“When astroparticles meet plasma turbulence”

micro-physics of cosmic-ray transport in the Galaxy

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LAGRANGE

OBSERVATOIRE DE LA CÔTE D’AZUR
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Collaborators:

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Lagrange Seminar — 28 June 2022

Outline

1. **Context & Motivation**

    ☞ The cosmic-ray (CR) spectrum and its implications

2. **CR diffusion in magneto-hydrodynamic (MHD) turbulence**

    ☞ Alfvénic turbulence: inefficiency of CR scattering

    ☞ Contributions of other MHD modes

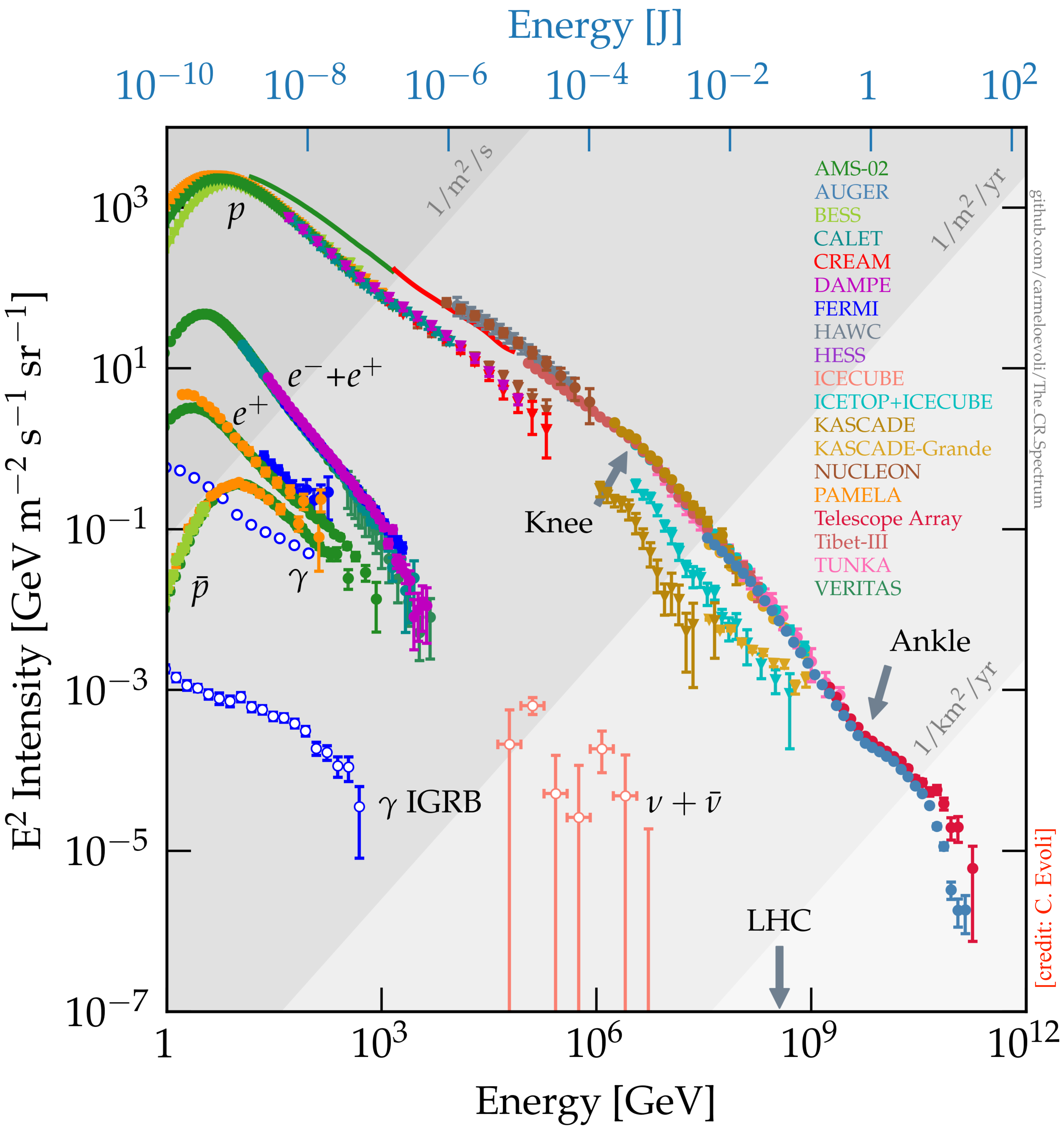
3. **Recent numerical results** [➥ Fornieri et al., MNRAS (2021)]

    ☞ Diffusion-coefficient solver for MHD modes

    ☞ The DRAGON code: global CR-transport simulations

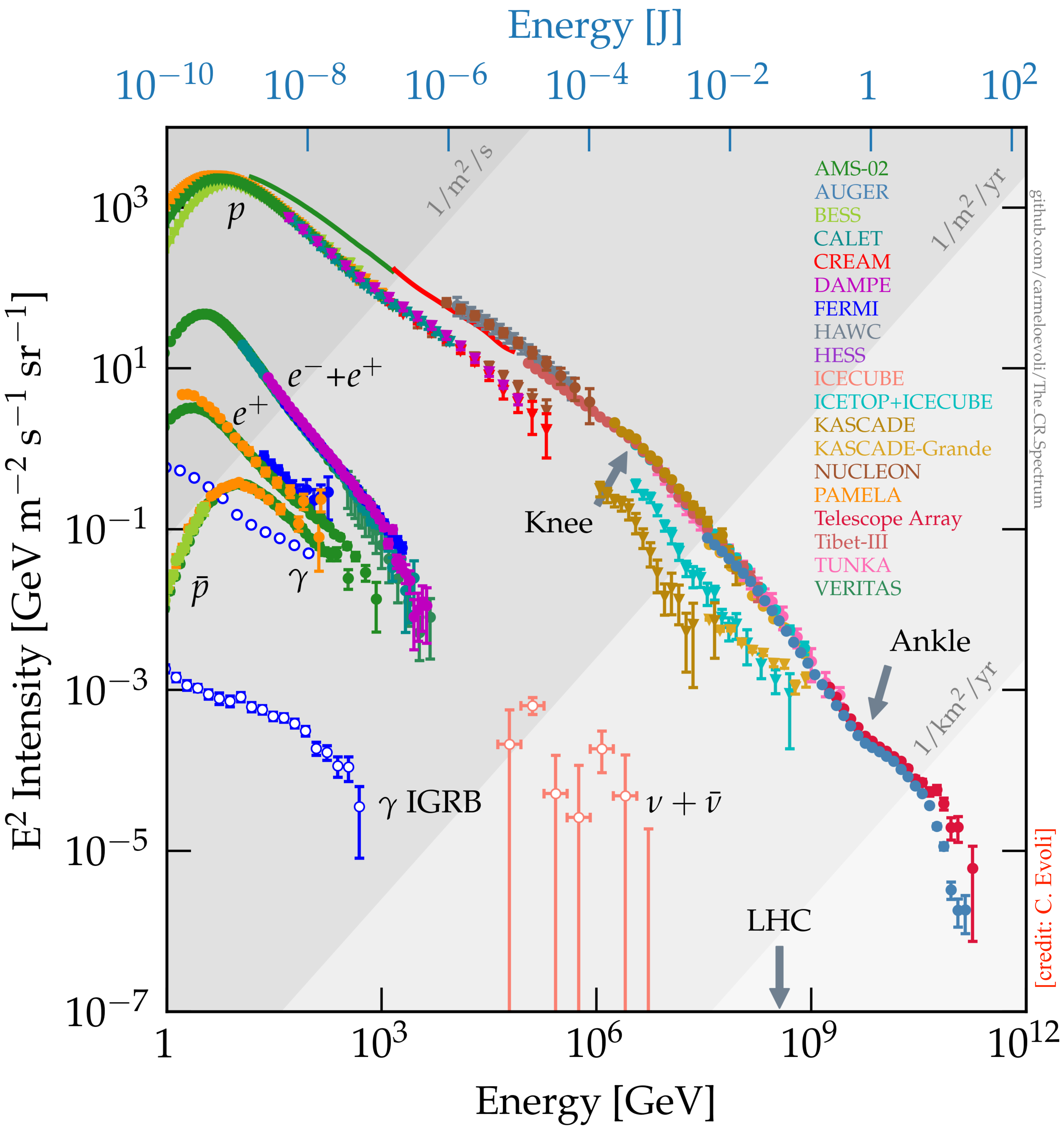
4. **Perspectives**

The cosmic-ray spectrum



- "Solar modulation" at E ≲ 30 GeV
- "Knee" at E ~ 10^6-10^7 GeV = 1-10 PeV
- "Ankle" at E ~ 10^9-10^{10} GeV = 1-10 EeV

The cosmic-ray spectrum



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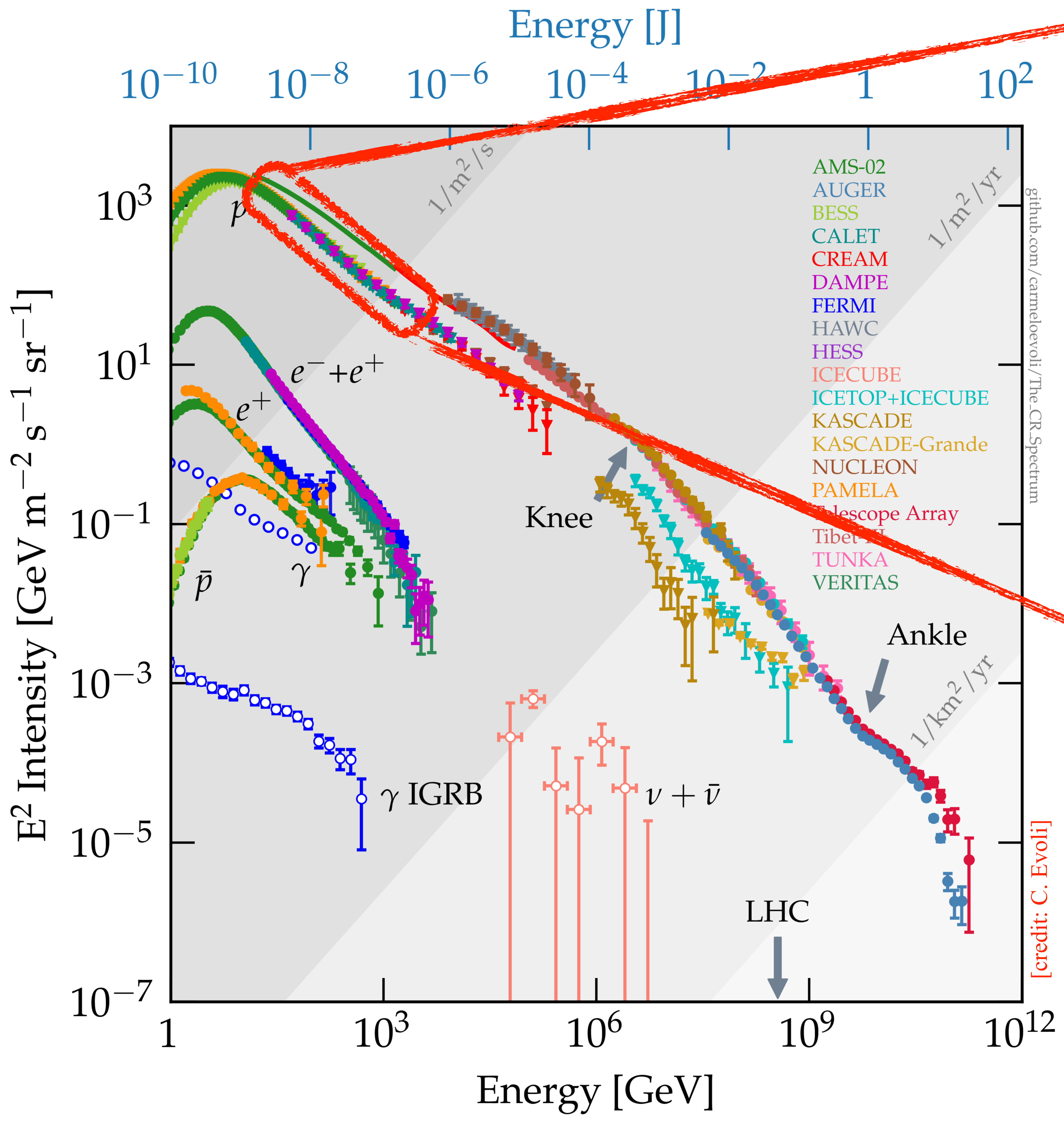
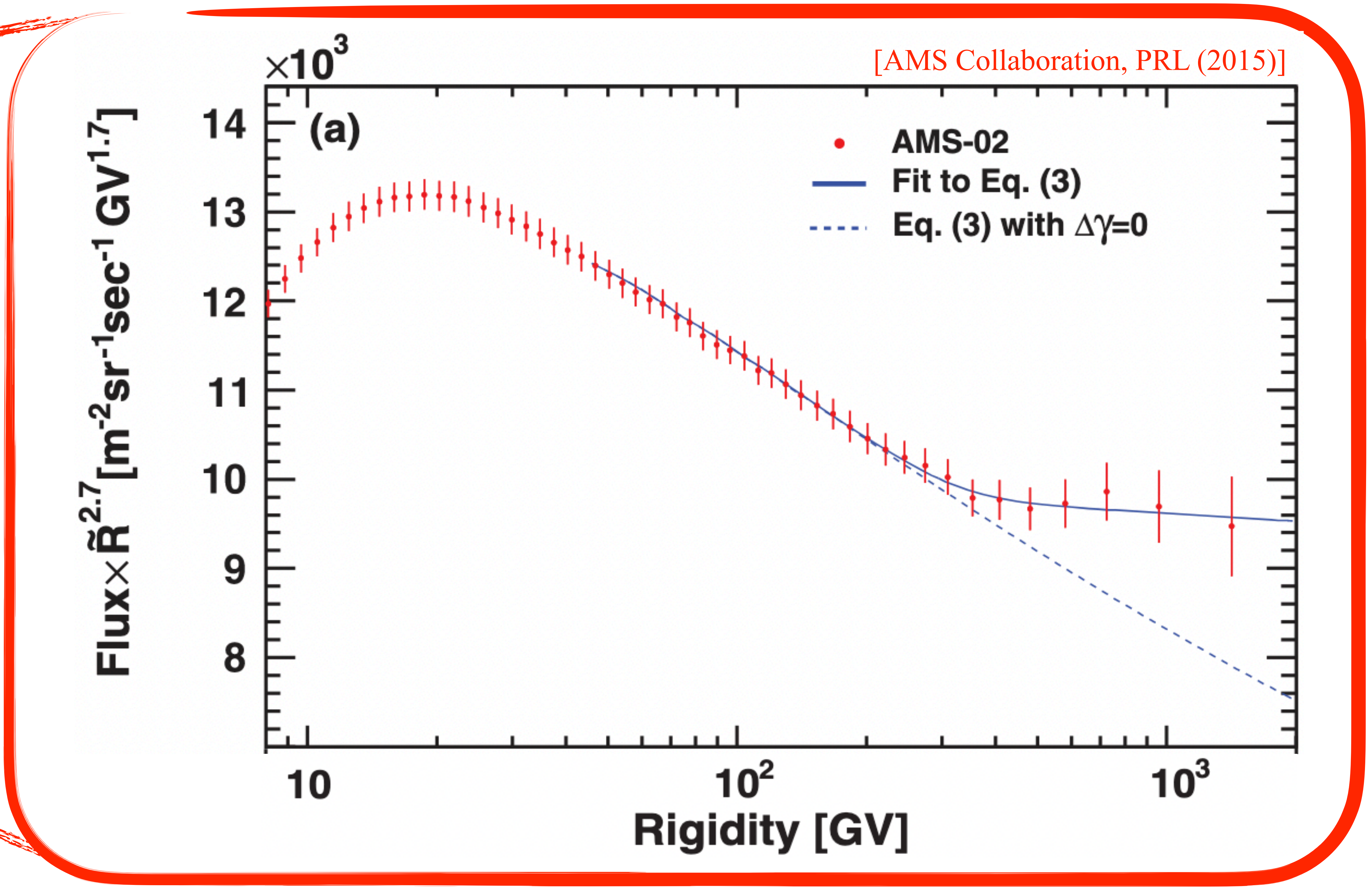
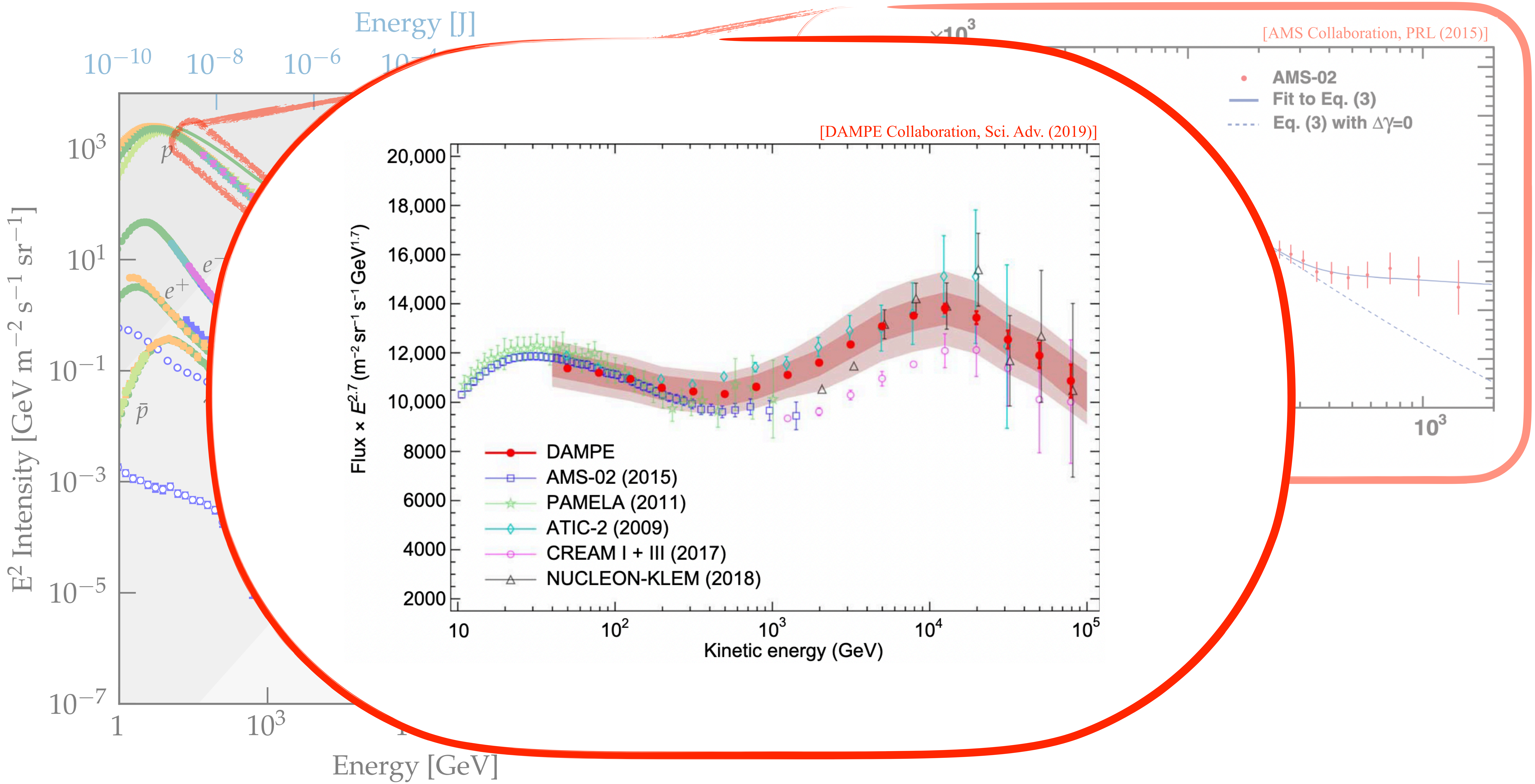
…but the devil is in the details

