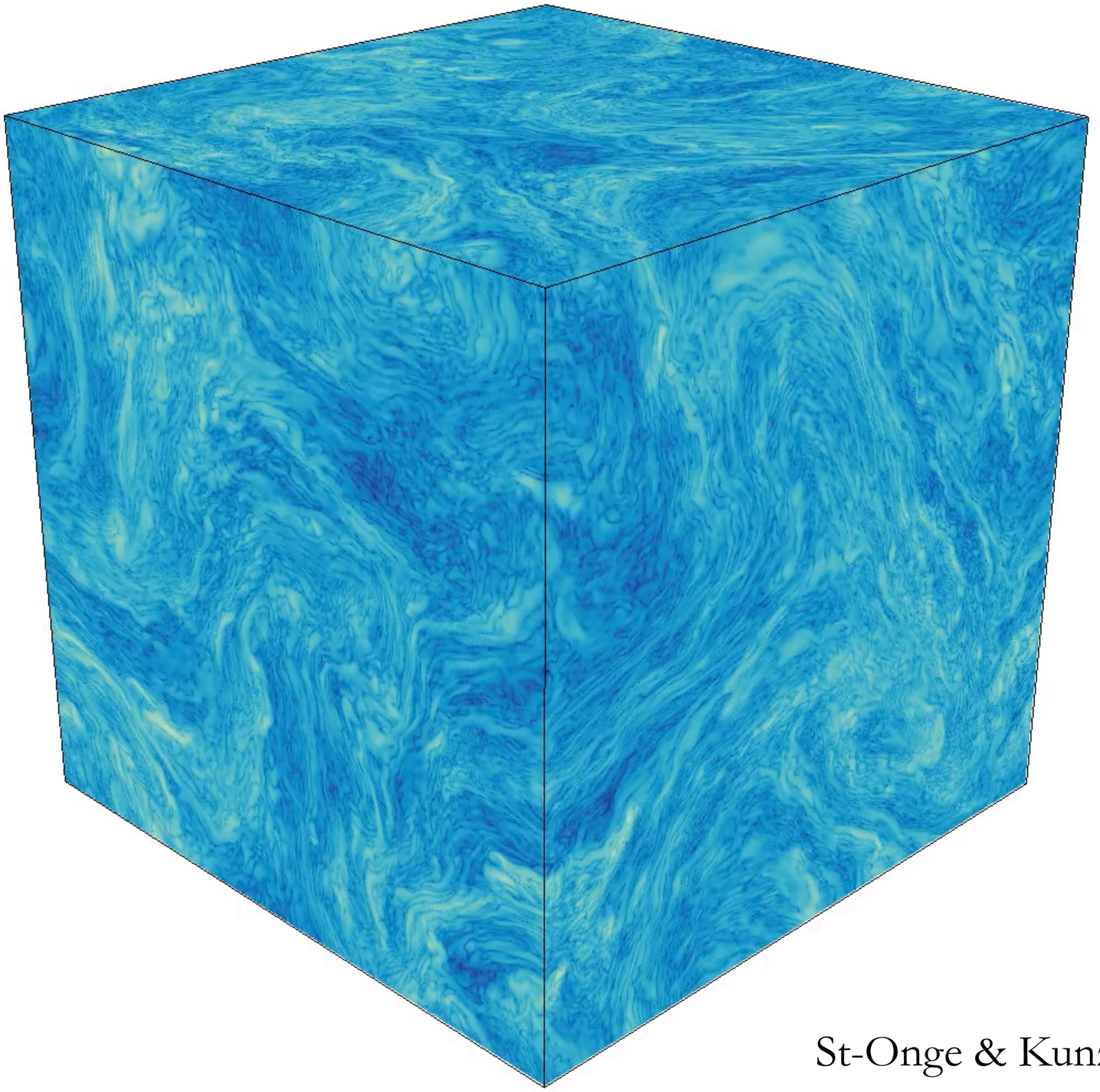
+ hyper-resistivity

zero-net-flux magnetic field at the largest box scales, subject to time-correlated incompressible, subsonic stirring

St-Onge & Kunz 2018, *ApJL*



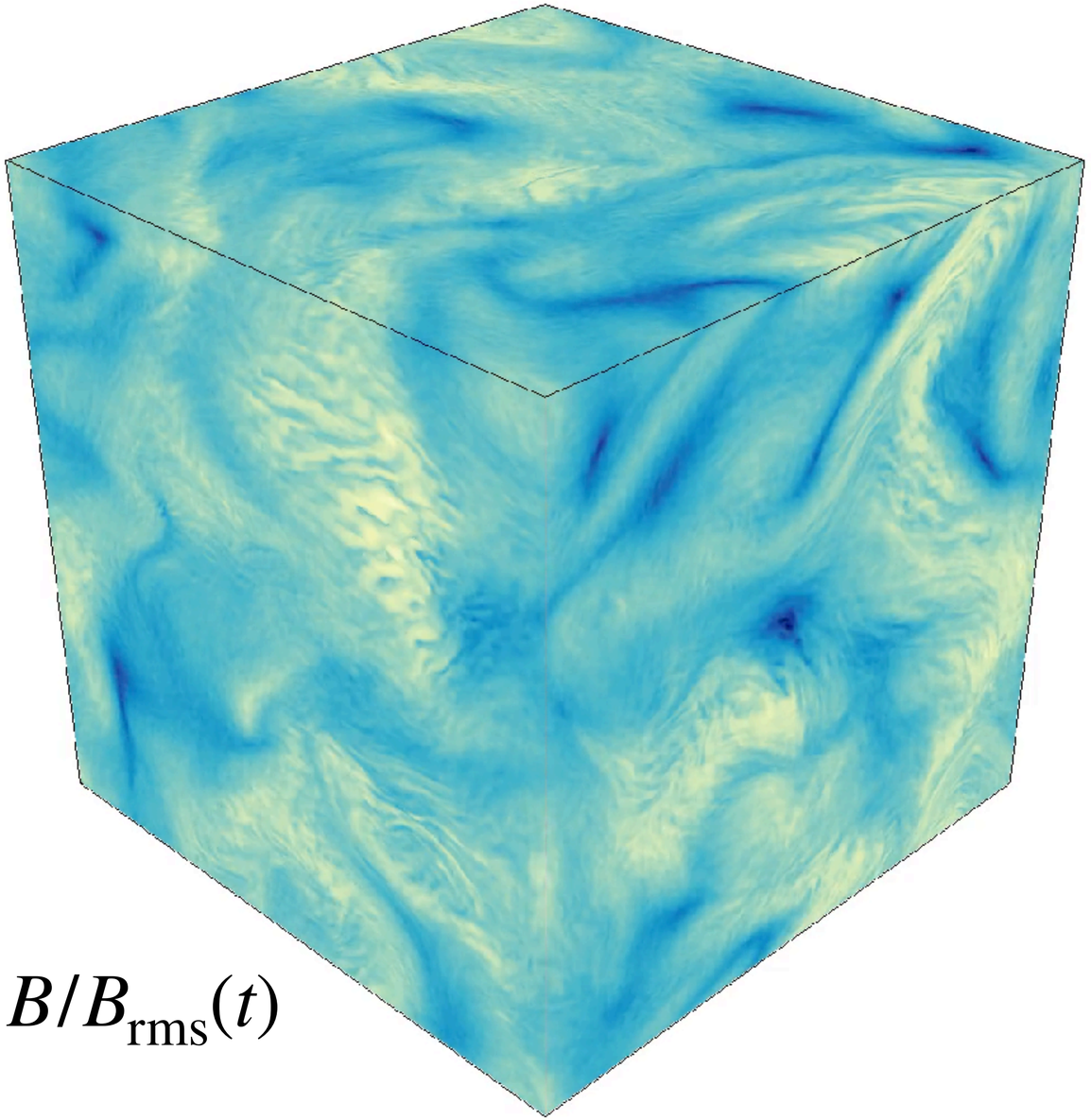




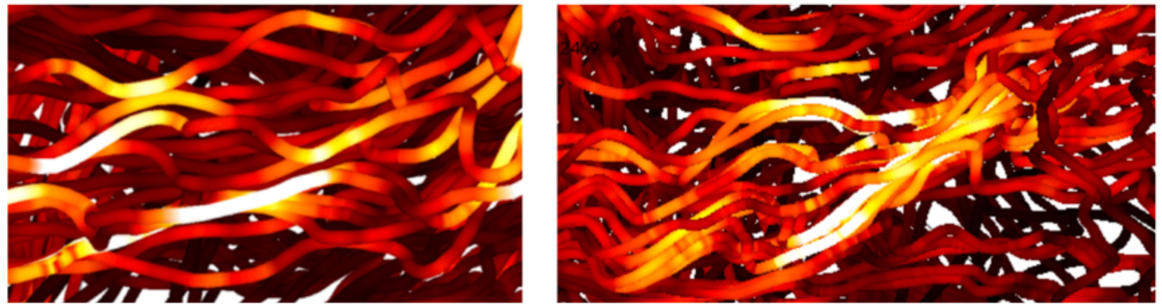
B

St-Onge & Kunz, 2018 *ApJL*

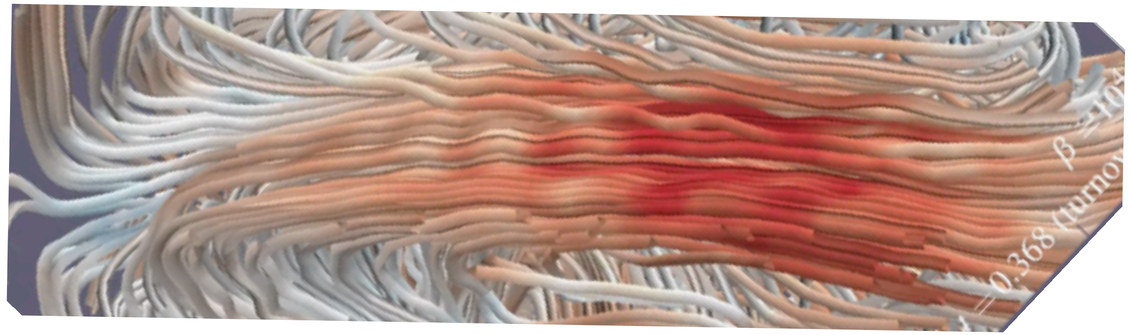
1. Turbulent amplification of magnetic fields to dynamically important strengths is possible in a collisionless plasma. Ion-Larmor-scale kinetic instabilities supply effective collisionality and increase $\mathrm{Re}_{\parallel}$.