The cosmic-ray spectrum



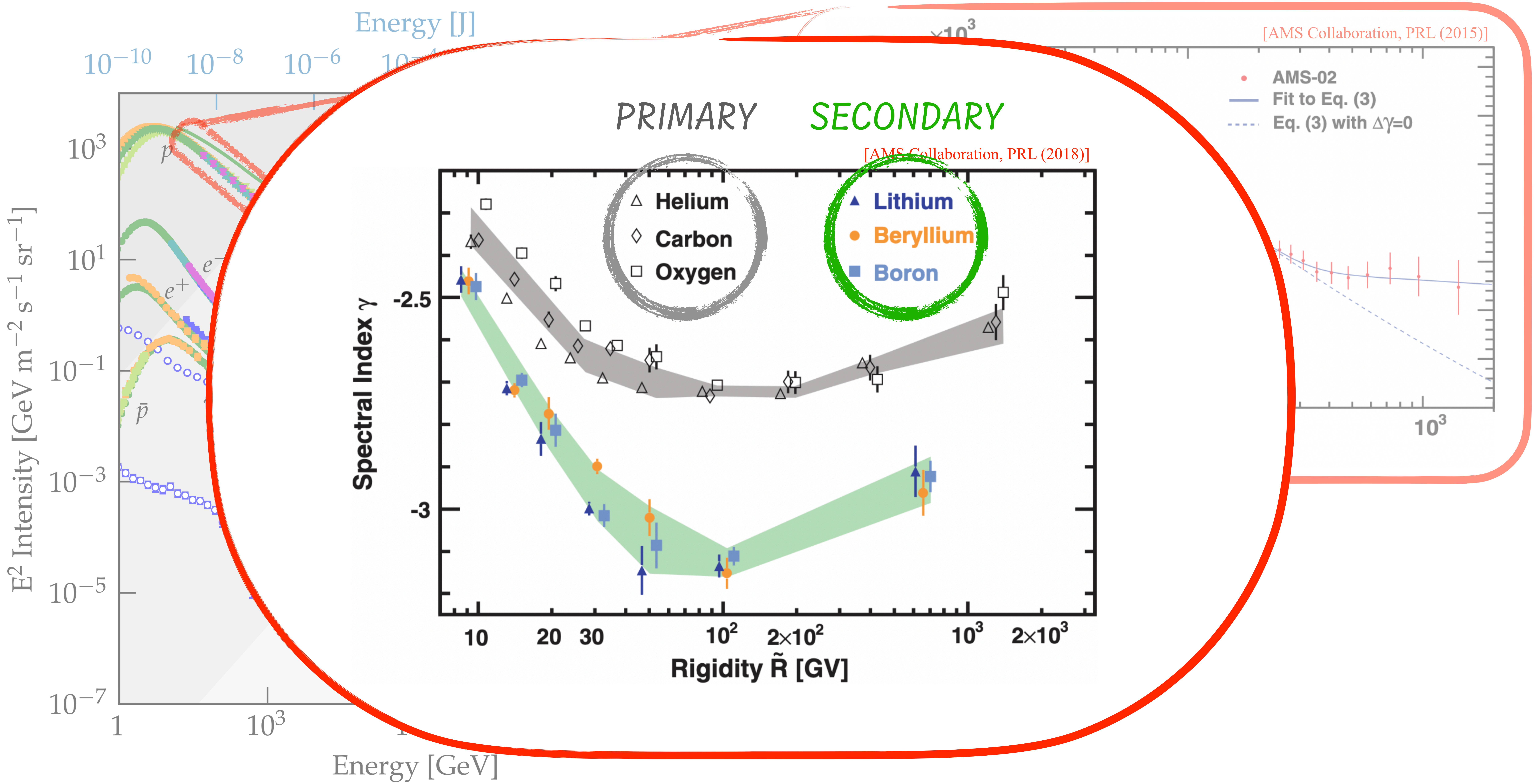


The cosmic-ray spectrum



The cosmic-ray spectrum

The cosmic-ray spectrum

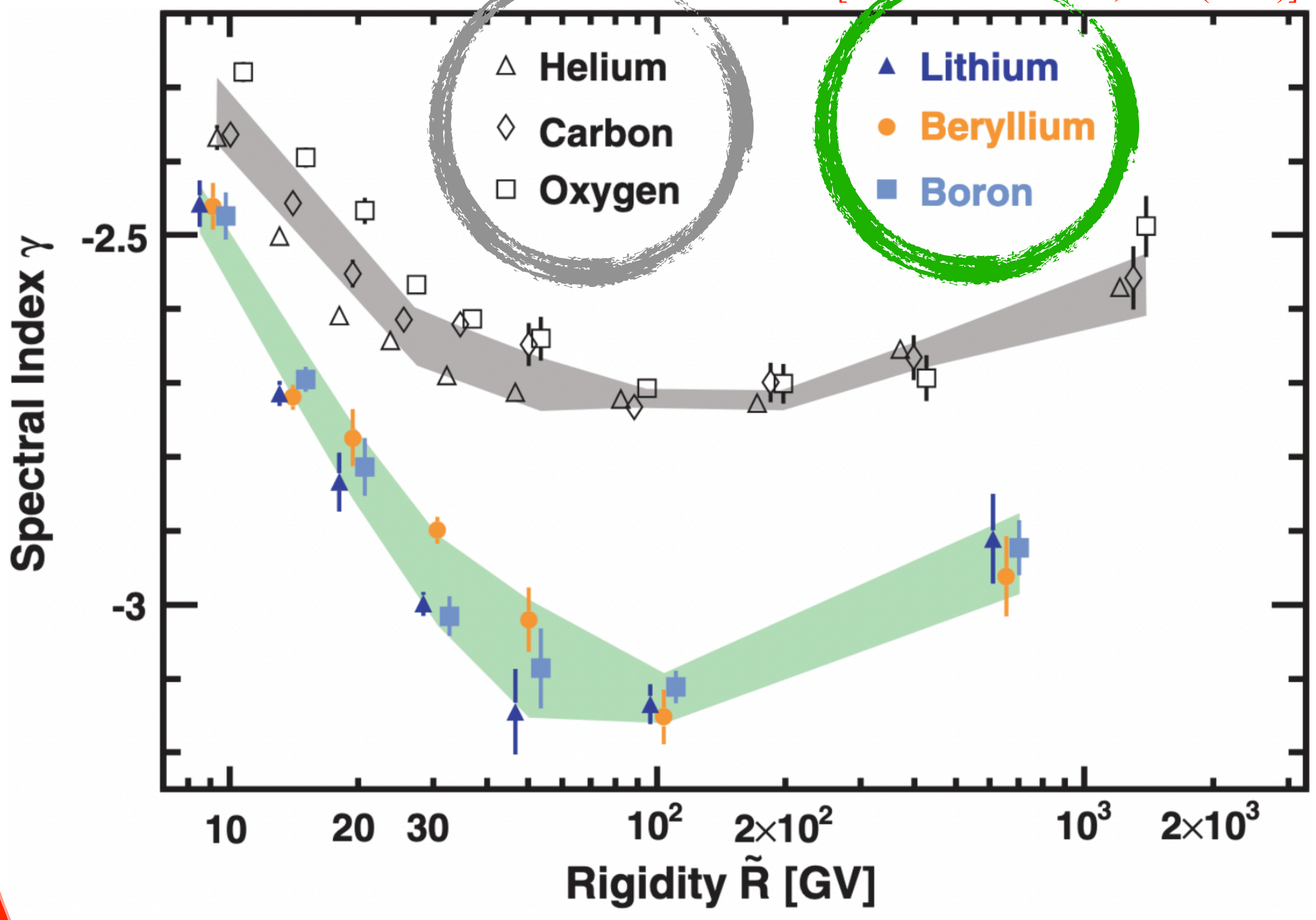


The cosmic-ray spectrum

PRIMARY

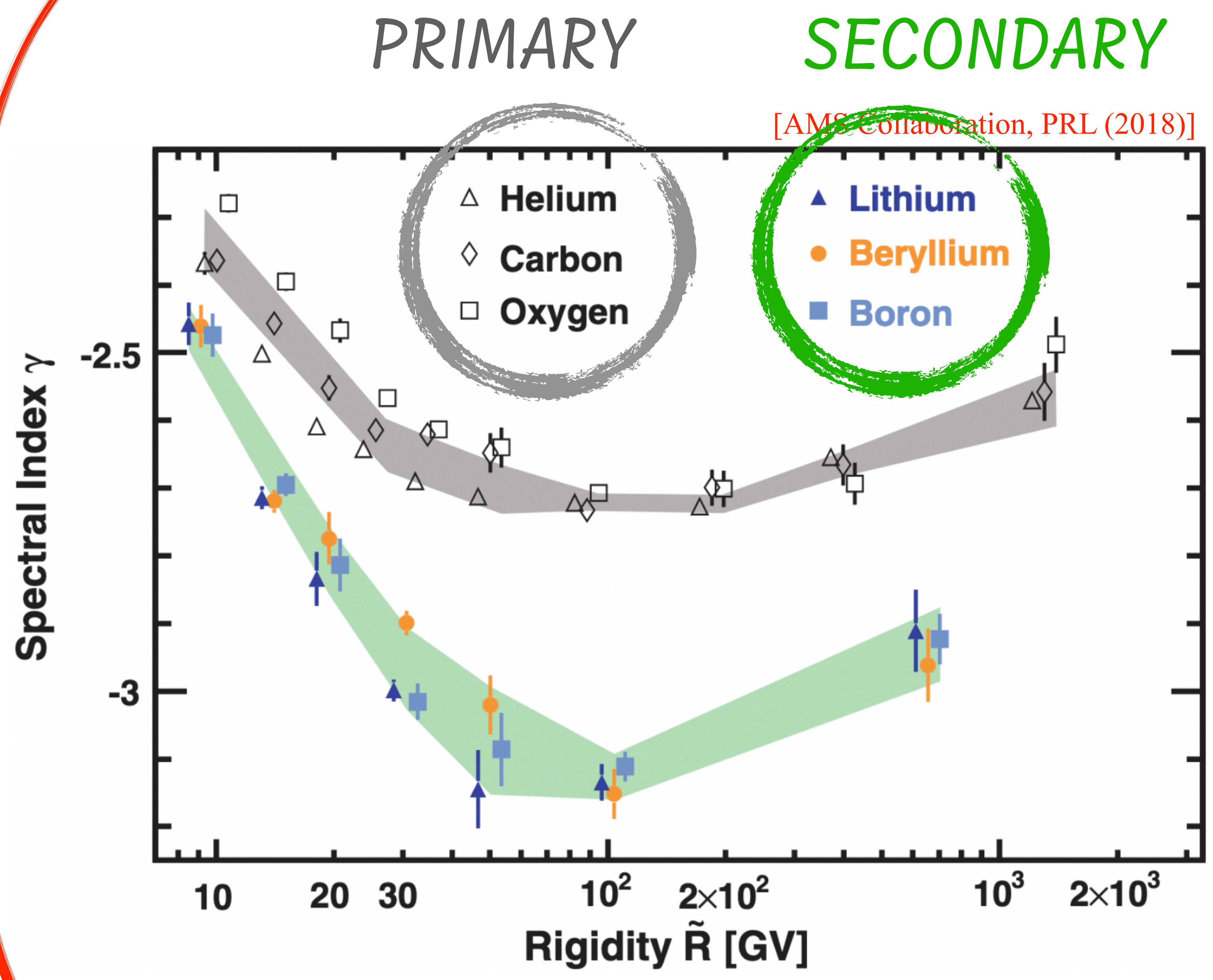
SECONDARY

[AMS Collaboration, PRL (2018)]



$$N_{\rm inj}(E) \propto E^{-\gamma_{\rm inj}}$$

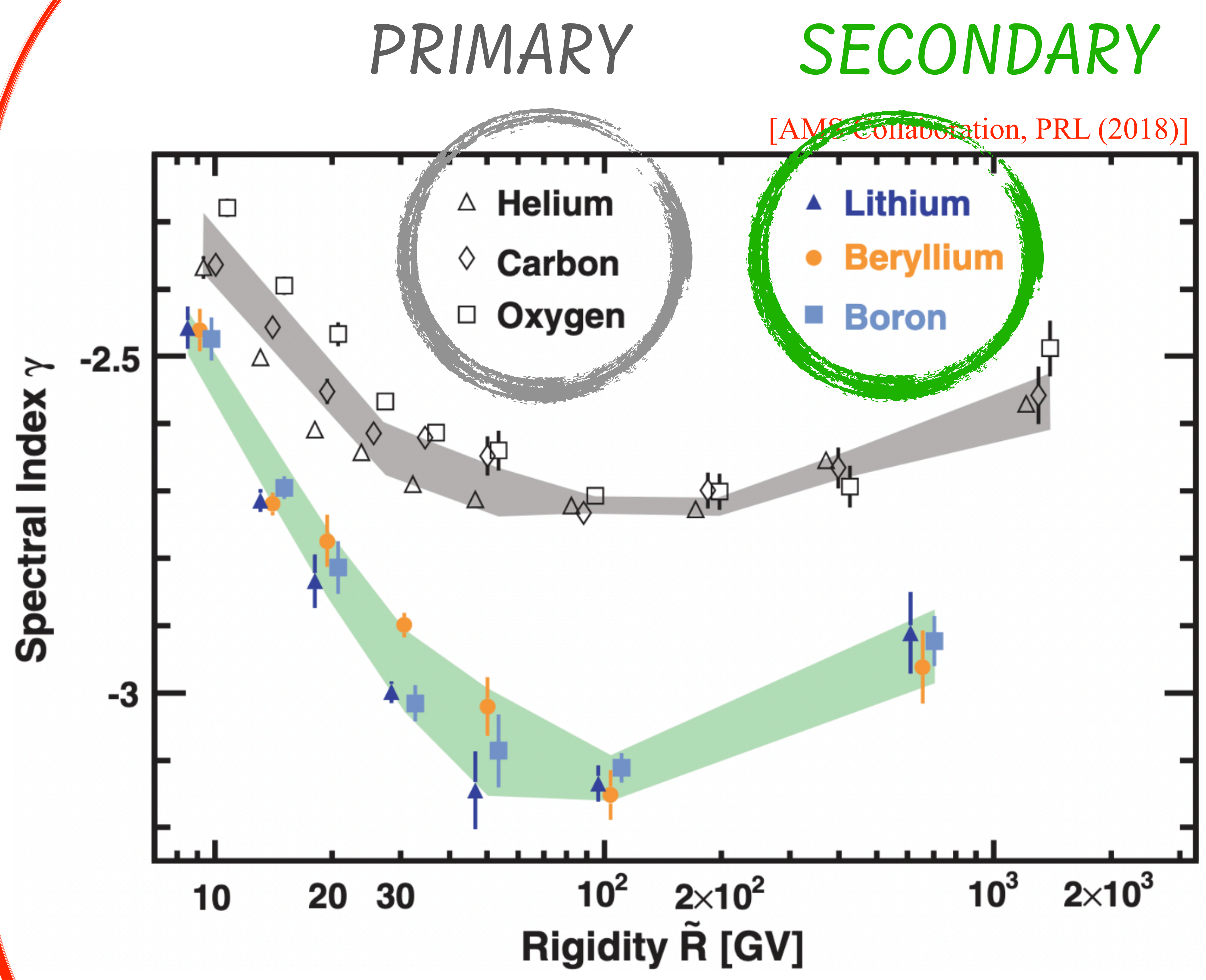
The cosmic-ray spectrum



$$N_{\rm inj}(E) \propto E^{-\gamma_{\rm inj}}$$

Plasma micro-physics related to
injection/acceleration at SNRs shocks
[✗ *not treated in this talk*]

The cosmic-ray spectrum



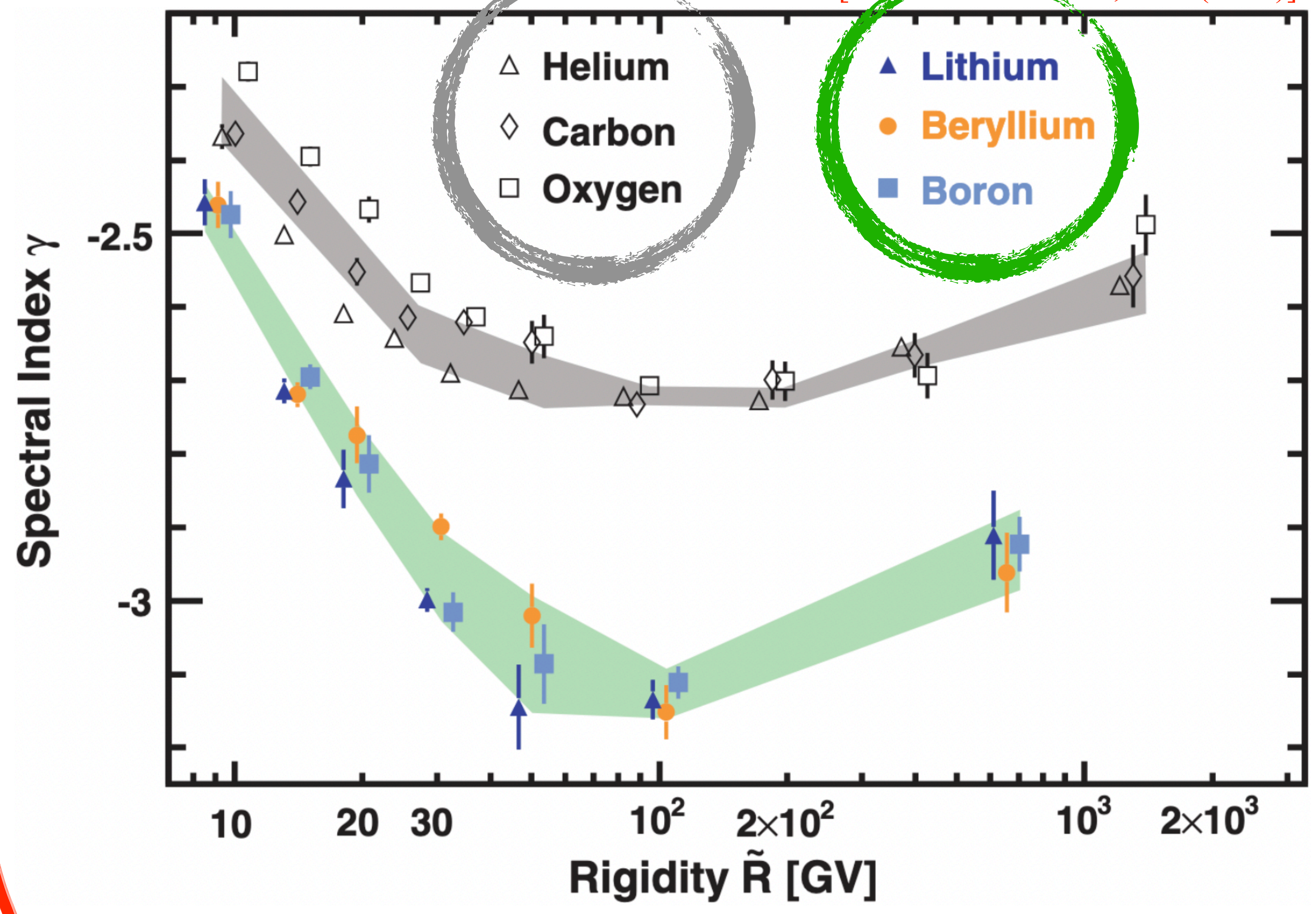
$$N_{\text{inj}}(E) \propto E^{-\gamma_{\text{inj}}}$$

$$D(E) \propto E^{\delta}$$

The cosmic-ray spectrum

PRIMARY SECONDARY

[AMS Collaboration, PRL (2018)]



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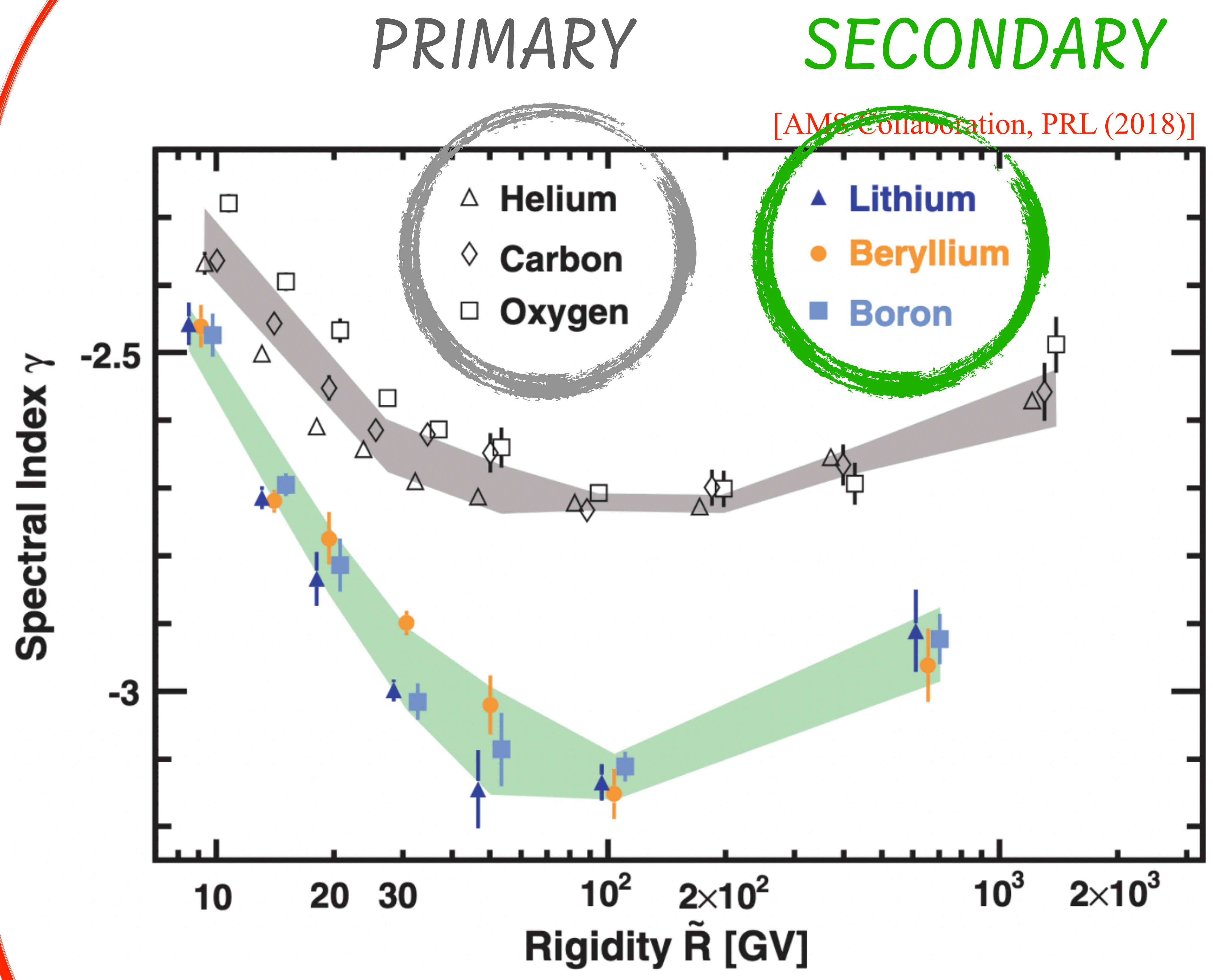
Plasma micro-physics related to
CR-turbulence interaction

The cosmic-ray spectrum



$$N_{\rm inj}(E) \propto E^{-\gamma_{\rm inj}}$$

$$D(E) \propto E^{\delta}$$

Plasma micro-physics related to
CR-turbulence interaction

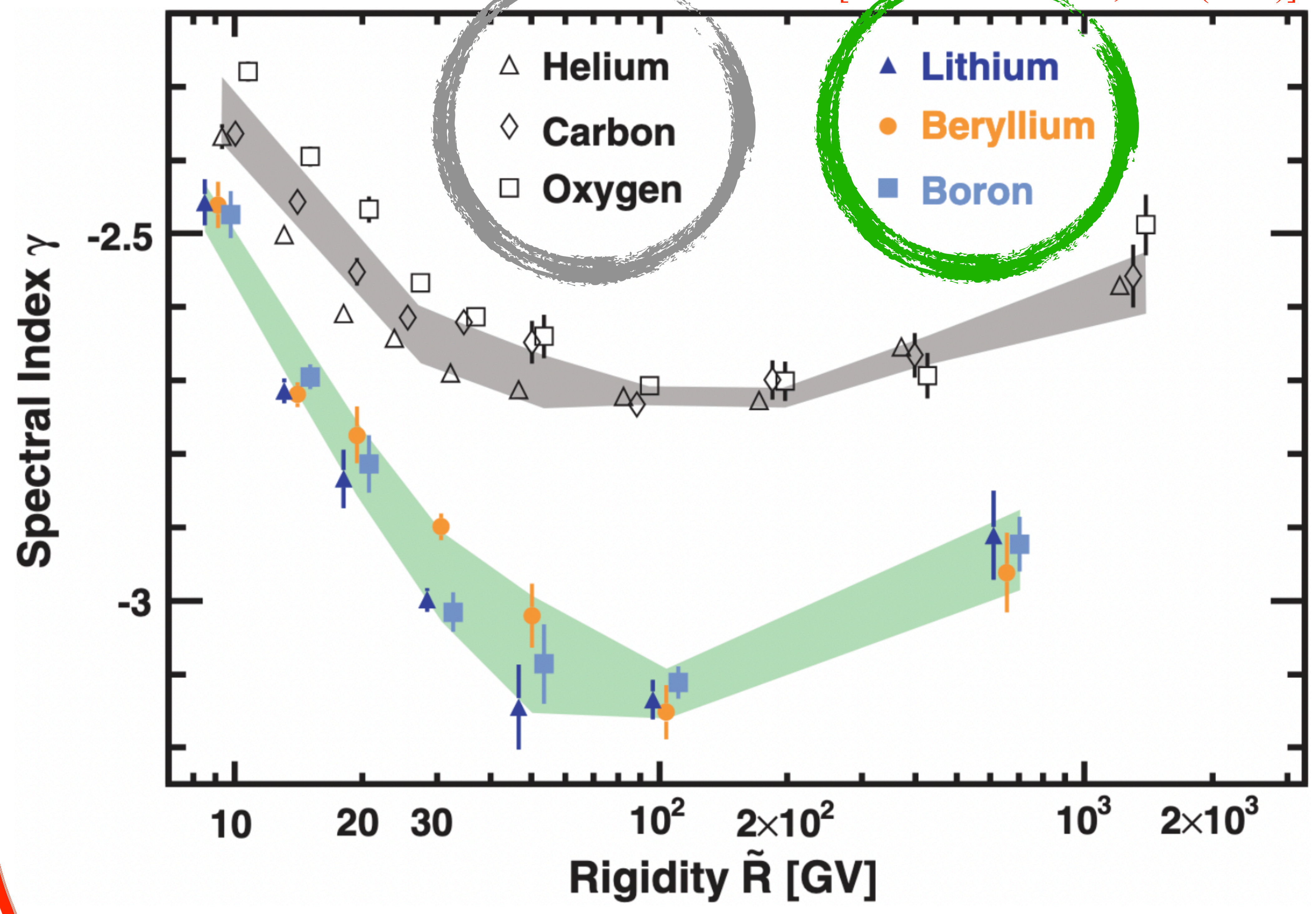
- self-generated turbulence (E ≲ 200 GeV)
  [✗ *not treated in this talk*]
- ***pre-existing turbulence*** (E ≳ 200 GeV)
  [✔ *this talk will focus on this*]