$B/B_{\mathrm{rms}}(t)$

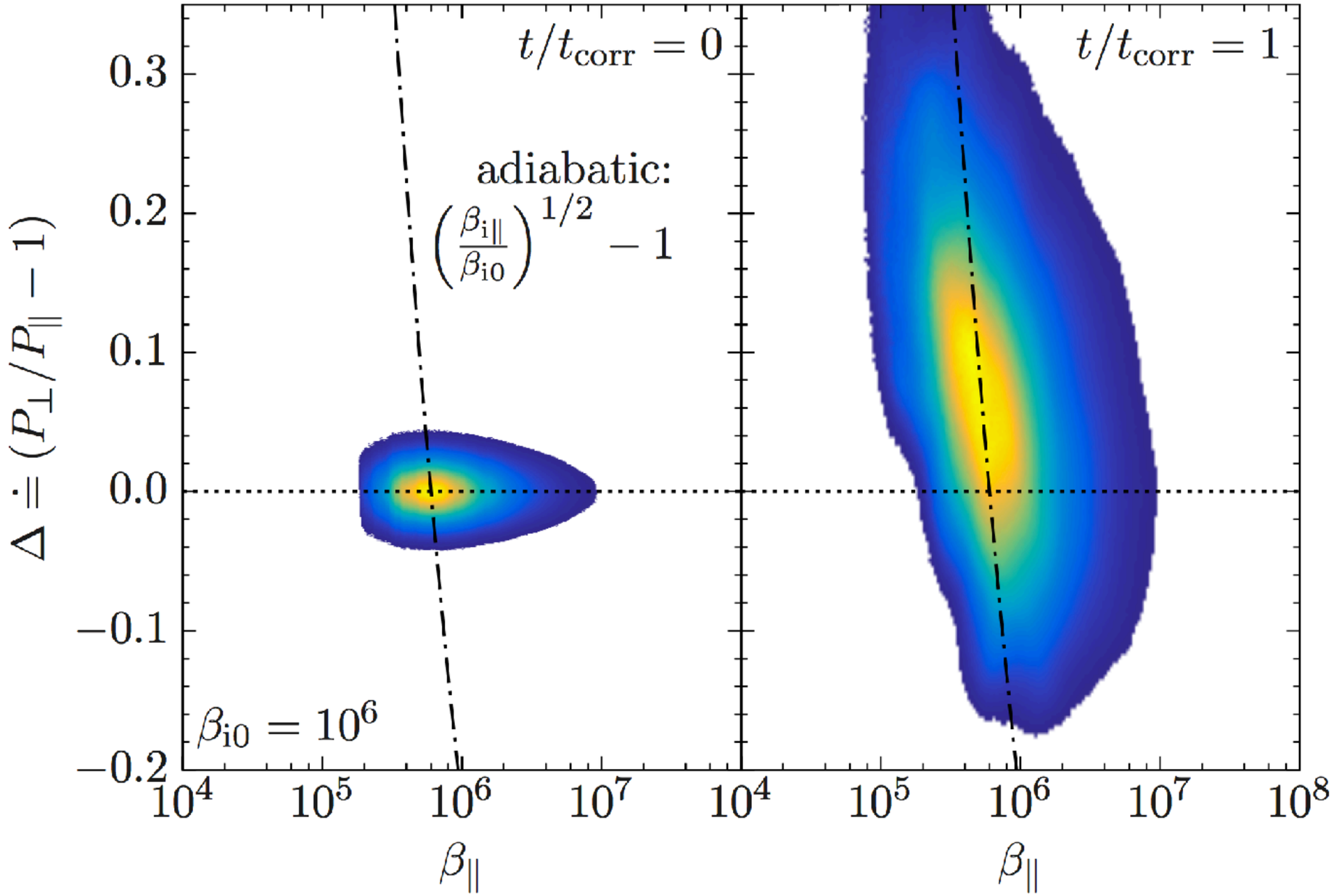


magnetic-field lines from a **Pegasus** simulation of plasma dynamo showing ion-Larmor-scale mirror instabilities (light/dark = weak/strong magnetic field)

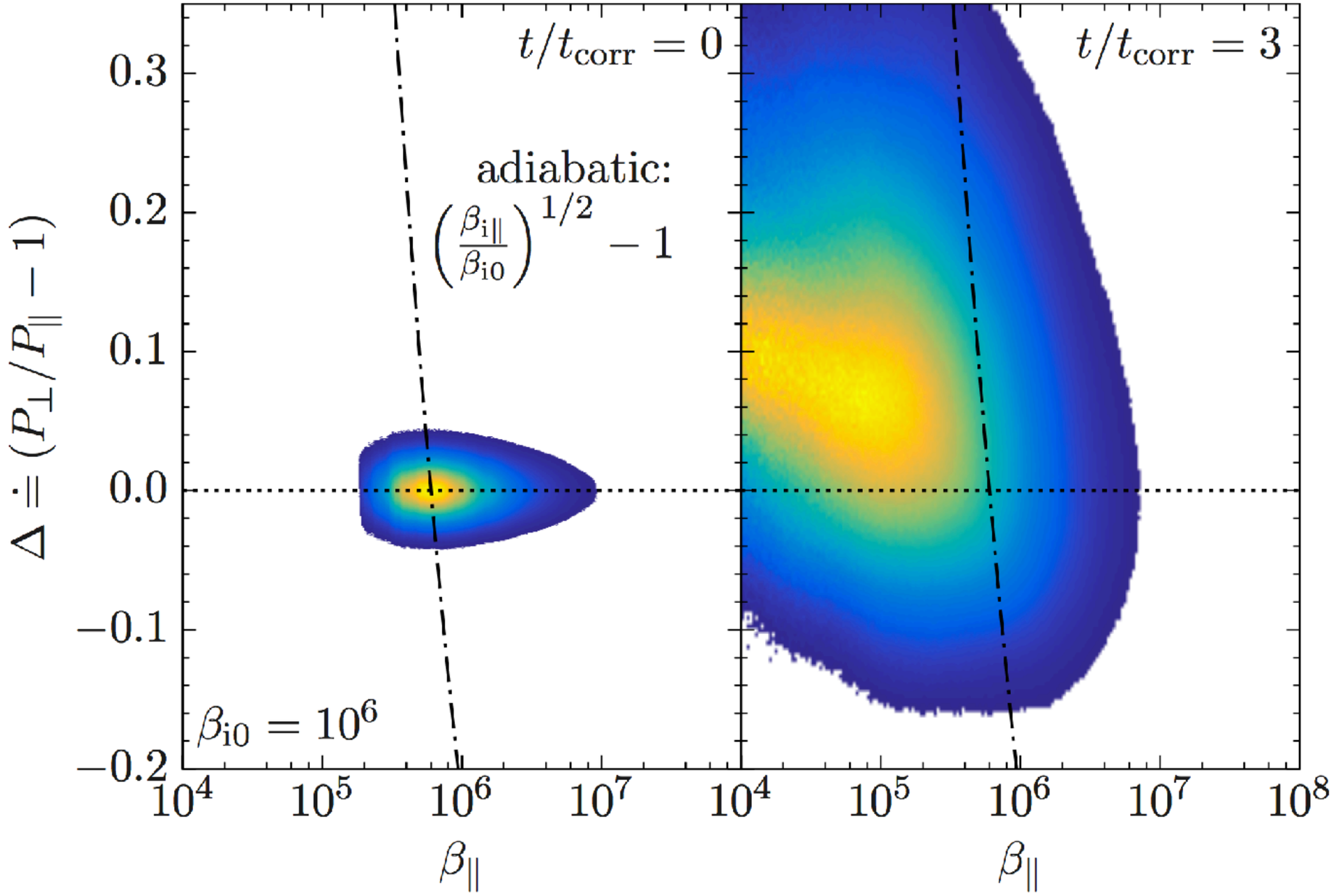


also seen in François Rincon's (2016, *PNAS*) plasma dynamo simulations

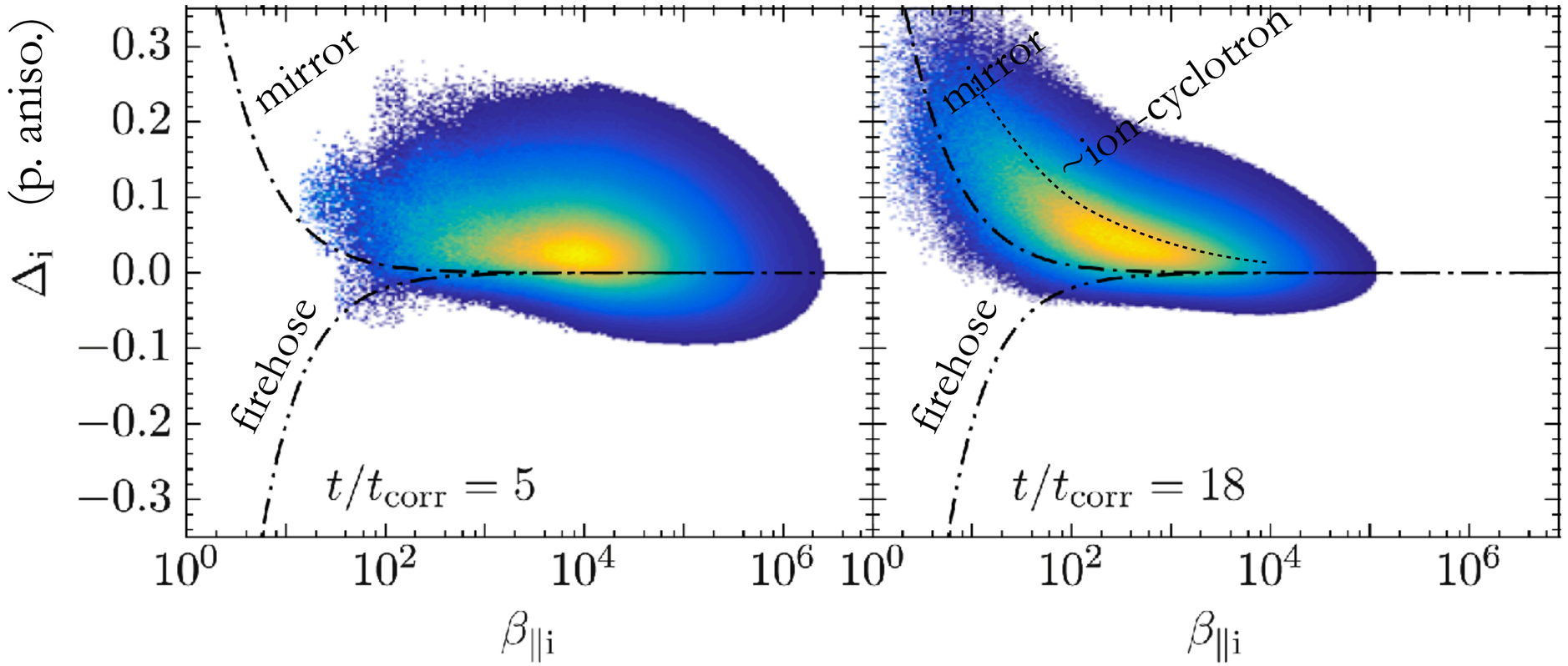
adiabatic evolution produces pressure anisotropy…



…which is relaxed by firehose/mirror instabilities.



throughout exponential-growth phase, p. aniso. knows about thresholds



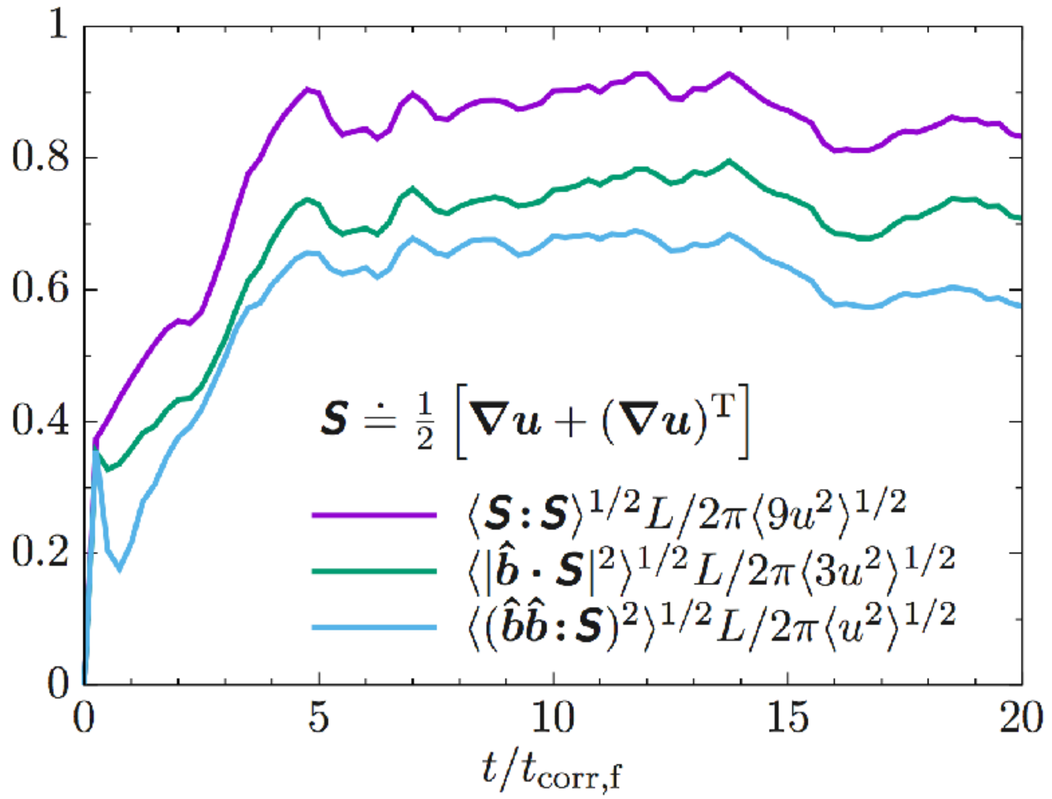
firehose/mirror instabilities limit

(though not completely)

departures from thermodynamic equilibrium

2. There is no true "kinematic" phase… *B*, no matter how weak, influences the flow.

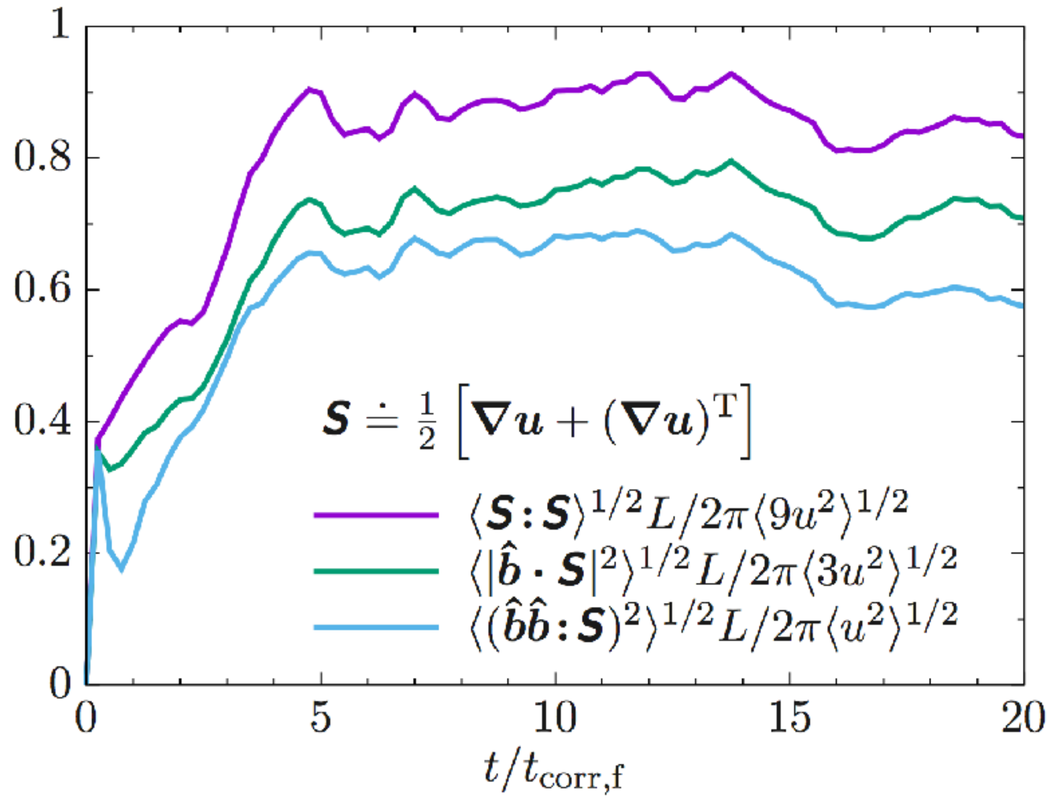
total rate of strain ≫ **field-parallel rate of strain**



certain motions are preferred over others:
viscosity is **anisotropic**, as though the plasma were weakly collisional and magnetized
(à la Braginskii 1965)

2. There is no true "kinematic" phase… B, no matter how weak, influences the flow.

total rate of strain $\gg$ **field-parallel rate of strain**



certain motions are preferred over others: viscosity is **anisotropic**, as though the plasma were weakly collisional and magnetized
(à la Braginskii 1965)

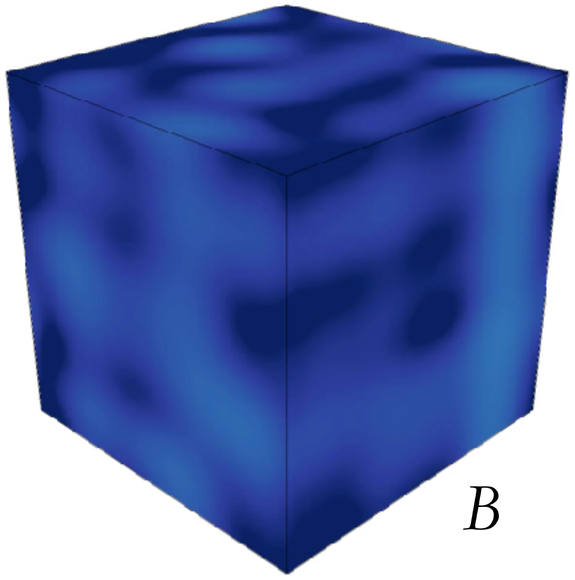
leveraged to:

i. perform & analyze (w. novel diagnostics) a large number of Braginskii-MHD simulations of plasma dynamo;

ii. construct analytic modified Kazantsev-Kraichnan model for field evolution
(supplants Malyshkin & Kulsrud 2001)

$$\langle u^i(t,r)u^j(t',r')\rangle = \delta(t-t')\,\kappa^{ij}(r-r')$$

(St-Onge, Kunz *et al.* 2020, *JPP*)

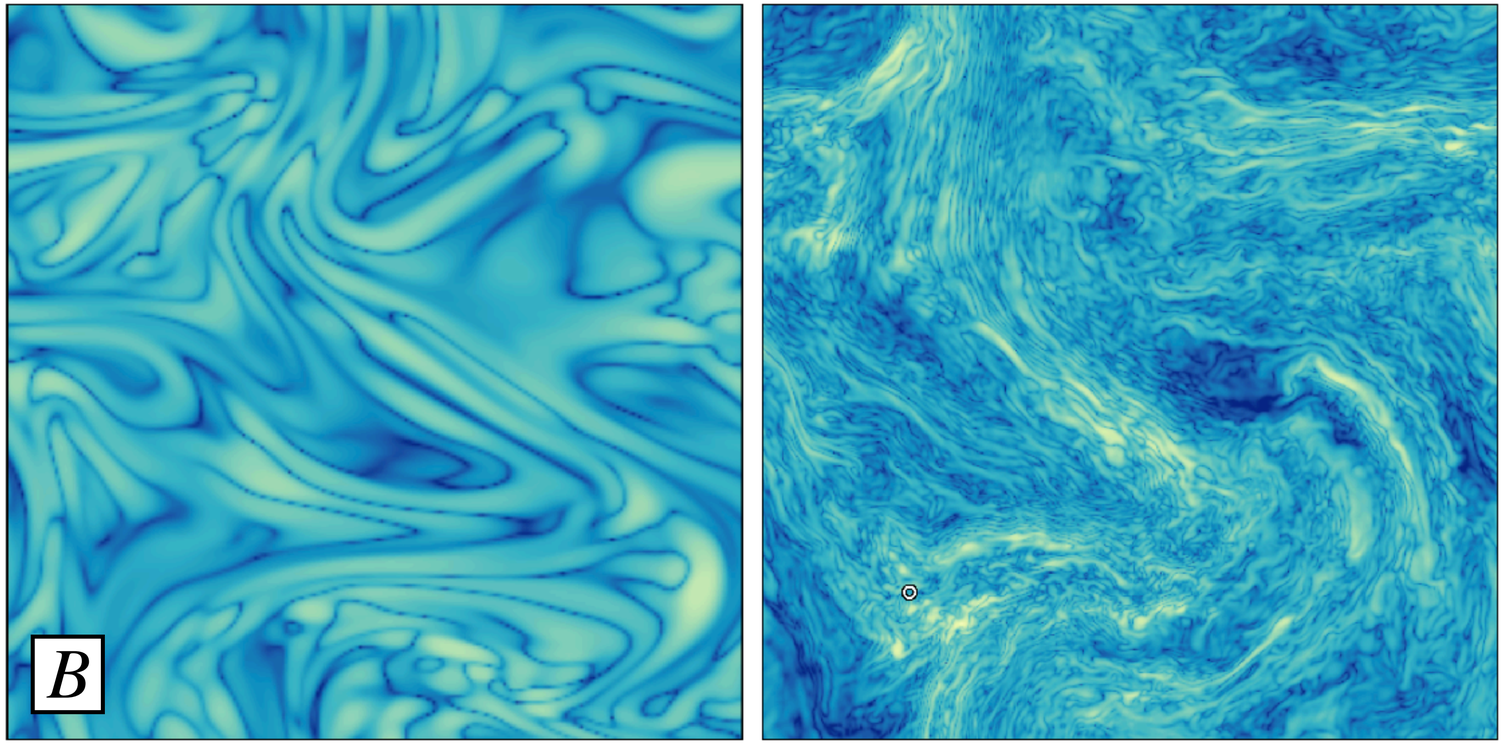


B

3. In many respects, behaves as though it were a Pm ≫ 1 MHD dynamo:
   exponential growth, Kazantsev-like spectrum, folded magnetic fields, saturates with $\langle B^2 \rangle \sim \langle u^2 \rangle$