The cosmic-ray spectrum

PRIMARY

SECONDARY

[AMS Collaboration, PRL (2018)]



$$N_{\rm inj}(E) \propto E^{-\gamma_{\rm inj}}$$

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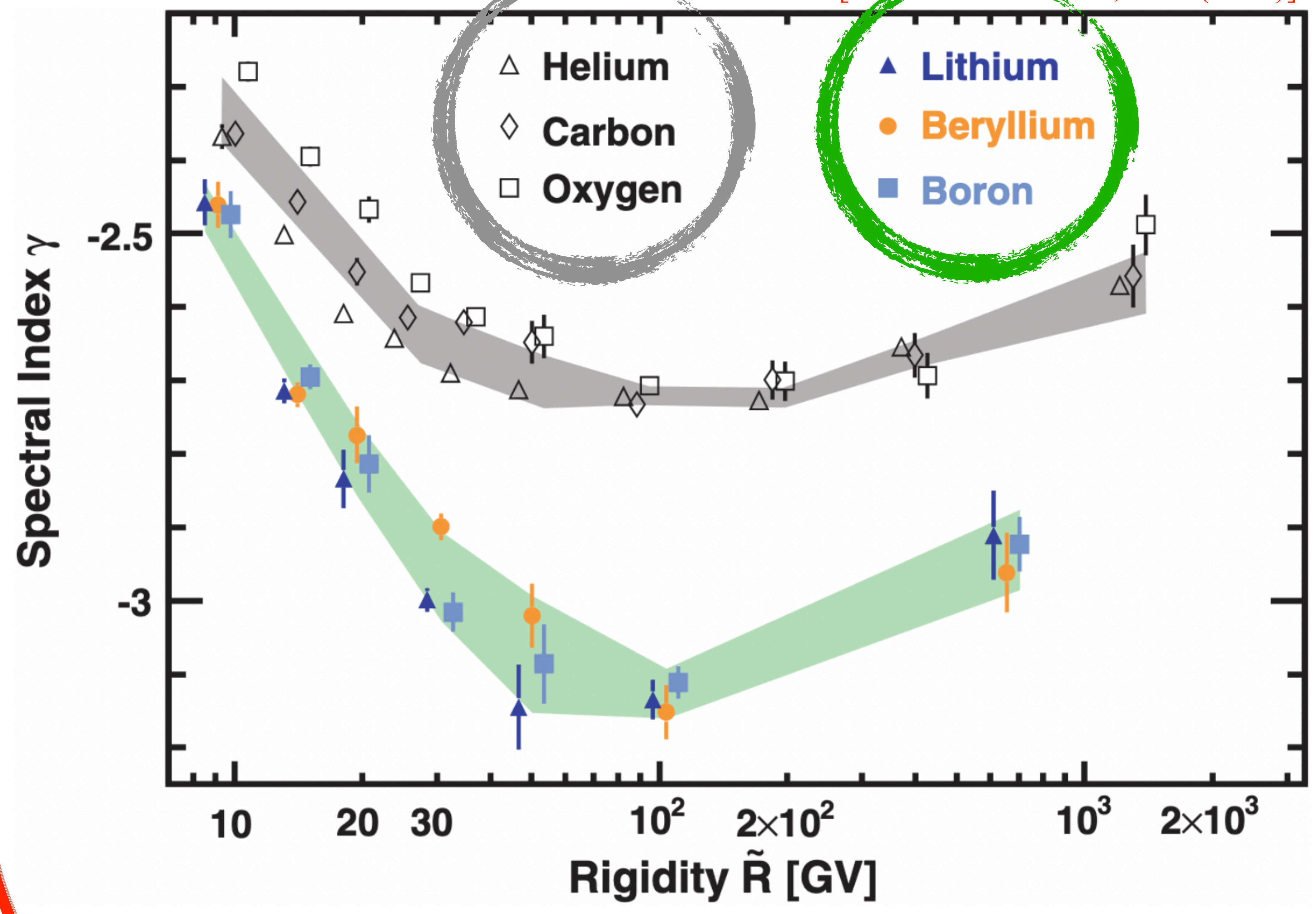
$$\Rightarrow \qquad F(E) \sim \frac{N(E)}{D(E)}$$

The cosmic-ray spectrum

PRIMARY SECONDARY

[AMS Collaboration, PRL (2018)]



$$N_{\rm inj}(E) \propto E^{-\gamma_{\rm inj}}$$

$$D(E) \propto E^{\delta}$$

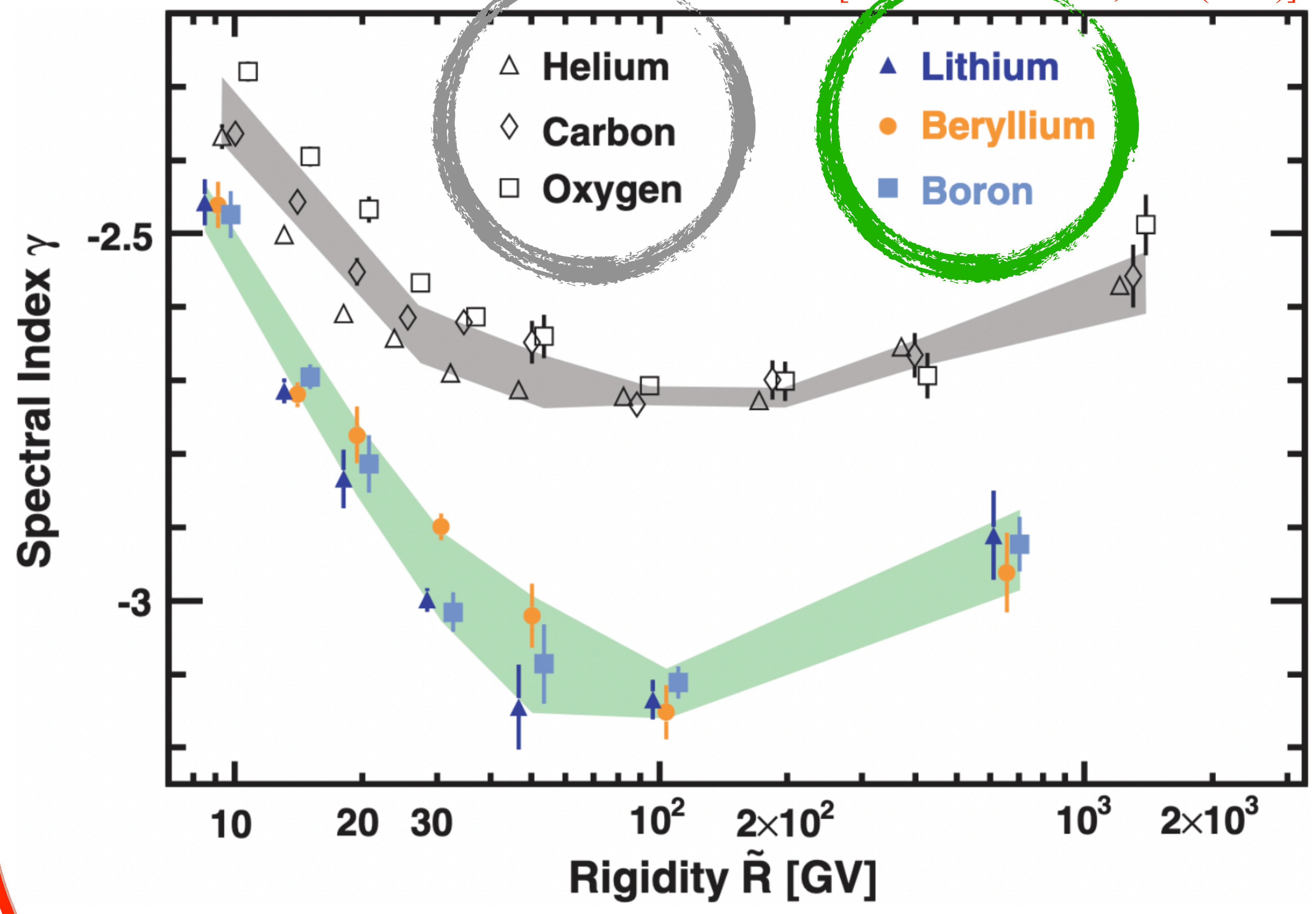
$$\Rightarrow \qquad F(E) \sim \frac{N(E)}{D(E)}$$

$$\Rightarrow \quad \boxed{\begin{aligned} F_{\rm pri}(E) &\sim \frac{N_{\rm inj}(E)}{D(E)} \propto E^{-\gamma_{\rm inj}-\delta} \\ F_{\rm sec}(E) &\sim \frac{E^{-\gamma_{\rm inj}-\delta}}{D(E)} \propto E^{-\gamma_{\rm inj}-2\delta} \end{aligned}}$$

The cosmic-ray spectrum

PRIMARY SECONDARY

[AMS Collaboration, PRL (2018)]



$$N_{\text{inj}}(E) \propto E^{-\gamma_{\text{inj}}}$$

$$D(E) \propto E^{\delta}$$

$$\Rightarrow \quad F(E) \sim \frac{N(E)}{D(E)}$$

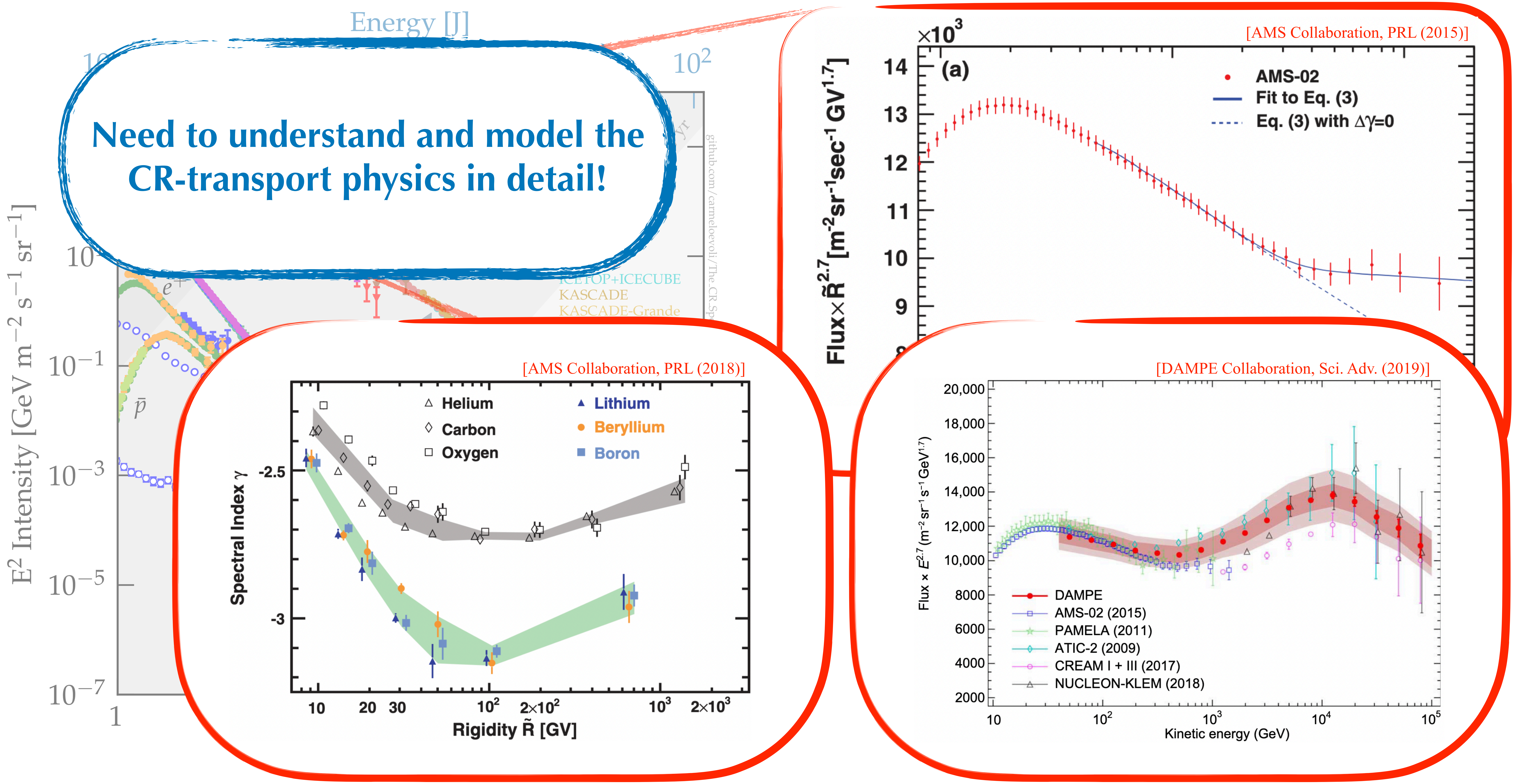
$$\Rightarrow \quad F_{\text{pri}}(E) \sim \frac{N_{\text{inj}}(E)}{D(E)} \propto E^{-\gamma_{\text{inj}}-\delta}$$

$$F_{\text{sec}}(E) \sim \frac{E^{-\gamma_{\text{inj}}-\delta}}{D(E)} \propto E^{-\gamma_{\text{inj}}-2\delta}$$

$$\Rightarrow \quad \frac{F_{\text{sec}}(E)}{F_{\text{pri}}(E)} \propto E^{-\delta}$$

The cosmic-ray spectrum

Need to understand and model the CR-transport physics in detail!



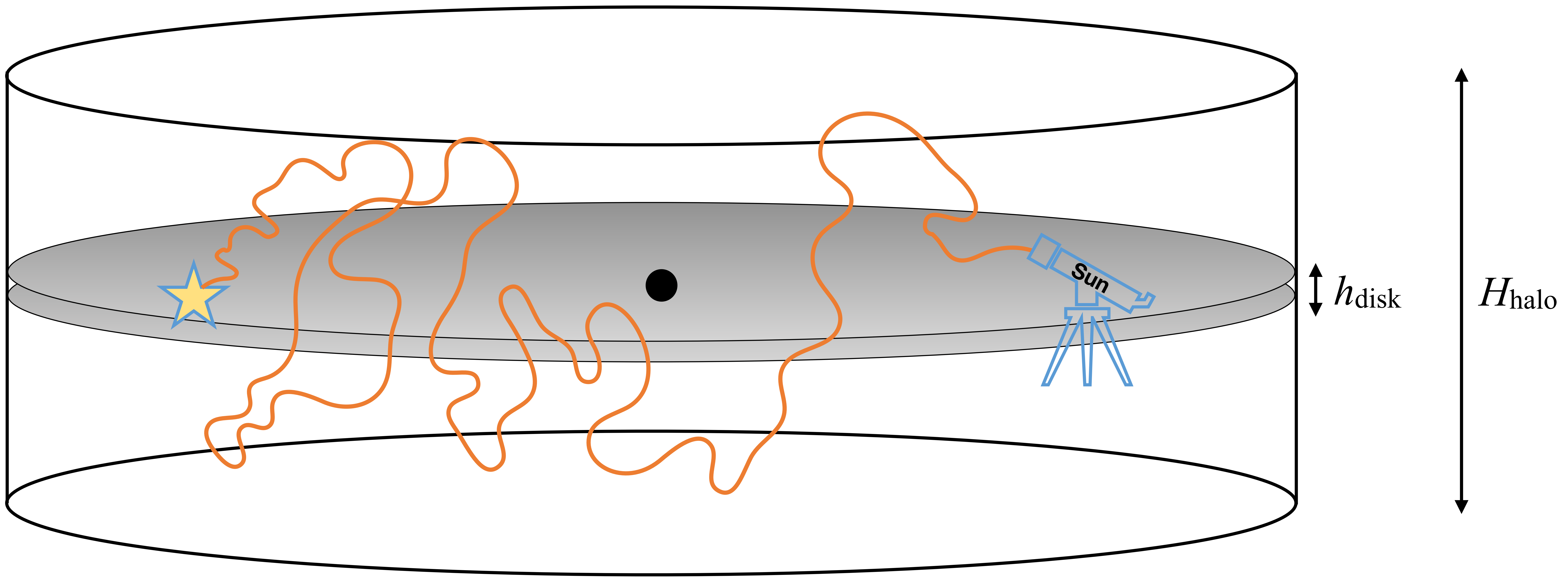
CR diffusion

Why “diffusion”?

CR diffusion

Why "diffusion"?