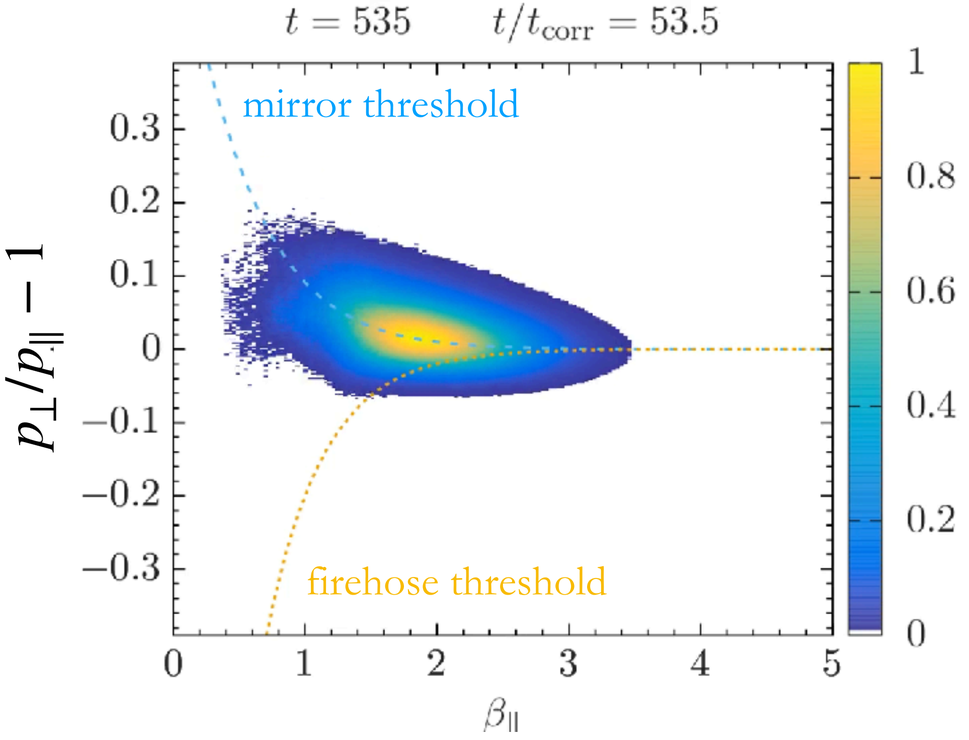
MHD (Pm = 500)

Kinetic

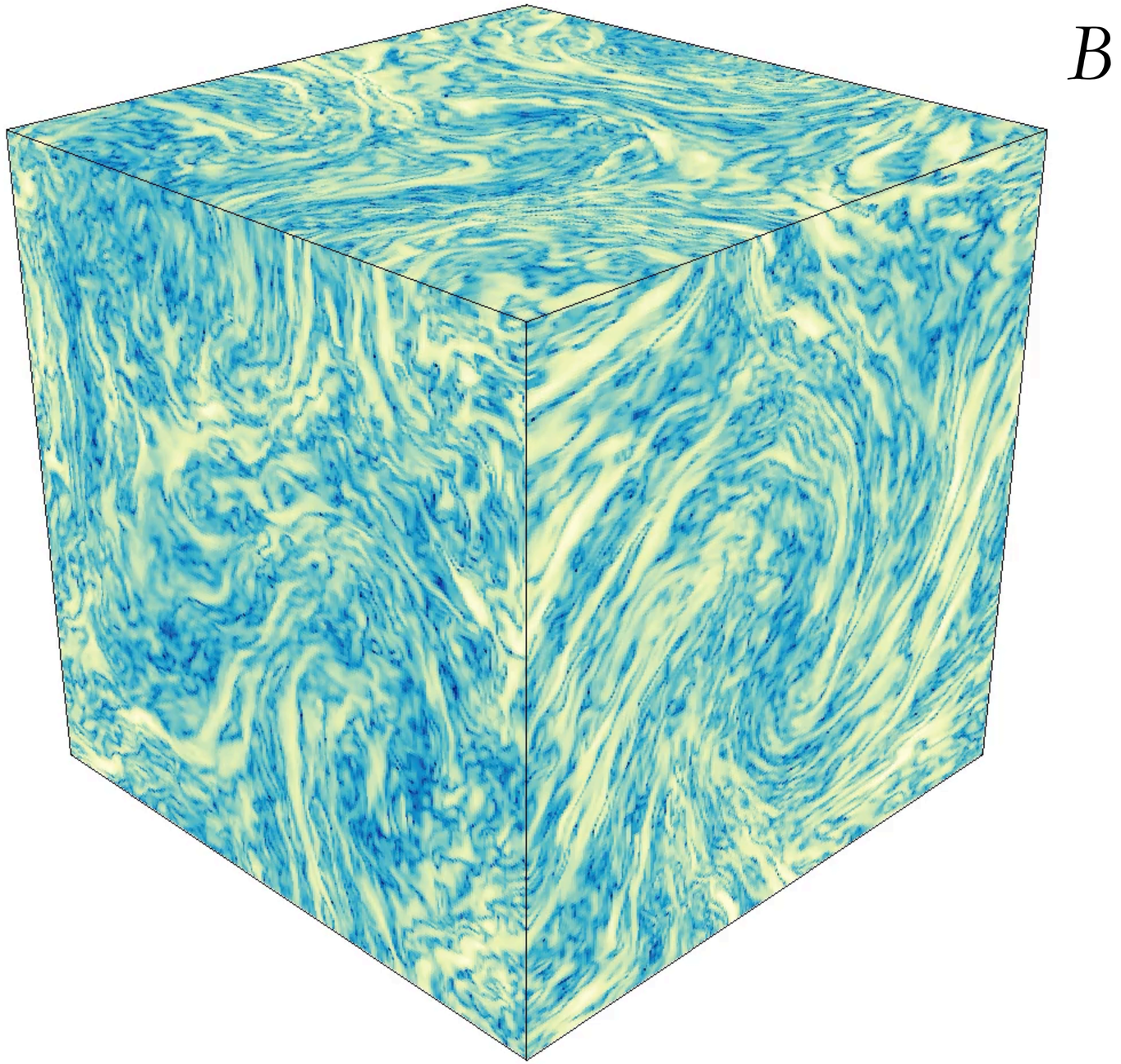


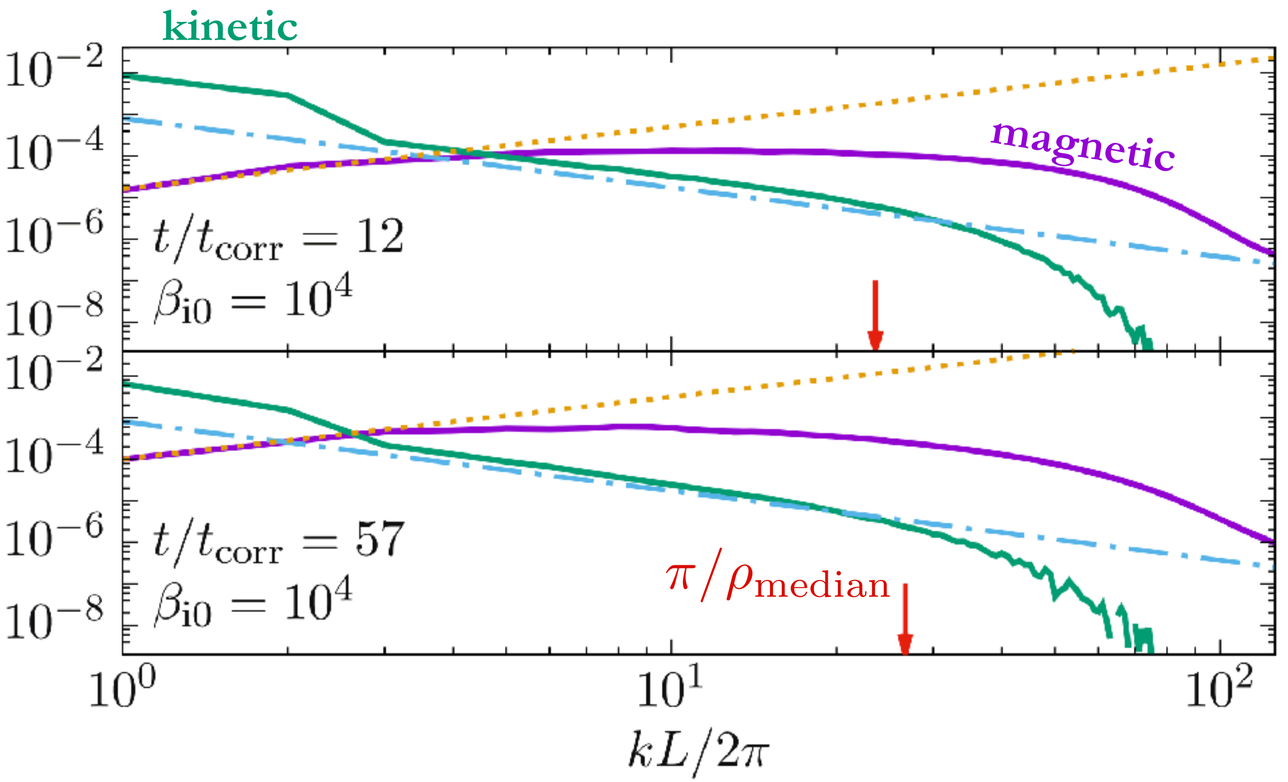
(take off your glasses)

…because pressure anisotropy is regulated



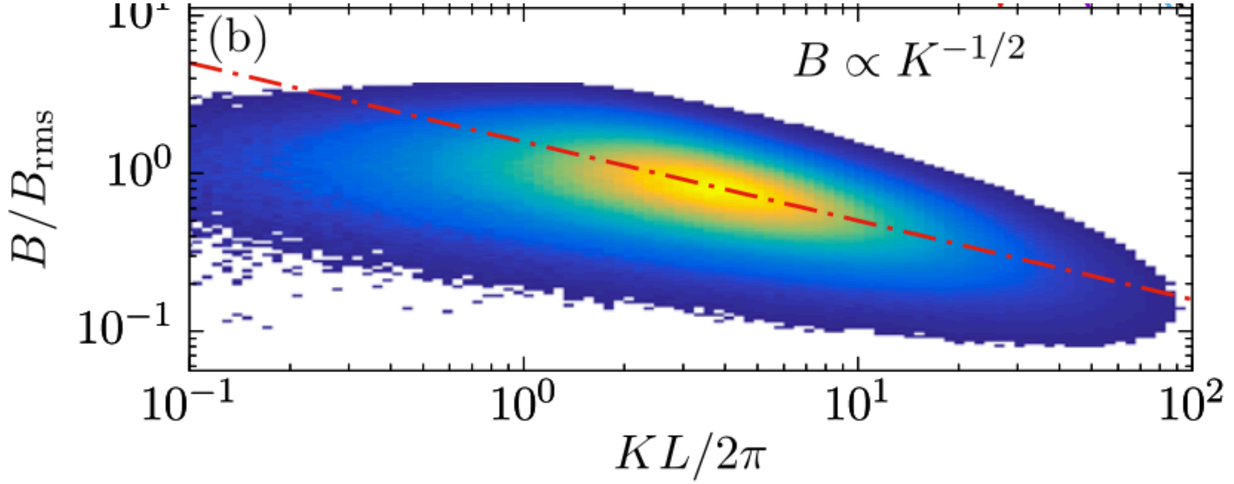
we did a separate run that reached nonlinear and saturation regimes