"**Grammage**" traversed by a CR with energy *E*: $$X(E) \sim \overline{m}_{\text{gas}}\, n_{\text{disk}}\, \frac{h_{\text{disk}}}{H_{\text{halo}}}\, v\, \tau_{\text{esc}}(E)$$



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☞ use typical Galactic values (e.g., $m_{\rm gas}$ ~ 1.4 $m_{\rm p}$, $n_{\rm disk}$ ~ 1 cm^{-3}, $h_{\rm disk}$ ~ 150 pc, $H_{\rm halo}$ ~ 3 kpc) and v ~ c:

$$\tau_{\rm esc}(10\,{\rm GeV/n}) \sim 10\text{--}100\,{\rm Myr} \quad \gg \quad \tau_{\rm bal} \sim \frac{H}{c} \sim 1\text{--}10\,{\rm kyr}$$

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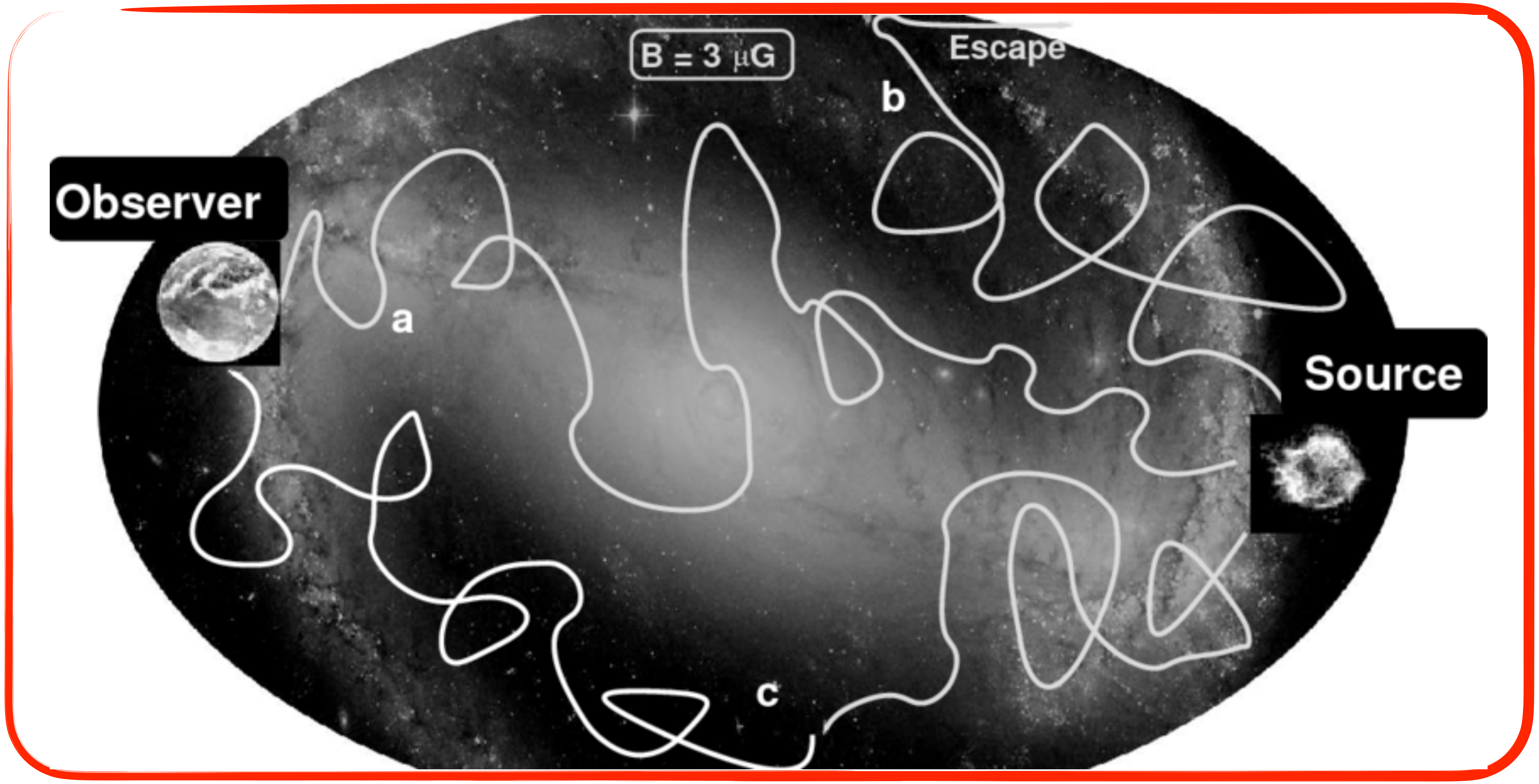
$$\tau_{\rm esc}(10\,{\rm GeV/n}) \sim 10\text{–}100\,{\rm Myr} \quad \gg \quad \tau_{\rm bal} \sim \frac{H}{c} \sim 1\text{–}10\,{\rm kyr}$$

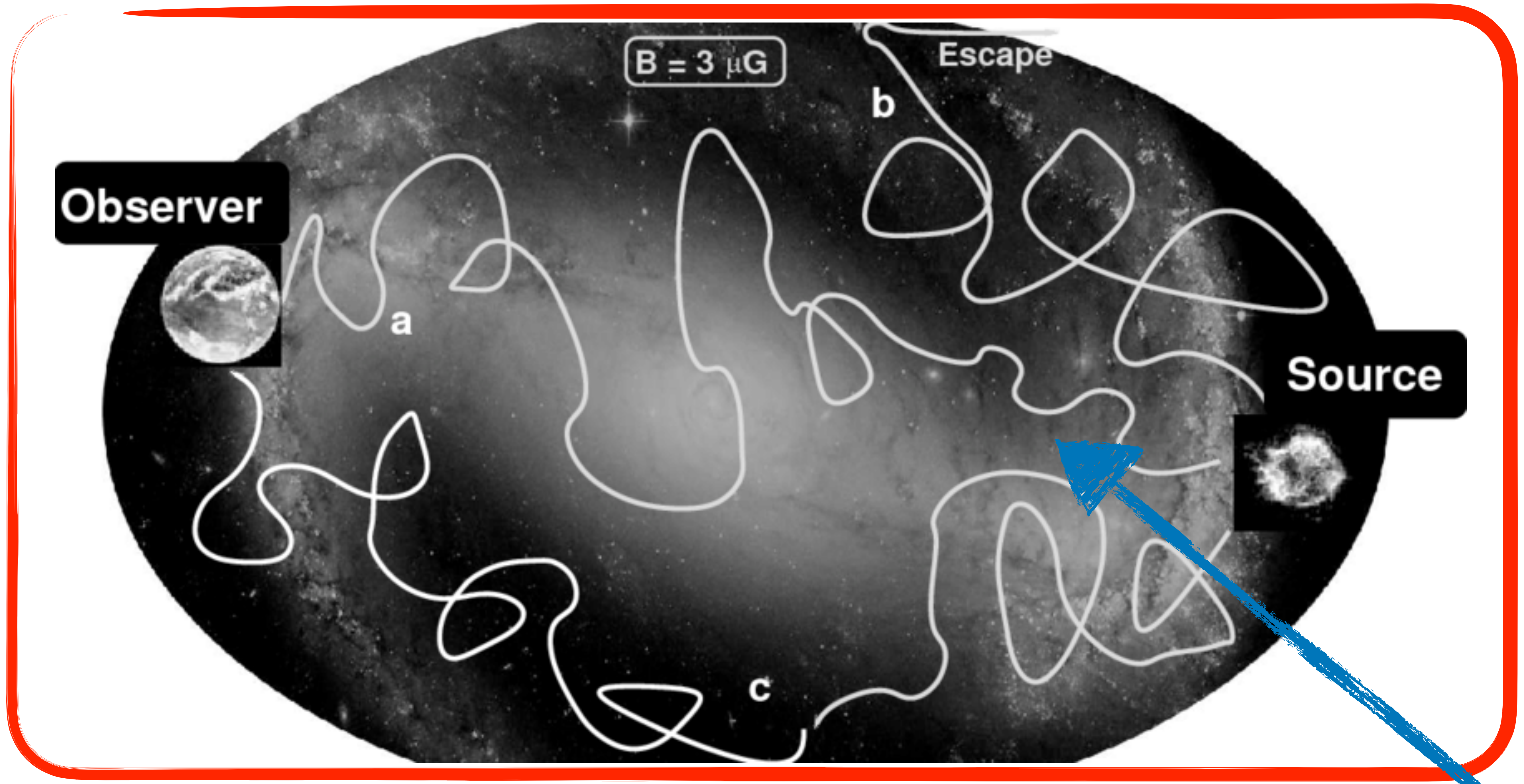
⇒ there is a mechanism that confines CRs within the Galaxy for long time before they can escape

CR diffusion



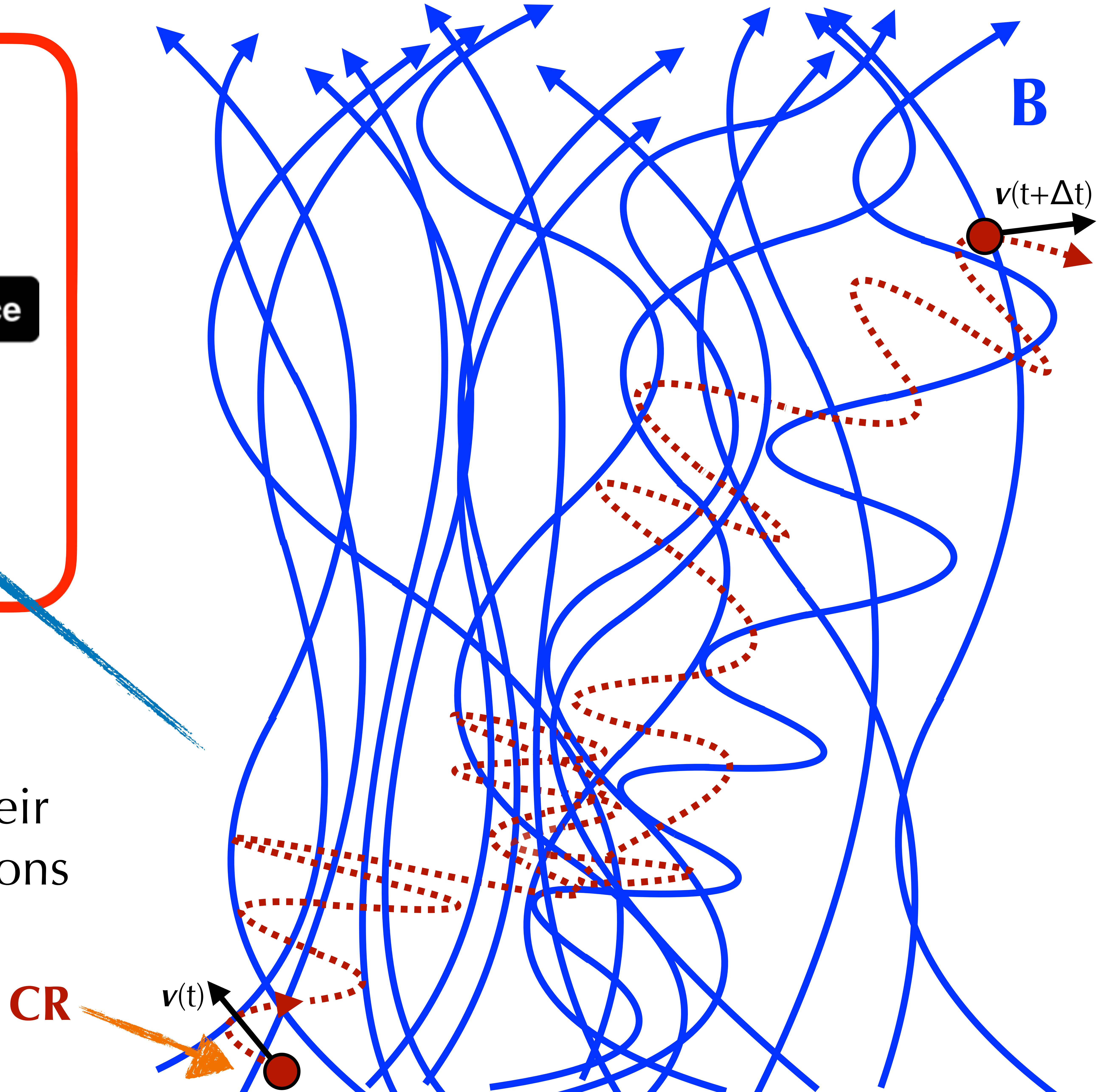
CR diffusion

CR diffusion



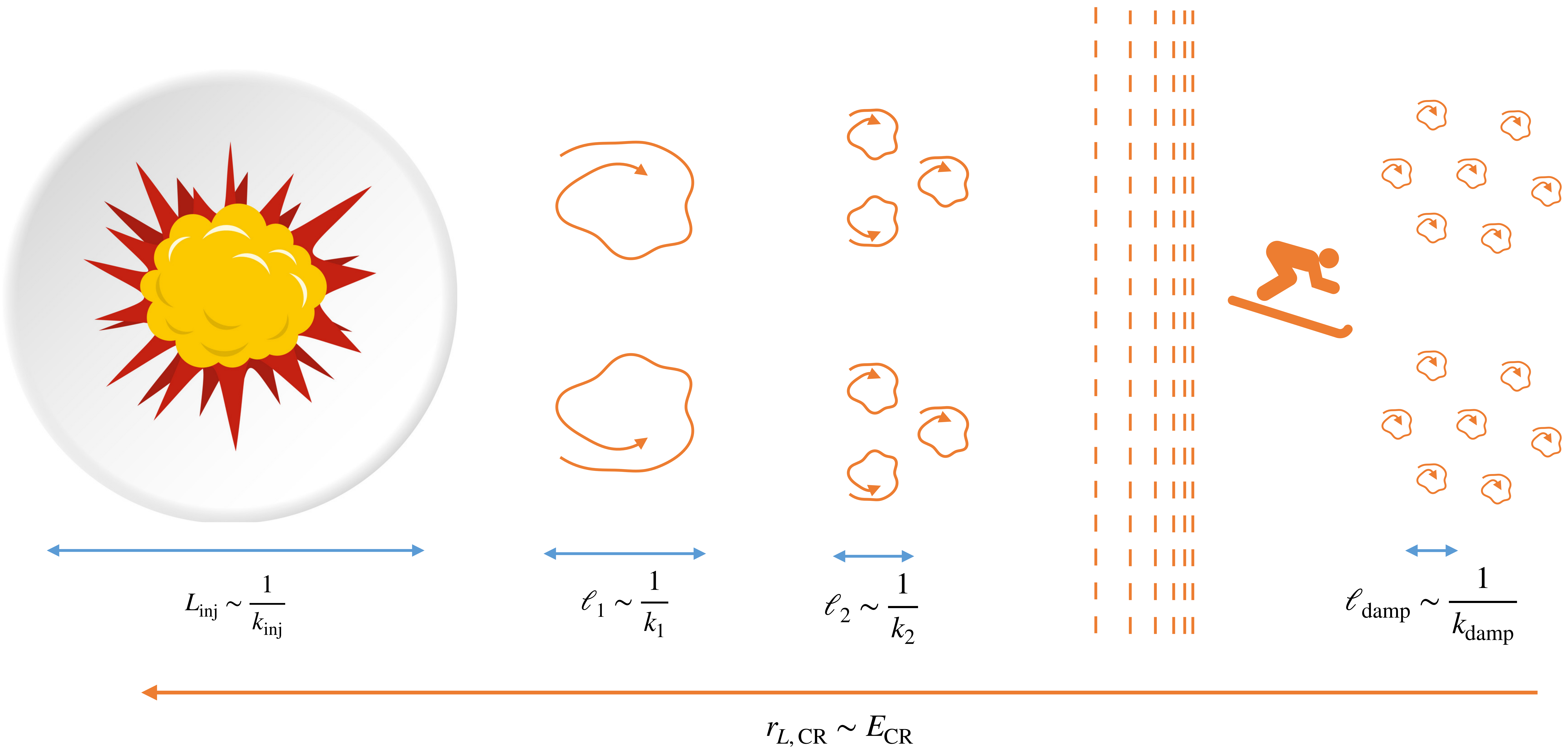
CR diffusion

random motion of charged particles due to their scattering on turbulent magnetic-field fluctuations



CR diffusion

Turbulent cascade



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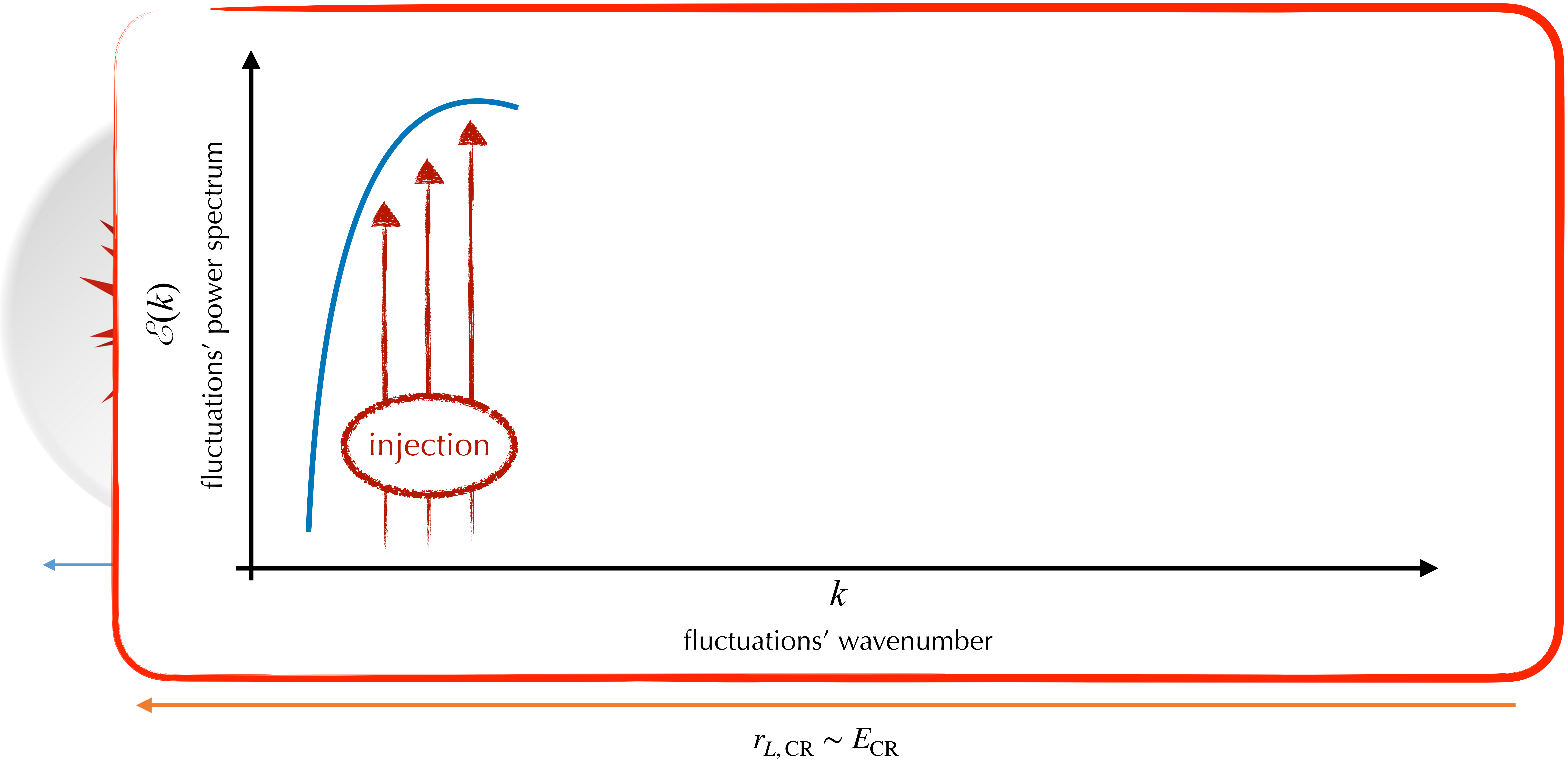