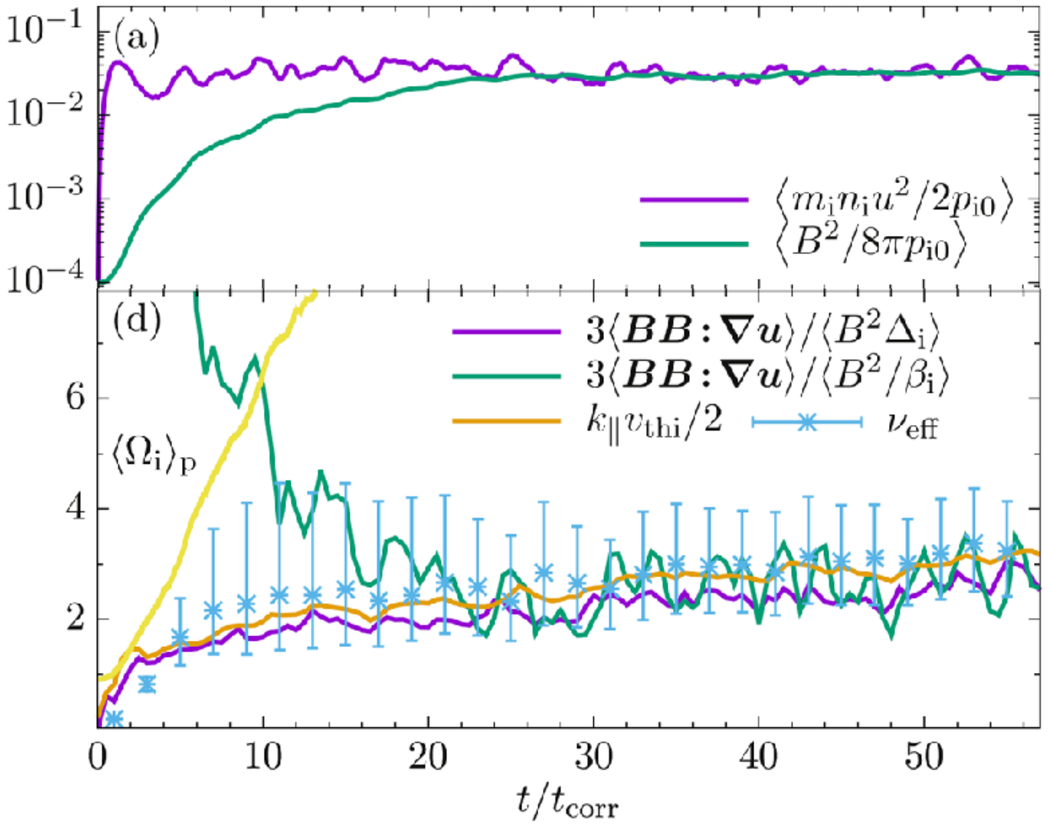




magnetic power migrates to larger scales, above resistive and Larmor

anti-correlation between B and field-line curvature K (i.e., folds, as in Pm ≫ 1)

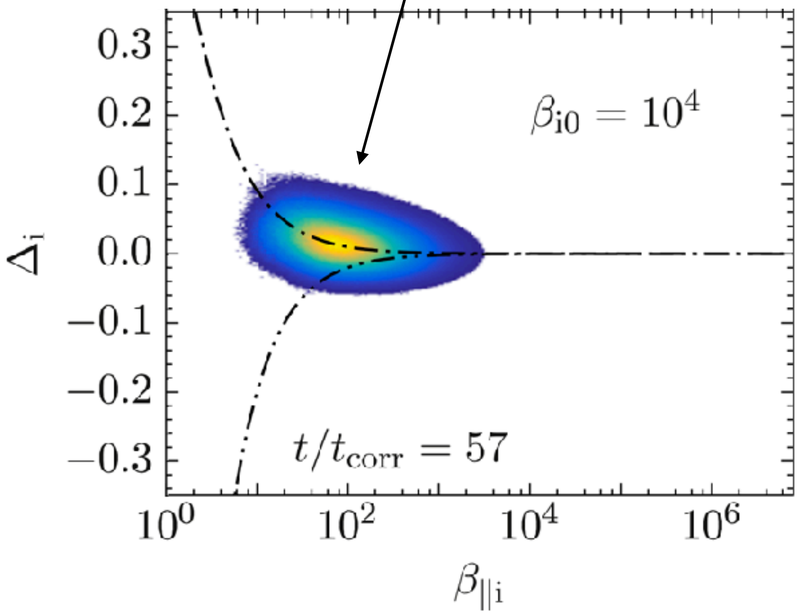




$\langle B^2\rangle \sim \langle u^2\rangle$

collisionality ~ **$\parallel$ flow shear** × β

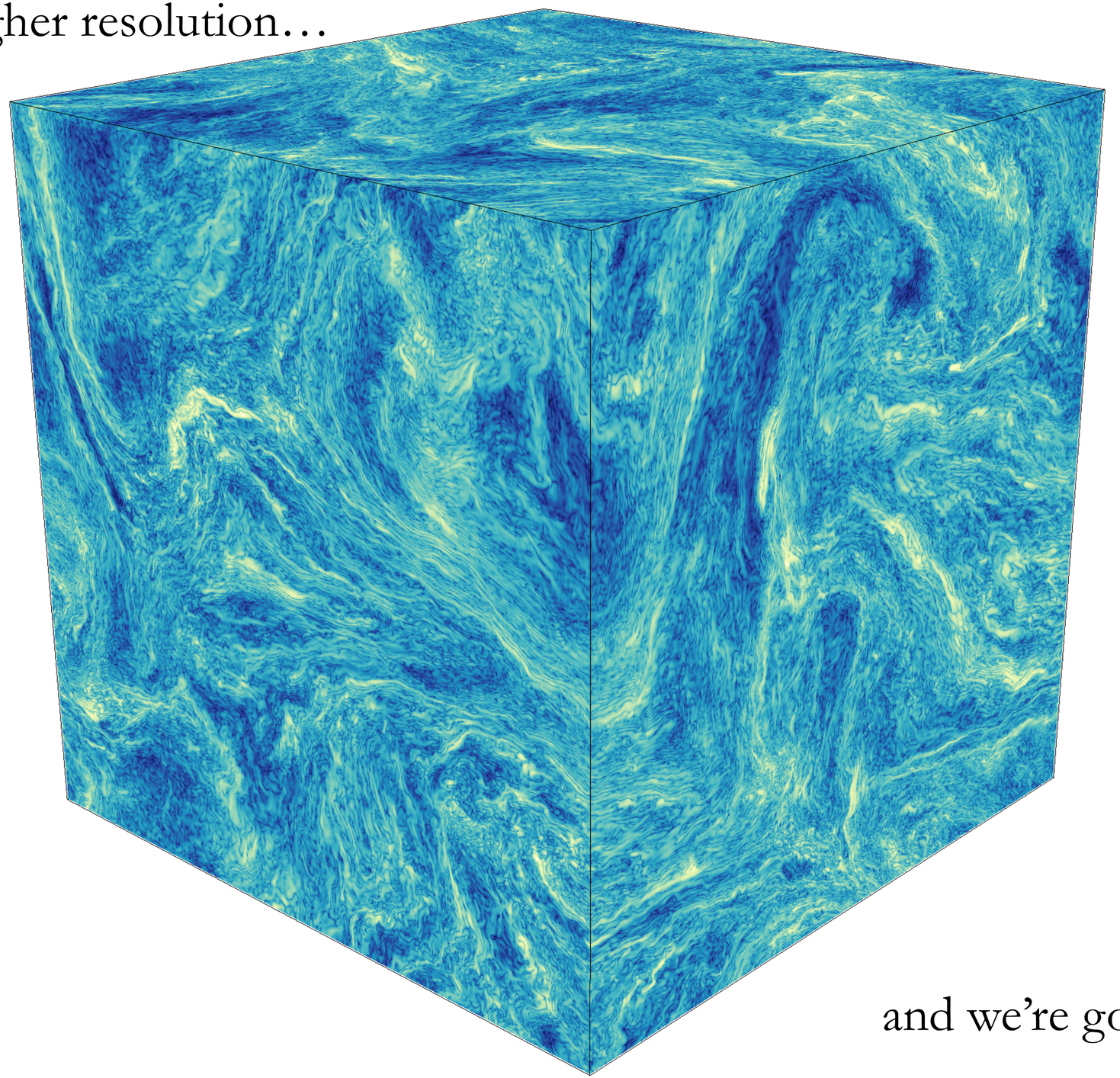
implies tight regulation of p. aniso.
which is indeed seen



<u>problem for testing explosive growth idea</u>:
explosive growth is predicted to onset
in this run just as saturation occurs,
an issue of limited scale separation

at even higher resolution…

1008^3



B

and we're going to ~2000^3 now

some take-aways on **plasma dynamo**

- Turbulent dynamo works in a collisionless plasma (see also Rincon *et al.* 2016), a non-trivial statement! Dynamically strong cosmic magnetic fields owe their existence to kinetic plasma instabilities.

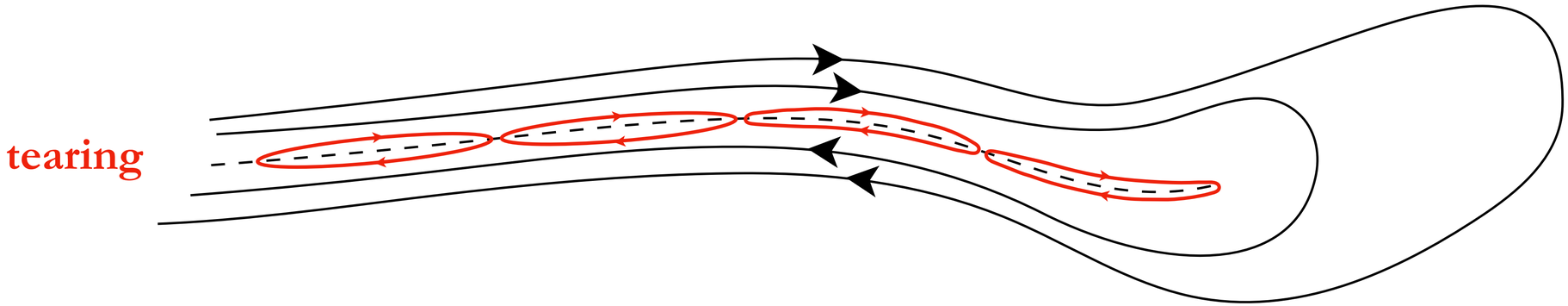
- In many respects, collisionless magnetized plasma behaves as though it were weakly collisional, magnetized fluid with $\mathrm{Pm} \gg 1$. Larmor-scale instabilities easily triggered, induce effective collisionality, and limit departures from LTE; wave-particle interactions supplant particle-particle interactions.

- β-dependent effective viscosity. Modified viscous scale $\ell_{\parallel,\mathrm{visc}} \sim L M_{\mathrm{A}}^{-3} \sim 3$ kpc in Coma
  (which explains I. Zhuravleva's X-ray observations of extended turbulence spectrum …
  galaxy clusters are becoming informative laboratories for frontier plasma physics!)

- β-dependent effective viscosity implies fast turbulent generation of ICM magnetic field
  (we are continuing investigation of *magnetogenesis and dynamo across cosmic time* through NSF CAREER award)

- Hybrid-kinetic and Braginskii-MHD simulations performed and analyzed. Some fun new diagnostics. Some aspects of Braginskii-MHD simulations match behavior in kinetic runs (closure?)

A recent wrinkle in the story…



view of $\mathrm{Pm} \gg 1$ dynamo as efficient generator of thin, elongated current sheets

Simple theory for tearing disruption of folds:

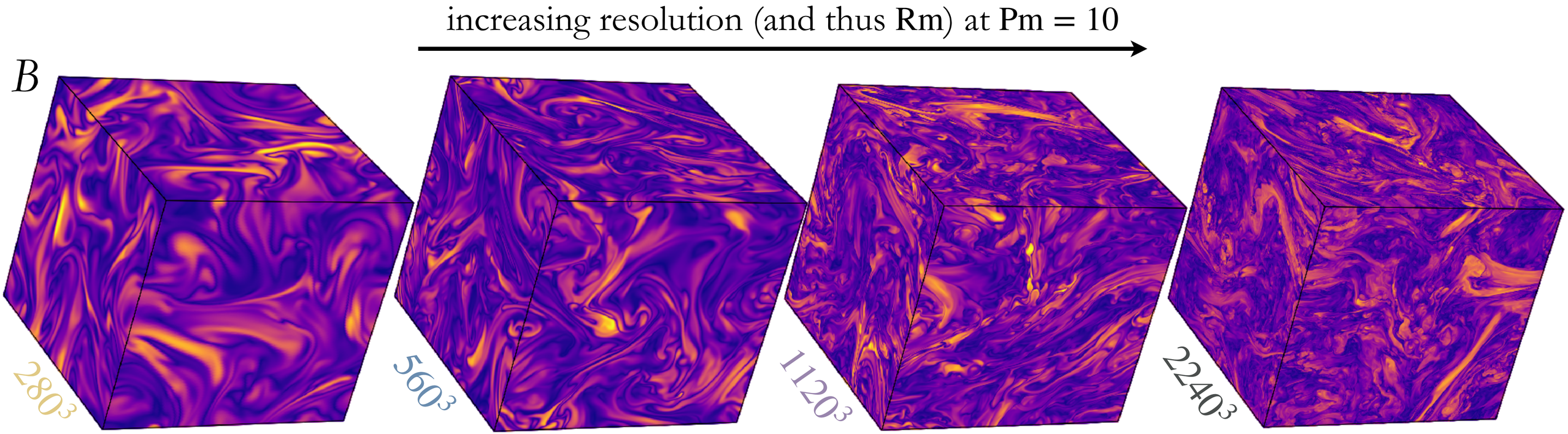
timescale for disruption $\lesssim$ min(lifetime of magnetic fold, diffusive timescale) ?

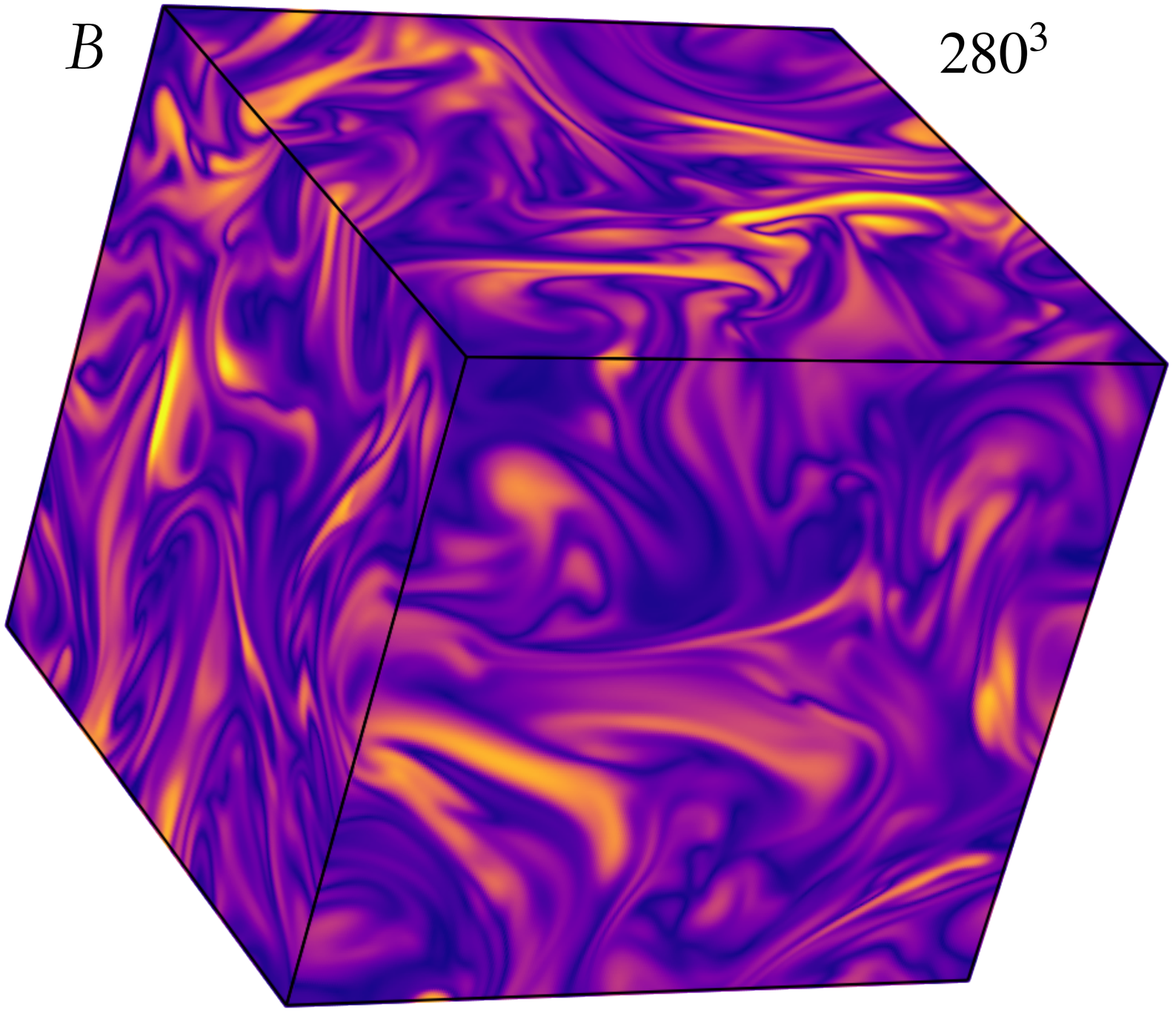
(Galishnikova, Kunz & Schekochihin 2022; Schekochihin, *JPP* review of MHD turbulence)

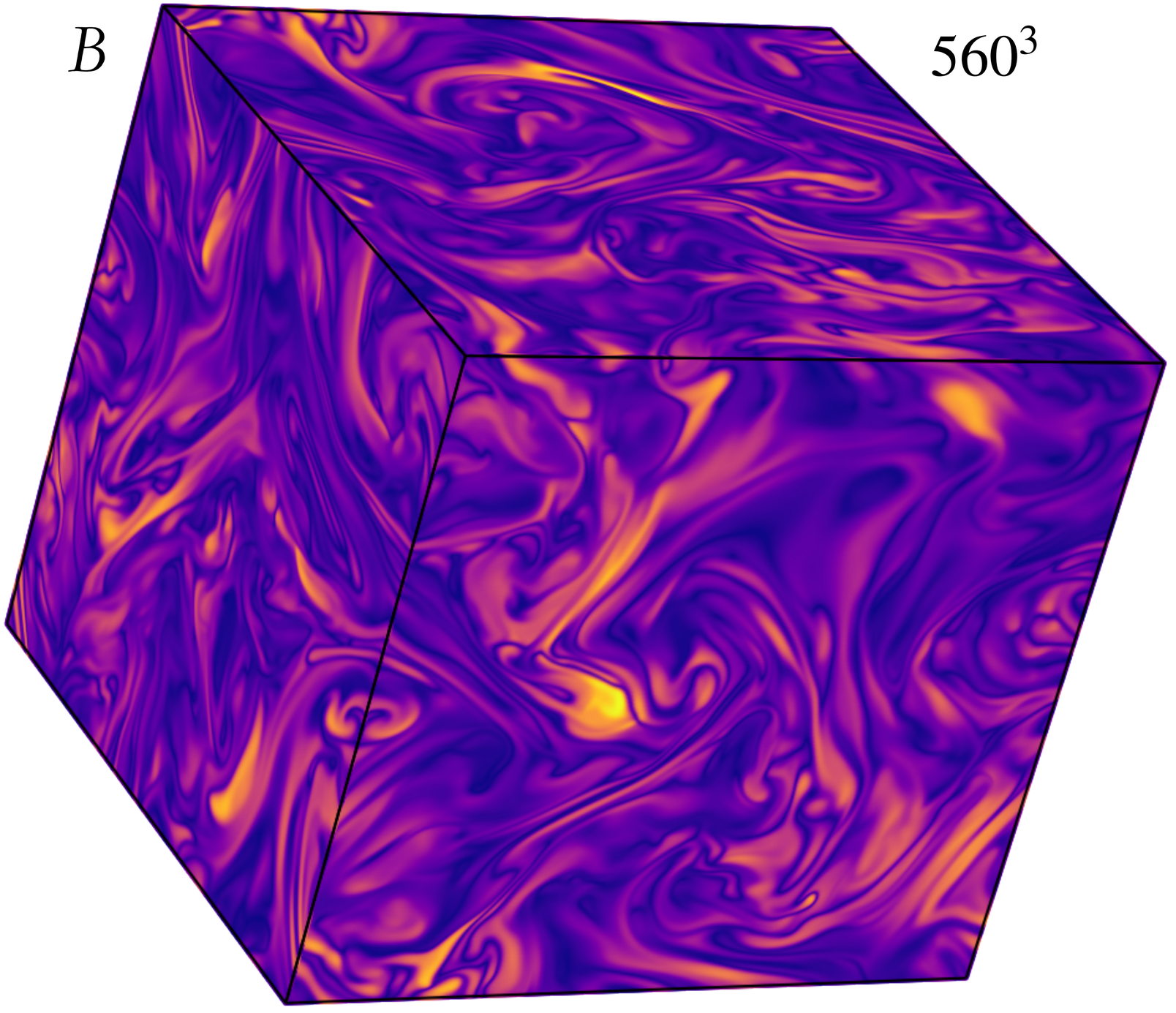
Can show that tearing is *always* slower than resistive diffusion during kinematic stage, but that $\gamma_\mathrm{t}\,\tau_\eta \gtrsim 1$ in nonlinear stage.