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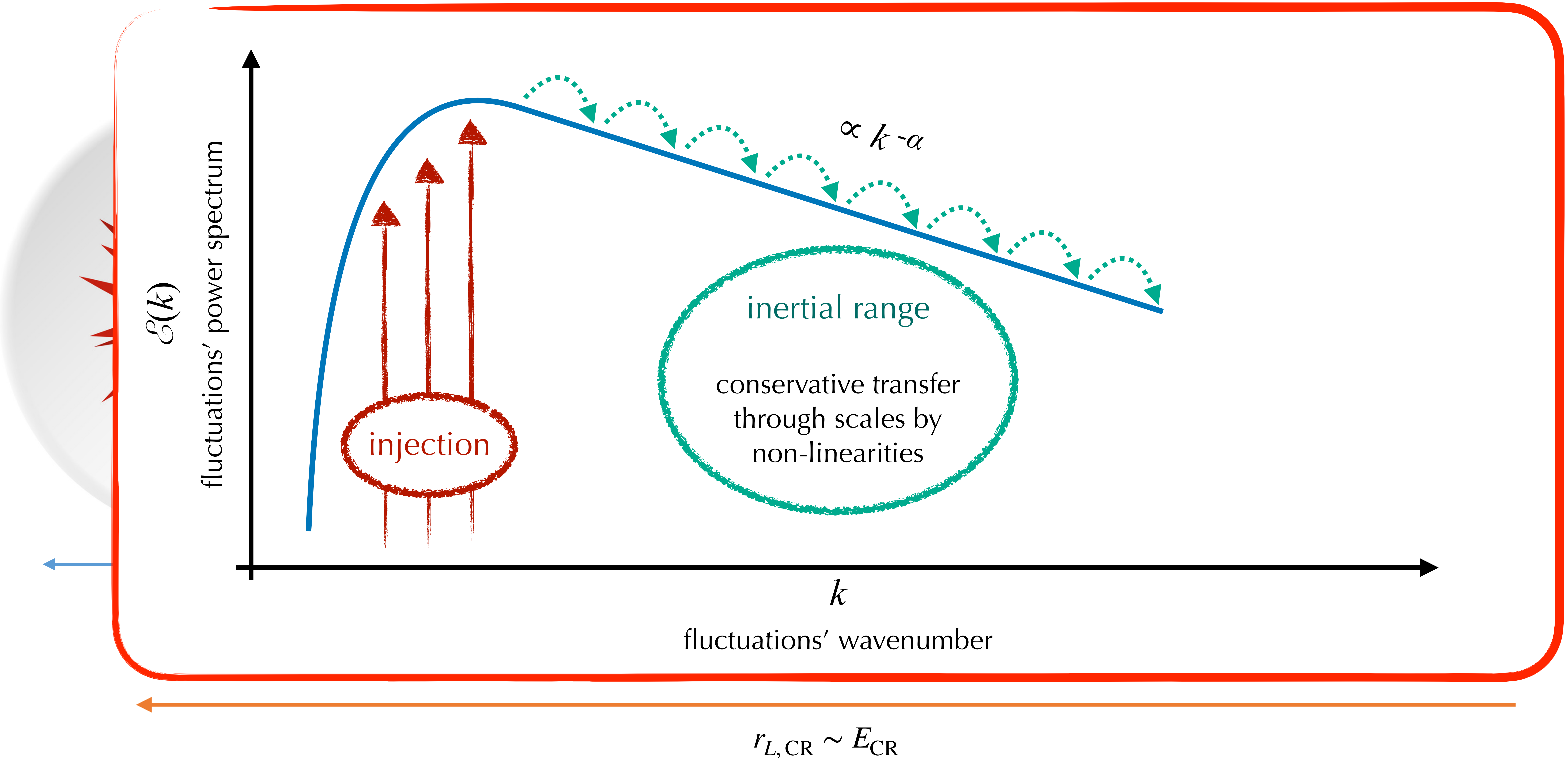

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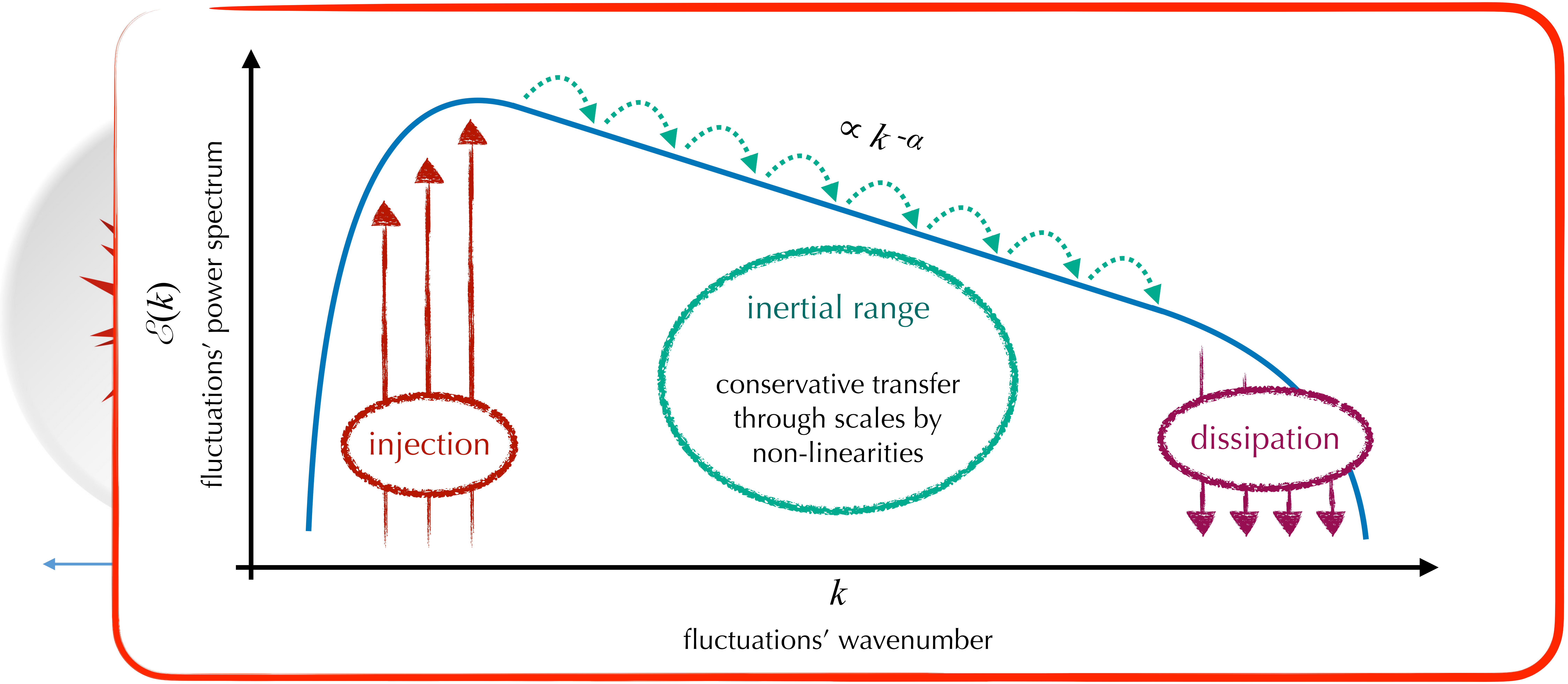
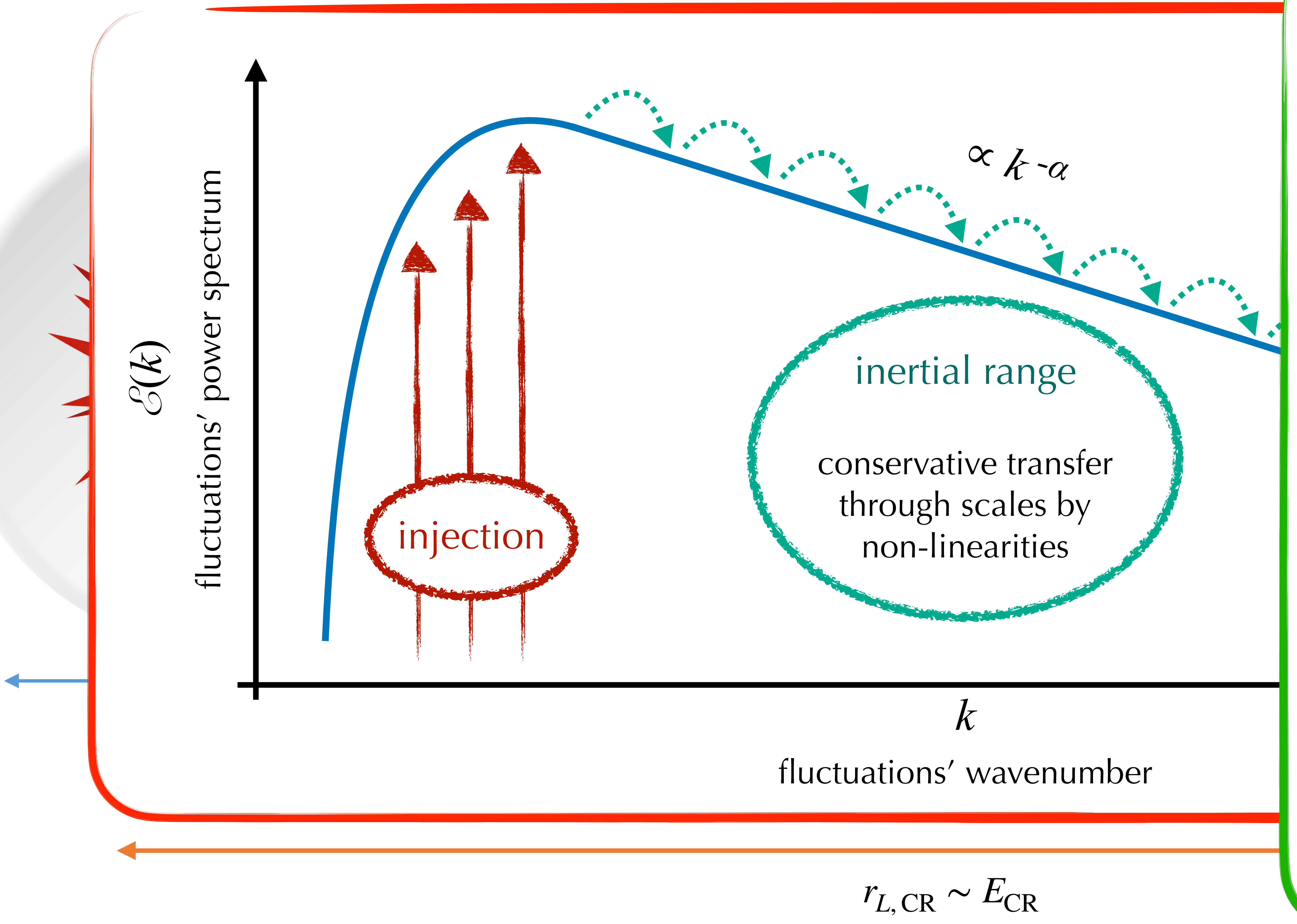


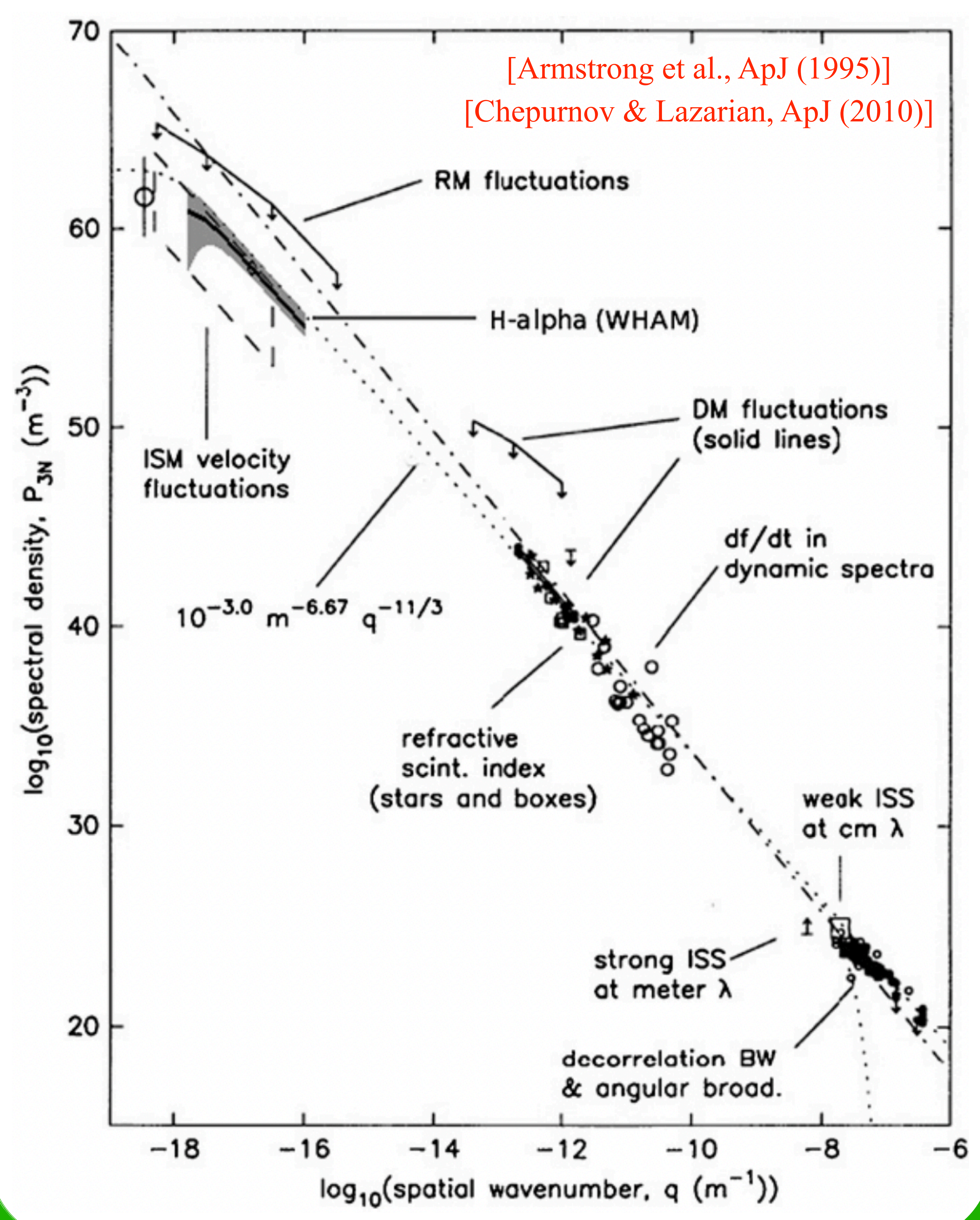
$r_{L,\mathrm{CR}} \sim E_{\mathrm{CR}}$

CR diffusion

Turbulent cascade



and yes, the ***interstellar medium (ISM)*** is indeed ***turbulent***



CR diffusion

Turbulent cascade of Alfvénic fluctuations

☞ incompressible MHD equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\frac{\boldsymbol{\nabla} P_{\rm tot}}{\varrho_0} + \frac{(\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{B}}{4\pi\varrho_0} + \nu \nabla^2 \boldsymbol{u}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{B} = (\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{u} + \eta \nabla^2 \boldsymbol{B}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

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☞ rewrite them in the Elsässer formulation ($\eta = \nu$):

$$\frac{\partial \boldsymbol{z}^+}{\partial t} + (\boldsymbol{z}^- \cdot \boldsymbol{\nabla})\boldsymbol{z}^+ = -\boldsymbol{\nabla} \widetilde{P}_{\rm tot} + \eta \nabla^2 \boldsymbol{z}^+$$

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CR diffusion

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Alfvén waves traveling "up" or "down" the magnetic field **B**

CR diffusion

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non-linear interaction only between counter-propagating Alfvén waves

CR diffusion

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incompressible Alfvénic cascade ~ non-linear interaction of counter-propagating AWs

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CR diffusion

Turbulent cascade of Alfvénic fluctuations

☞ split into "background + Alfvénic fluctuations":

$$\boldsymbol{B} = \boldsymbol{B}_0 + \delta \boldsymbol{B}_\perp$$

$$\boldsymbol{u} = \cancel{\boldsymbol{u}}_0 + \delta \boldsymbol{u}_\perp$$

$$\boldsymbol{v}_\mathrm{A} = \frac{\boldsymbol{B}_0}{\sqrt{4\pi\varrho_0}}$$

$$\delta \boldsymbol{z}^\pm = \delta \boldsymbol{u}_\perp \pm \frac{\delta \boldsymbol{B}_\perp}{\sqrt{4\pi\varrho_0}}$$

CR diffusion

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$$\Rightarrow \quad \left(\frac{\partial}{\partial t} \mp \boldsymbol{v}_\mathrm{A}\cdot\boldsymbol{\nabla}\right)\delta\boldsymbol{z}^\pm + (\delta\boldsymbol{z}^\mp\cdot\boldsymbol{\nabla})\delta\boldsymbol{z}^\pm = \ldots \text{ (!) turbulence needs finite dissipation!}$$

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linear frequency: $\omega_\mathrm{A} = k_\parallel v_\mathrm{A}$

non-linear frequency: $\omega_\mathrm{nl} = k_\perp \delta z^\mp$

CR diffusion

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linear frequency: $\omega_\mathrm{A} = k_\| v_\mathrm{A}$

non-linear frequency: $\omega_\mathrm{nl} = k_\perp \delta z^\mp$

$\Rightarrow$ non-linearity parameter: $\chi \doteq \dfrac{\omega_\mathrm{nl}}{\omega_\mathrm{A}} = \dfrac{k_\perp \delta z^\mp}{k_\| v_\mathrm{A}}$

<< 1 ("WEAK")

~ 1 ("STRONG")

CR diffusion

Turbulent cascade of Alfvénic fluctuations

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- << 1 ("WEAK")
- ~ 1 ("STRONG")

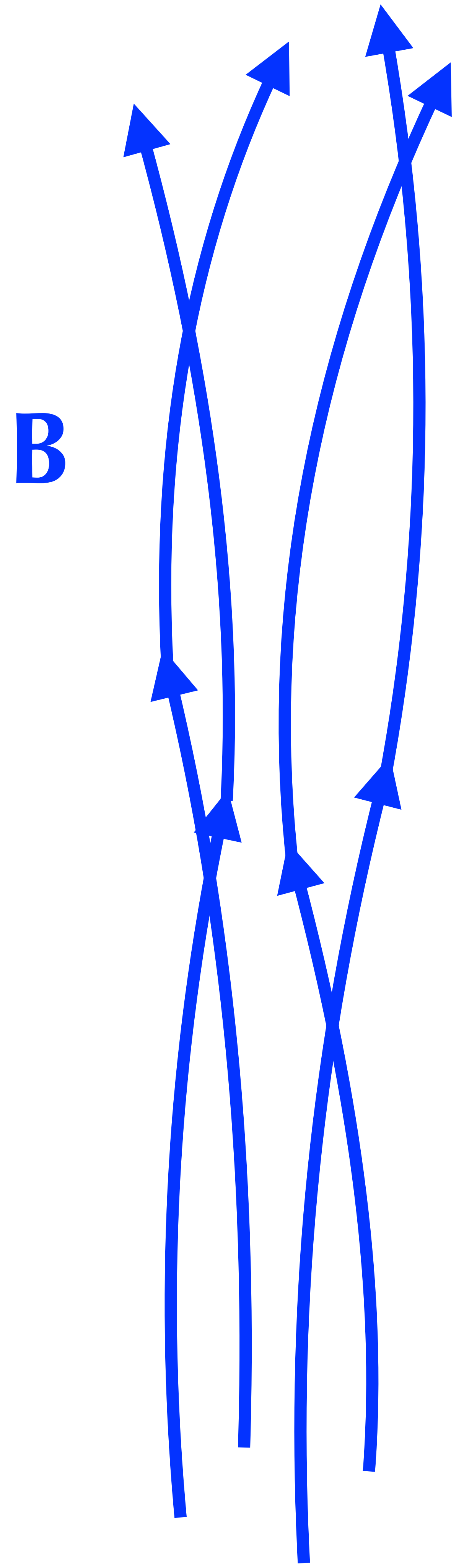
⚠ An initially weak Alfvénic cascade will increase ω_nl with decreasing perp. scale λ, and reach $\chi \sim 1$ at some scale λ_{CB}

[*Don't worry, I won't put you through all this!* but see, e.g., Schekochihin, arXiv:2010.00699, and references therein]

CR diffusion

critically balanced (strong) Alfvénic turbulence: a quick phenomenological derivation

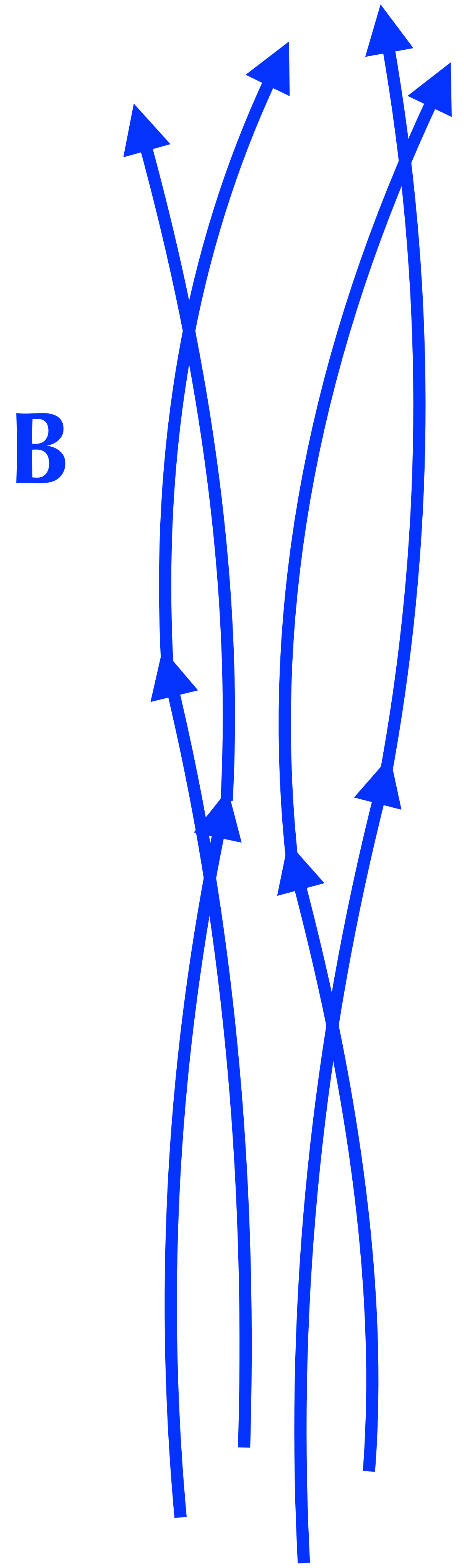
[Goldreich & Sridhar, ApJ (1995); see also: Oughton & Matthaeus, (2020); Schekochihin, arXiv:2010.00699]



CR diffusion

critically balanced (strong) Alfvénic turbulence: a quick phenomenological derivation

[Goldreich & Sridhar, ApJ (1995); see also: Oughton & Matthaeus, (2020); Schekochihin, arXiv:2010.00699]



☞ If you were to compute the cascade time for counter-propagating AWs:

$$\tau_{\rm casc} \sim N\,\tau_{\rm A} \sim \frac{\tau_{\rm nl}^2}{\tau_{\rm A}} = \frac{\tau_{\rm nl}}{\chi}$$

with $N \sim 1/\chi^2 \sim (\tau_{\rm nl}/\tau_{\rm A})^2$ [*# AW interactions*]

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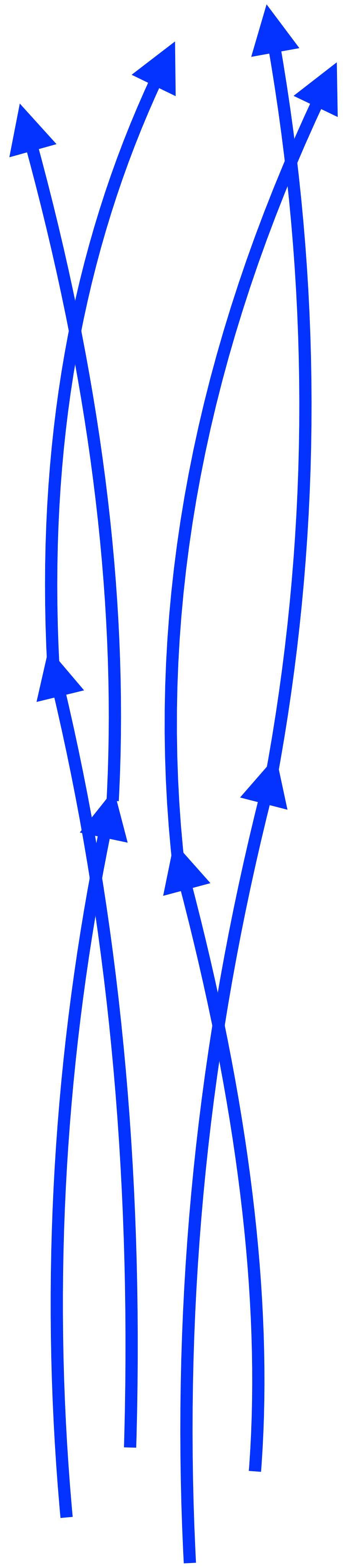

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☞ When $\chi = \tau_{\rm A}/\tau_{\rm nl} \sim 1$ is achieved, the linear, non-linear, and cascade timescales match each other:

$$\tau_{\rm nl} \sim \tau_{\rm A} \quad \Rightarrow \quad \tau_{\rm casc} \sim \tau_{\rm nl}$$

☞ **This is known as "critical balance" (CB)**

CR diffusion

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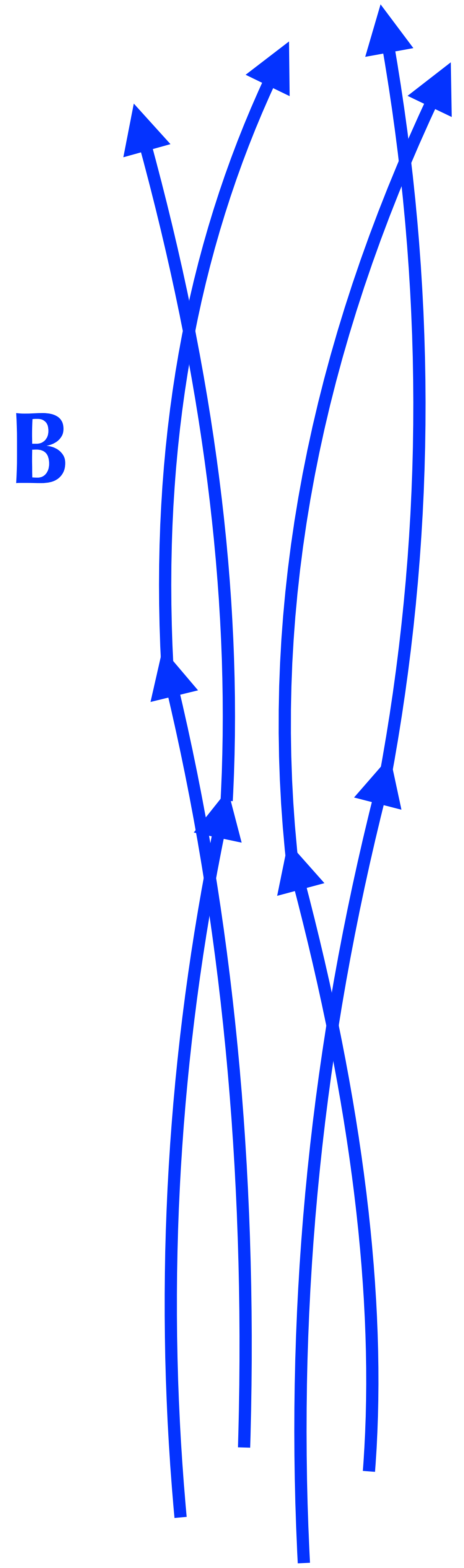

☞ So, the working hypothesis of a critically balanced Alfvénic cascade is:

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you can see the *``critical-balance condition" as the result of causality*:

CR diffusion

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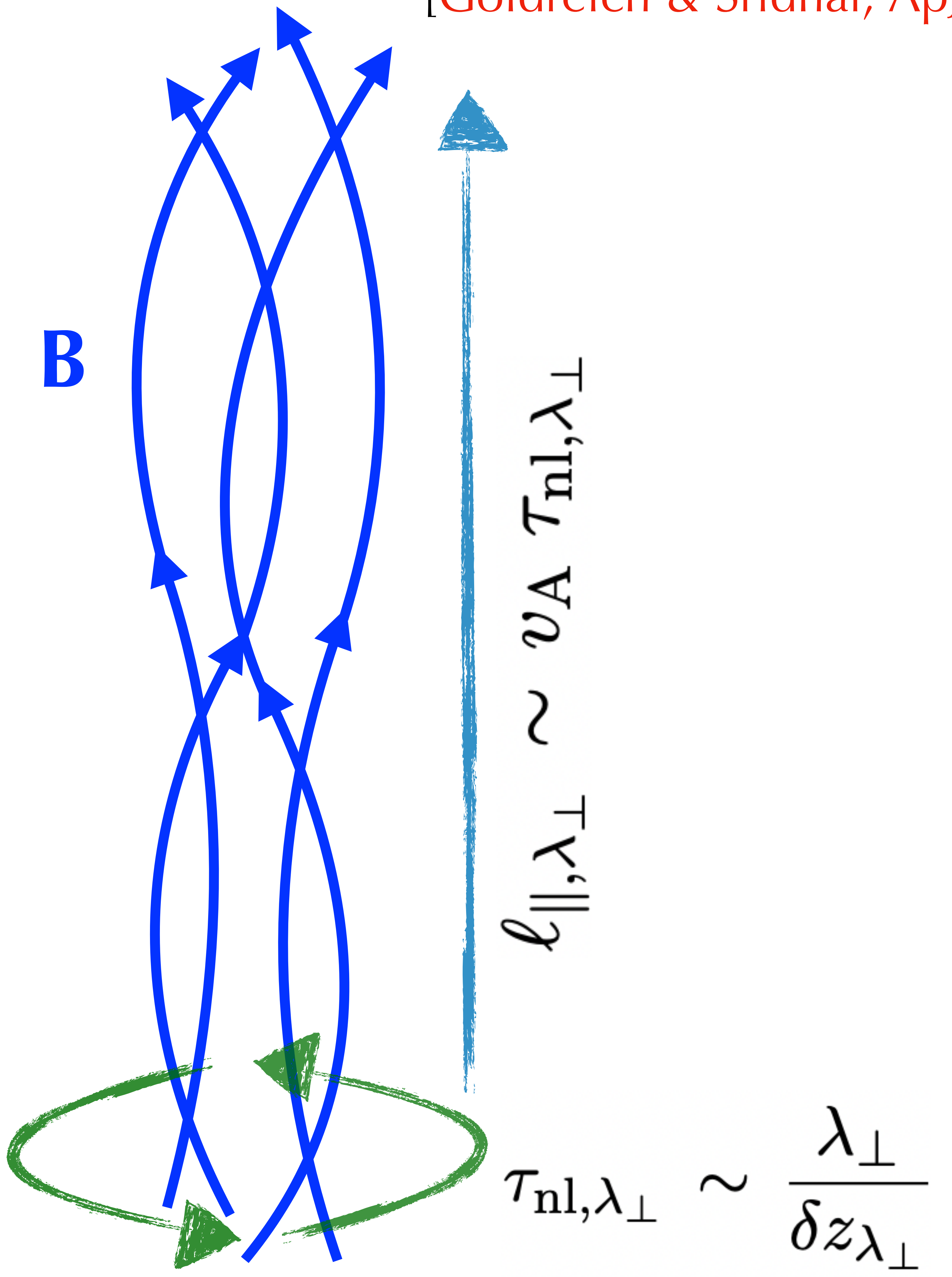

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the information about Alfvénic fluctuations decorrelating in the perpendicular plane over an eddy turn-over time τ_{nl} can only propagate along the field for a length $\ell_{\|}$ at maximum speed v_A.