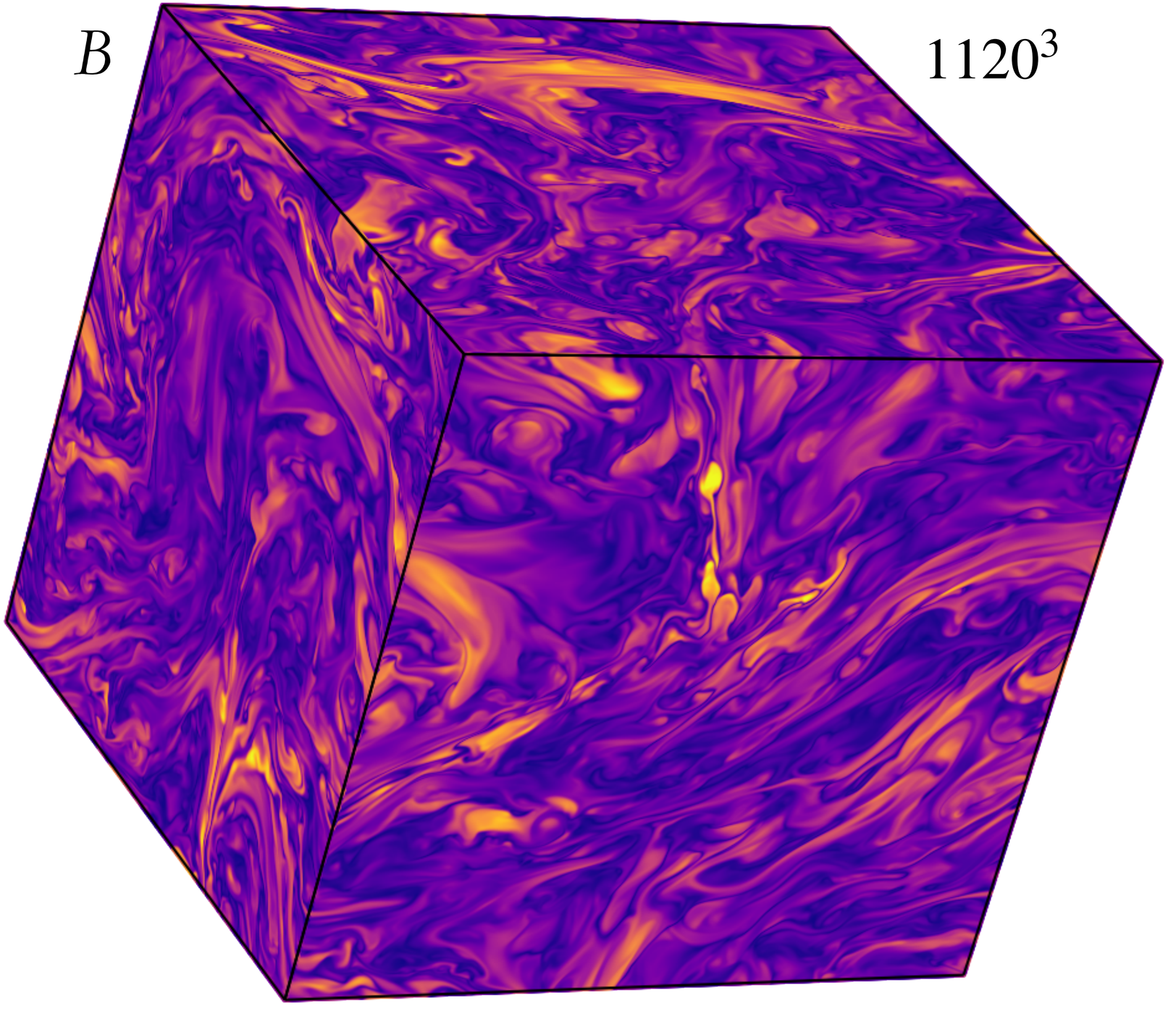
In saturated state, $\gamma_t \gtrsim U/L$ implies tearing of folds beneath "tearing scale" $\lambda_* \sim L\,\mathrm{Rm}^{-3/10}\,(1+\mathrm{Pm})^{-1/5}$

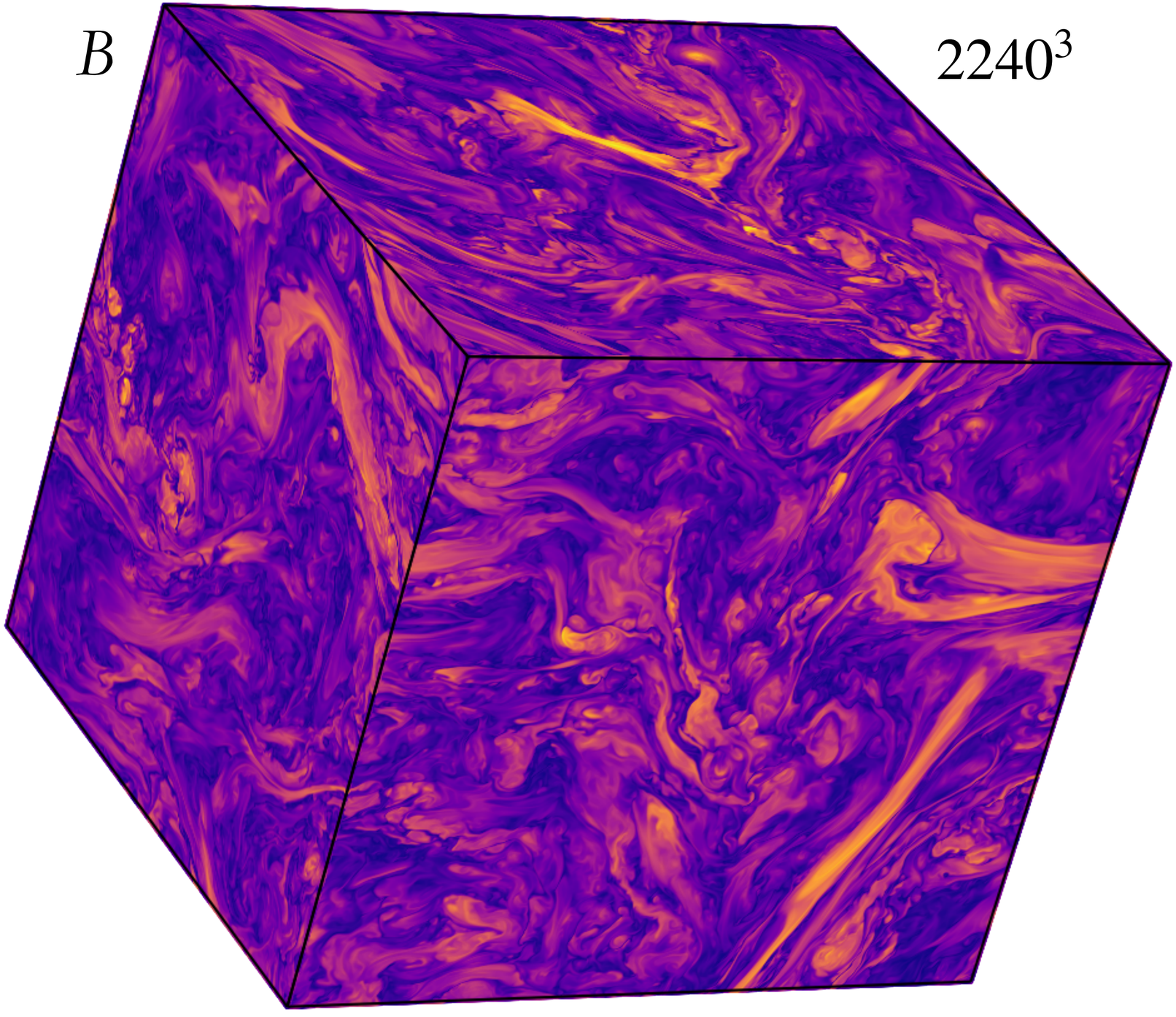
→ magnetic folds in saturation should be broken up into islands



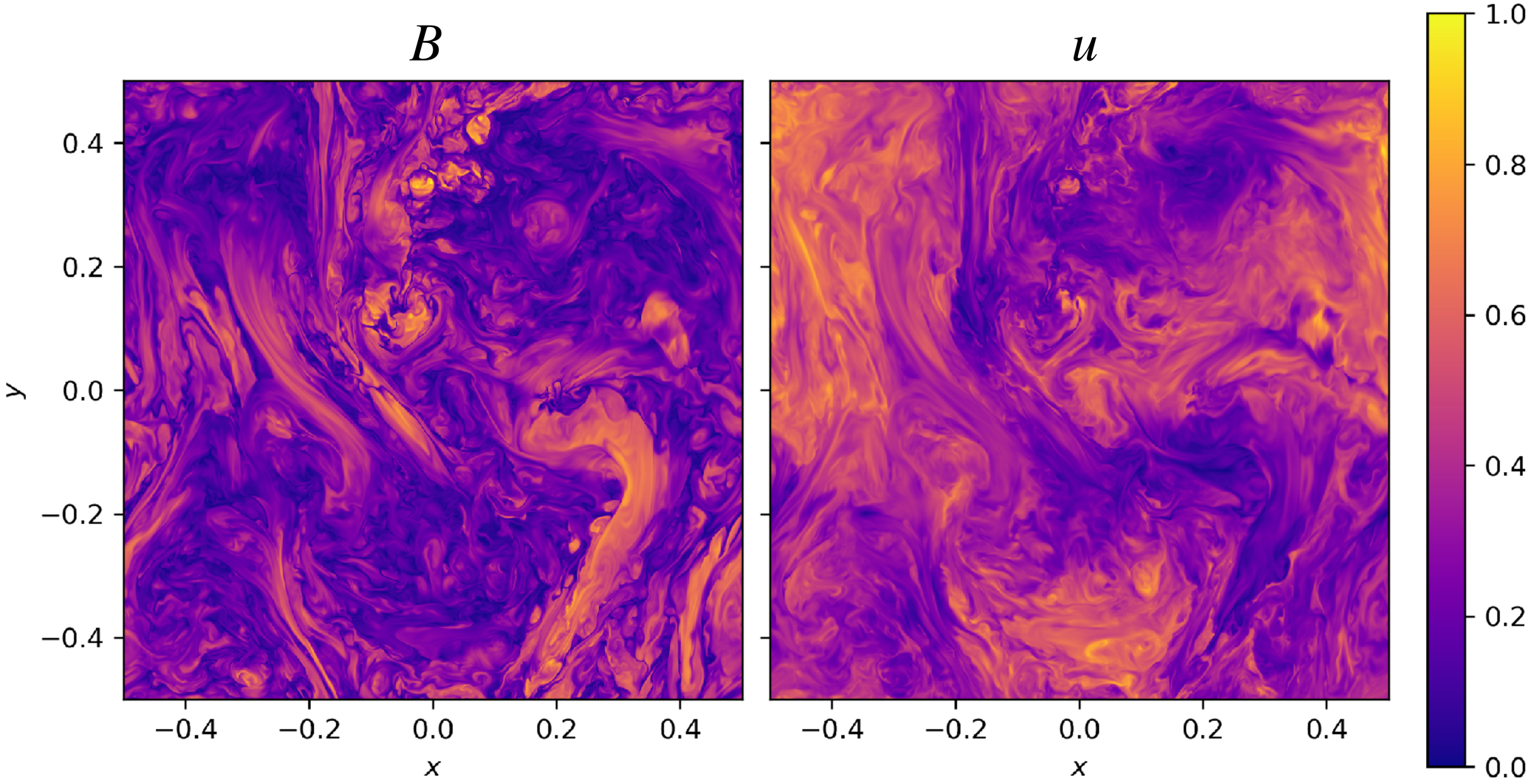

B
280^3


B
560^3

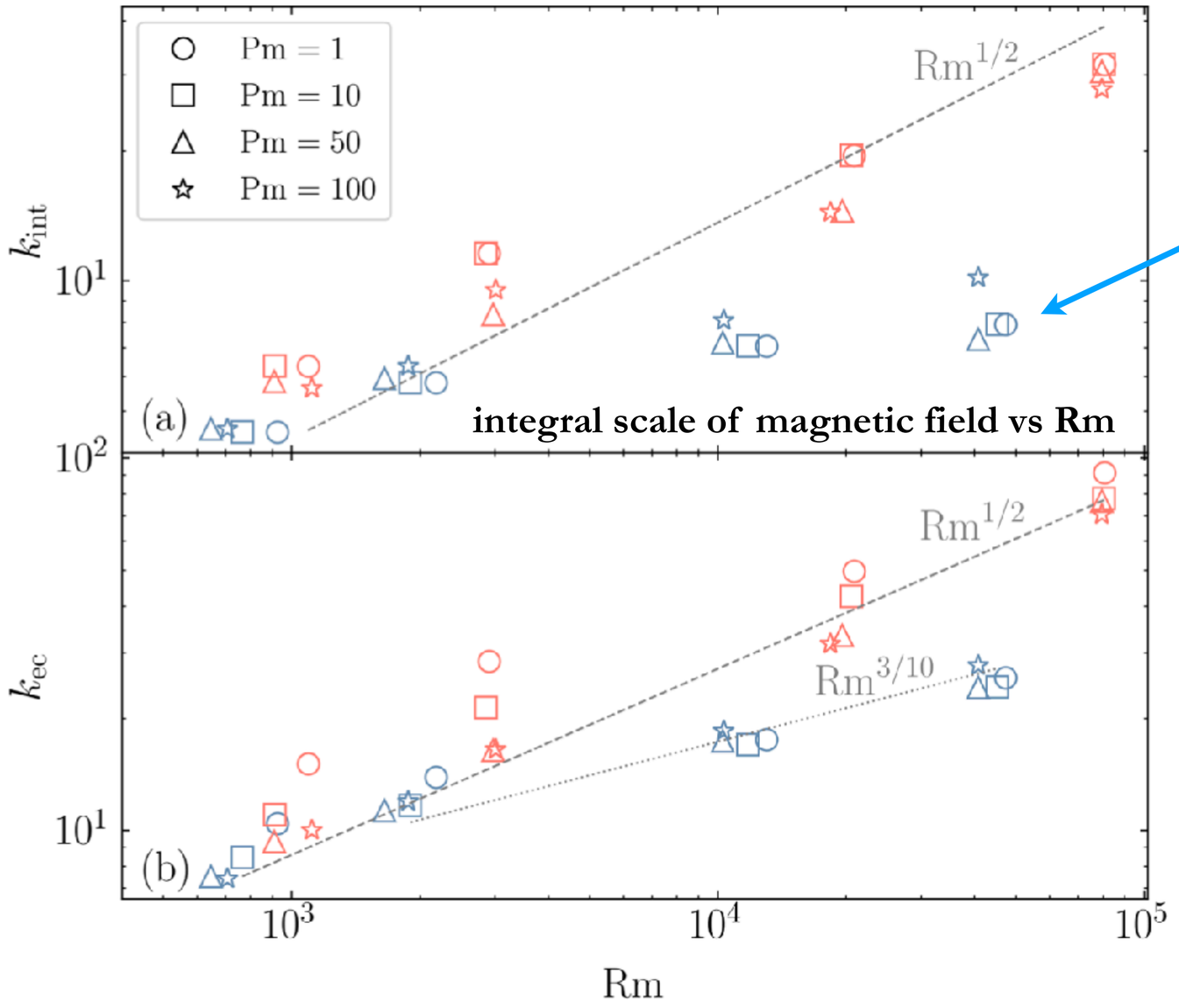

B
1120^3


B
2240^3

at 2240^3, Pm = 10



BONUS!

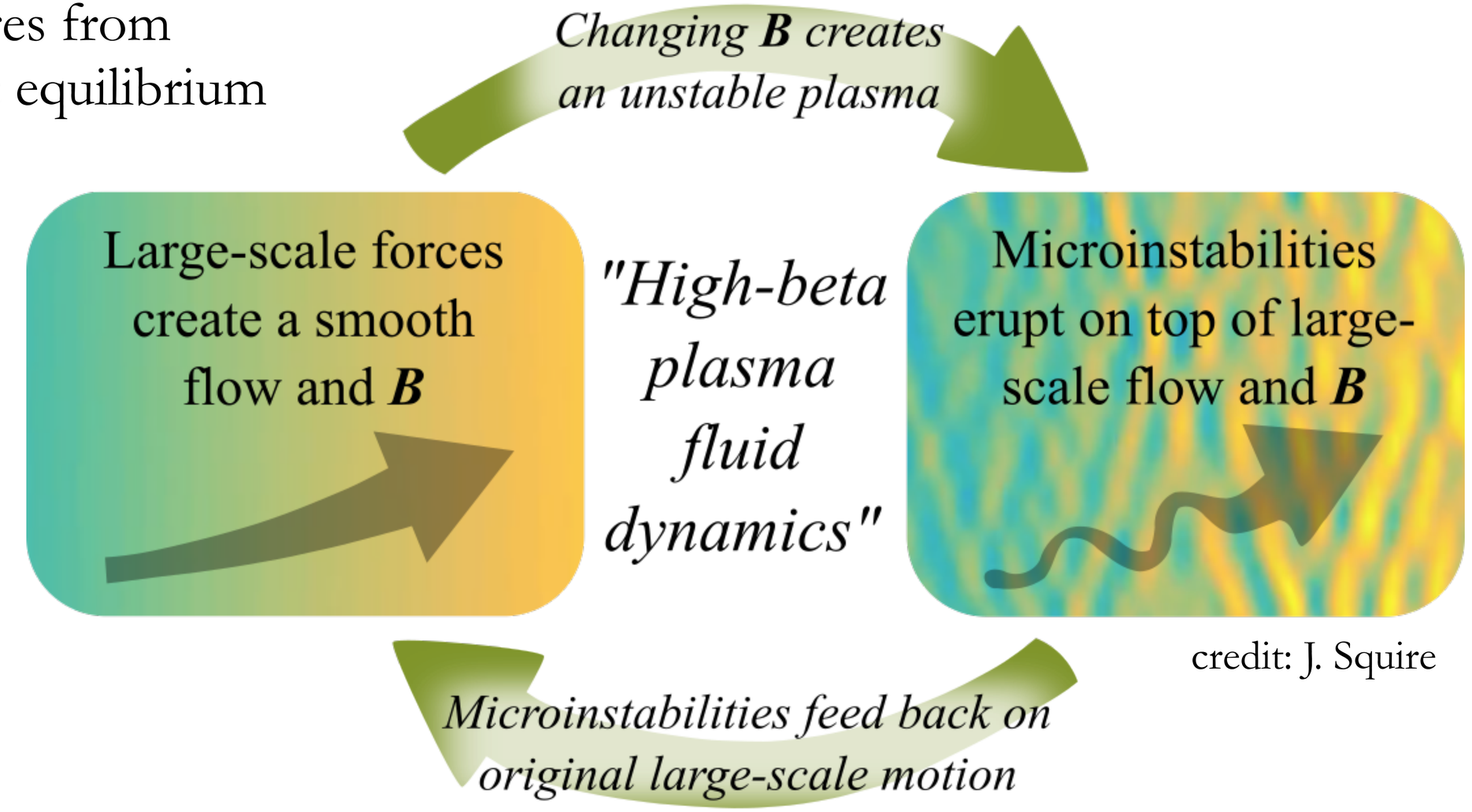


peak of the magnetic energy spectrum
in saturated state is
independent of Rm
at large Rm and low Pm,
within a factor of a few
of driving scale

new numerical result,
comports with old
theoretical suspicions
(Biermann & Schlüter, Subramanian, Beresnyak)
and ICM observations (what little there are…)

see Galishnikova, Kunz & Schekochihin for more

plasma dynamo is just one example:
kinetic Larmor-scale instabilities
restore fluid-like behavior to
collisionless systems by
limiting departures from
local thermodynamic equilibrium



credit: J. Squire

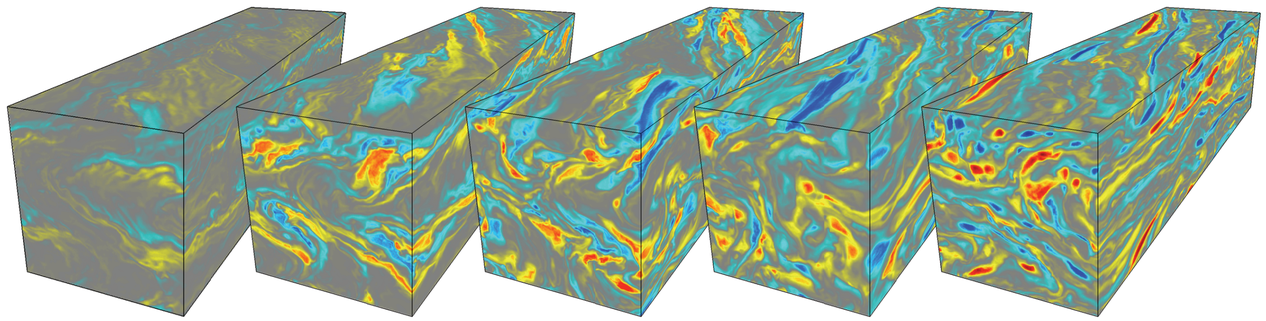
see US plasma decadal white paper by Kunz & Squire, et al.

arXiv:1903.04080

Balbus, Bale, Chen, Churazov, Cowley, Forest, Gammie, Quataert, Reynolds, Schekochihin, Sironi, Spitkovsky, Stone, Zhuravleva, Zweibel

facilitates enhanced angular-momentum transport in collisionless and weakly collisional accretion disks (direct connection between micro and macro):

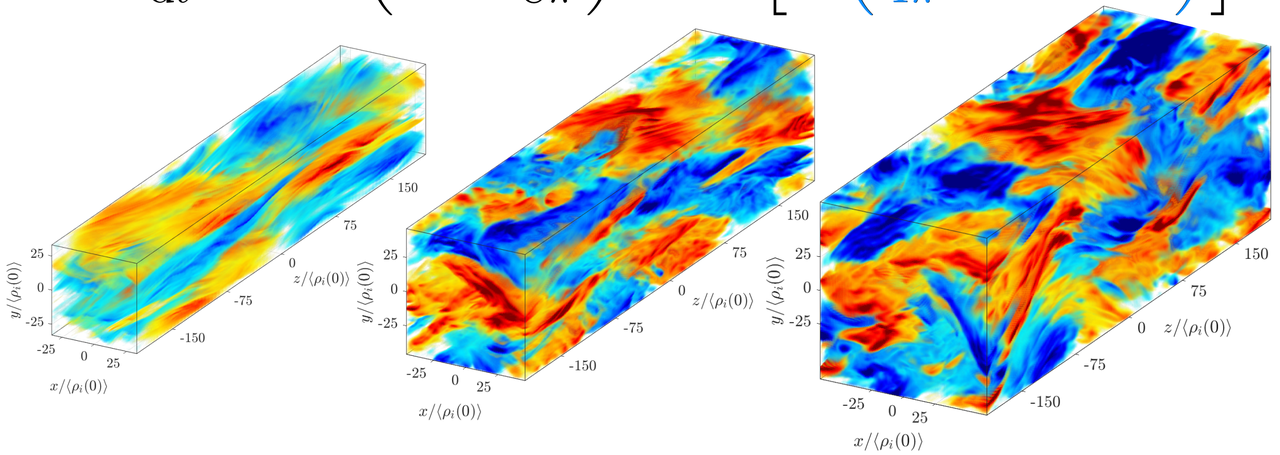
$$T_{R\phi} = \rho u_R u_\phi - \frac{B_R B_\phi}{4\pi}\left(1 + \frac{p_\perp - p_\parallel}{B^2/4\pi}\right)$$



Kunz, Stone & Quataert (2016, *PRL*)
Kempski, Quataert, Squire & Kunz (2019)

modifies critical balance in strong Alfvénic turbulence, application to expanding solar wind (direct connection between micro and macro):

$$mn\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = -\boldsymbol{\nabla}\left(p_\perp + \frac{B^2}{8\pi}\right) + \boldsymbol{\nabla}\cdot\left[\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\left(\frac{B^2}{4\pi} + p_\perp - p_\parallel\right)\right]$$



Bott, Arzamasskiy, Kunz, Quataert & Squire (2021, ApJL)