"So… CB is essentially AWs trying to keep up with the turbulent eddies…"

CR diffusion

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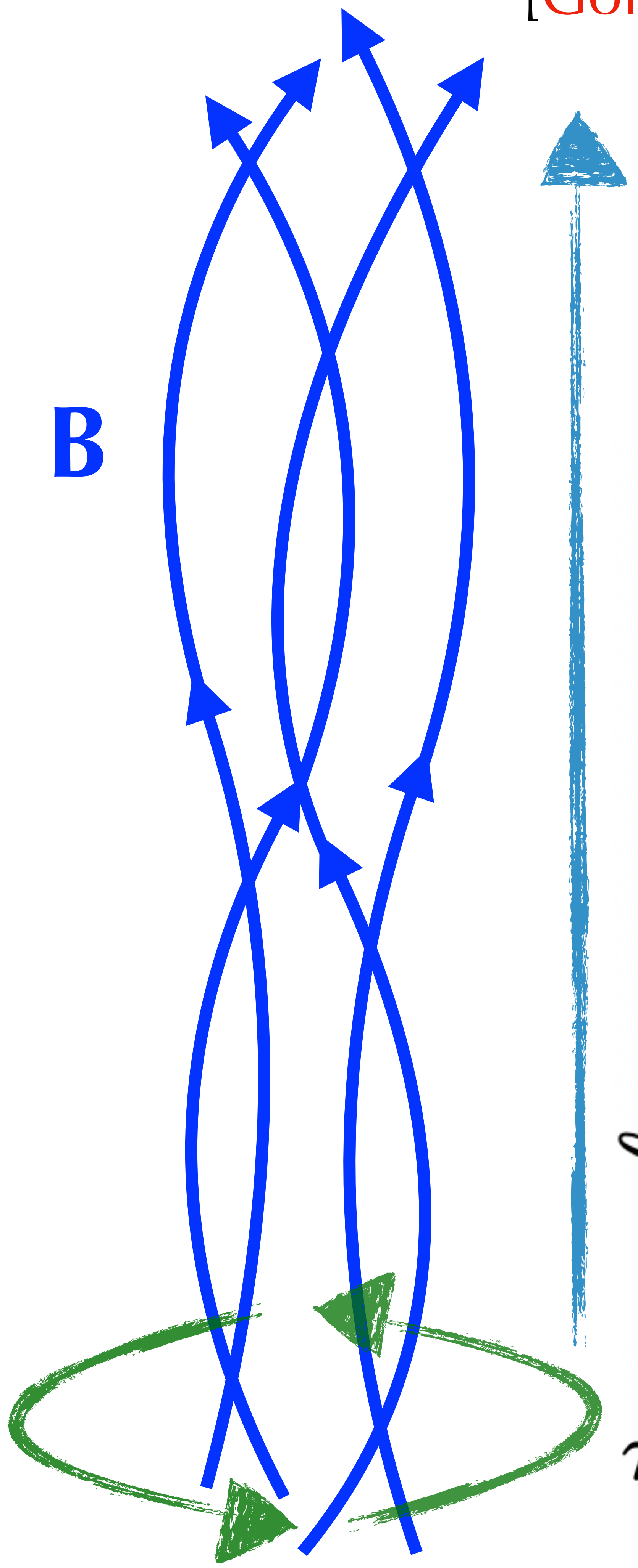

$$\ell_{\|,\lambda_\perp} \sim v_A \, \tau_{nl,\lambda_\perp}$$

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"So… CB is essentially AWs trying to keep up with the turbulent eddies…"

$$\tau_{nl,\lambda_\perp} \sim \frac{\lambda_\perp}{\delta z_{\lambda_\perp}}$$

Therefore, once $\tau_{nl} \sim \tau_A$ is reached, the balance is likely mantained.

(In principle, this could be done by continuing the cascade with τ_{nl} = const., or by generating smaller $\ell_{\|}$ such that $\tau_A \sim \ell_{\|}/v_A \sim \tau_{nl}$ keeps holding… it is the latter)

CR diffusion

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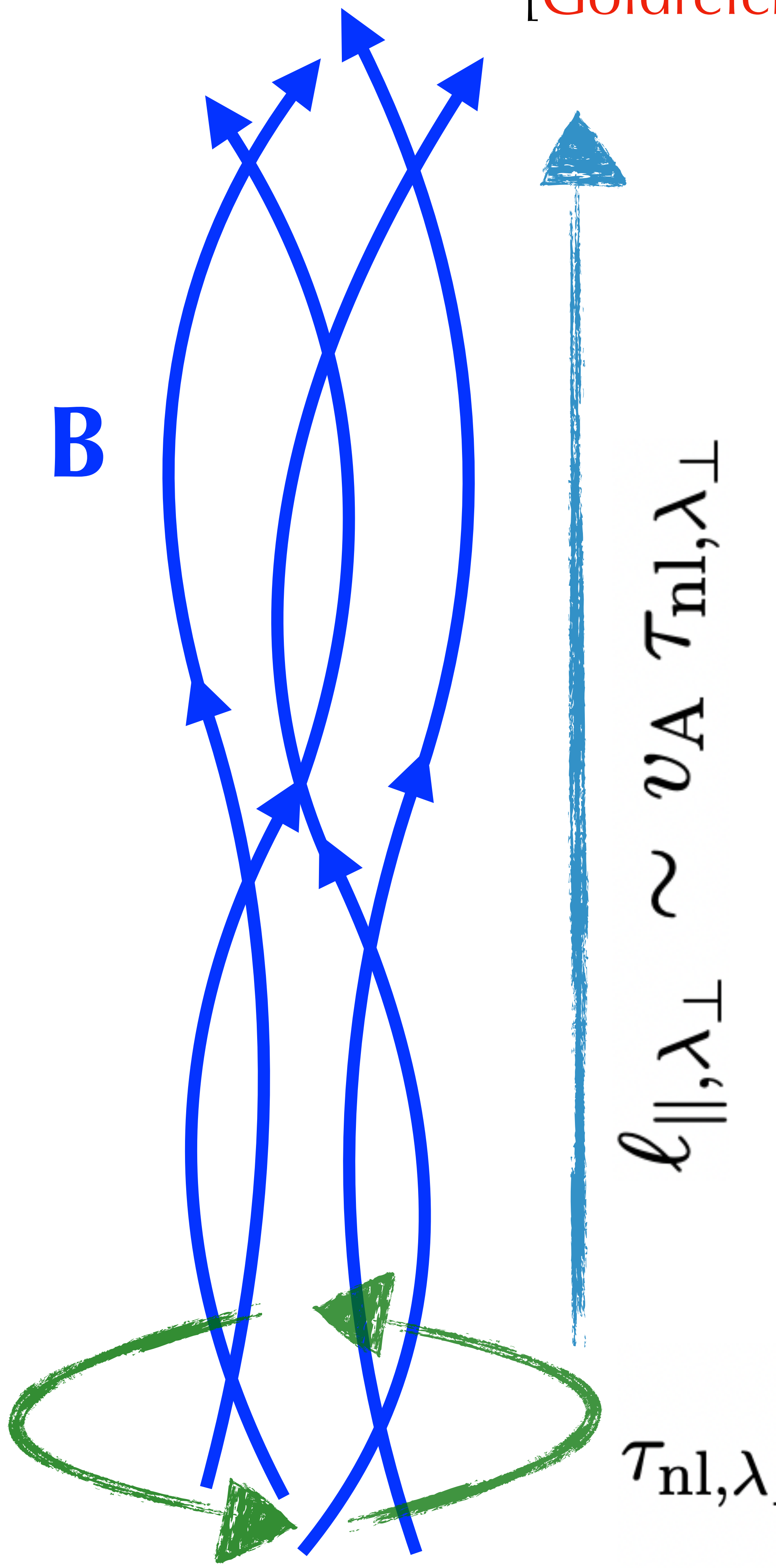

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☞ fluctuations' scaling and corresponding spectrum are obtained by ***assuming a constant energy flux*** ε ***through scales*** ("intertial range")

$$\frac{\delta z_{k_\perp}^2}{\tau_{nl,k_\perp}} \sim \varepsilon = \text{const.} \quad \Rightarrow \quad \boxed{\delta z_{k_\perp} \propto k_\perp^{-1/3} \quad \Rightarrow \quad \mathcal{E}_{\delta z}(k_\perp) \propto k_\perp^{-5/3}}$$

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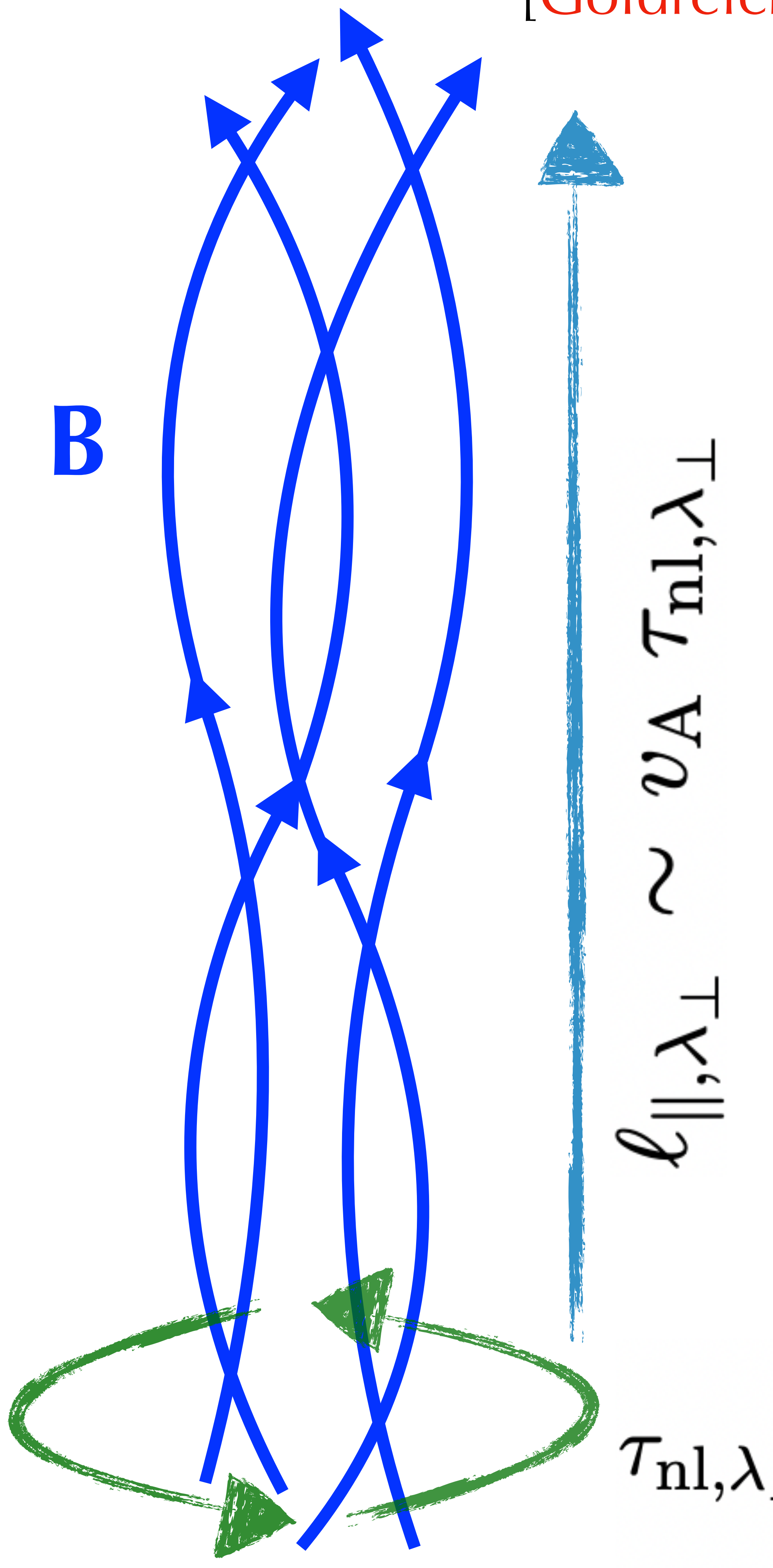

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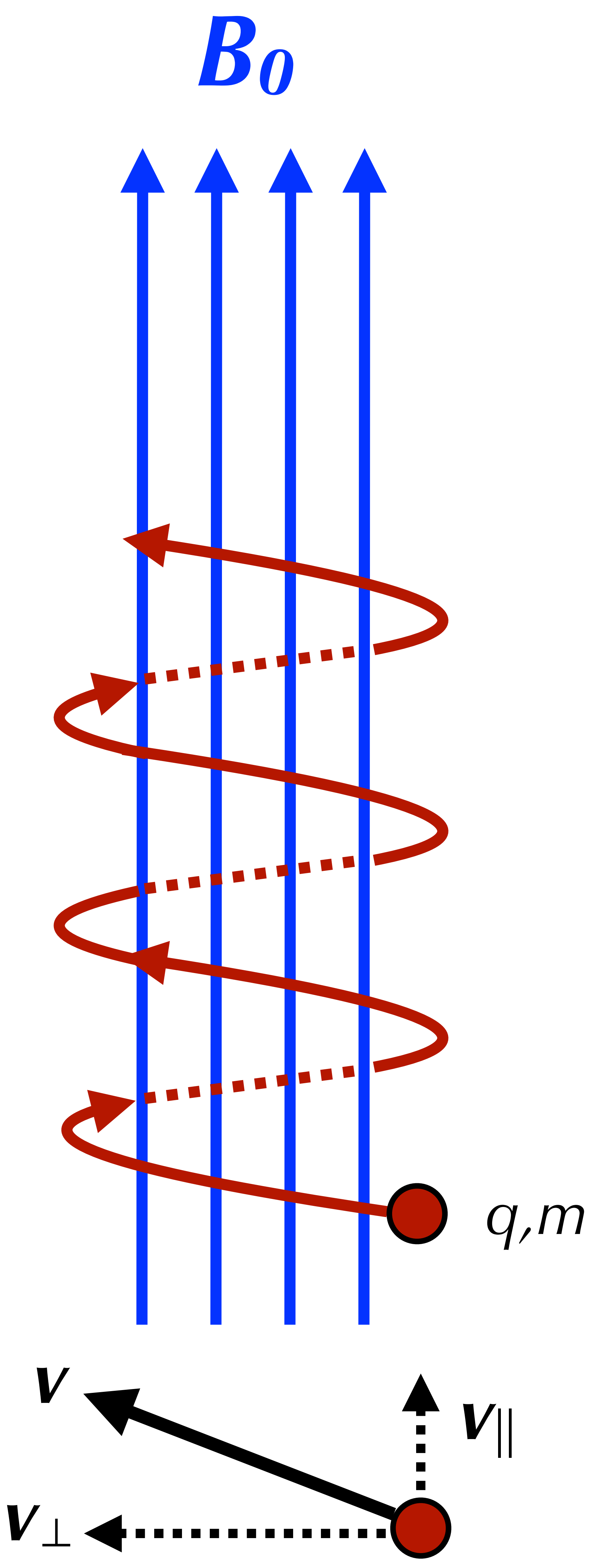
☞ now, you can compute the ***fluctuations' wavenumber anisotropy from CB***:

$$k_\perp \delta z_{k_\perp} \sim k_\| v_{\mathrm{A}} \quad \Rightarrow \quad \boxed{k_\| \propto k_\perp^{2/3}} \quad \left(\Rightarrow \quad \mathcal{E}_{\delta z}(k_\|) \propto k_\|^{-2}\right)$$

CR diffusion

Crude estimate of a diffusion coefficient (*quasi-linear approach on single mode*)

[see, e.g., Blasi, AAR (2013)]



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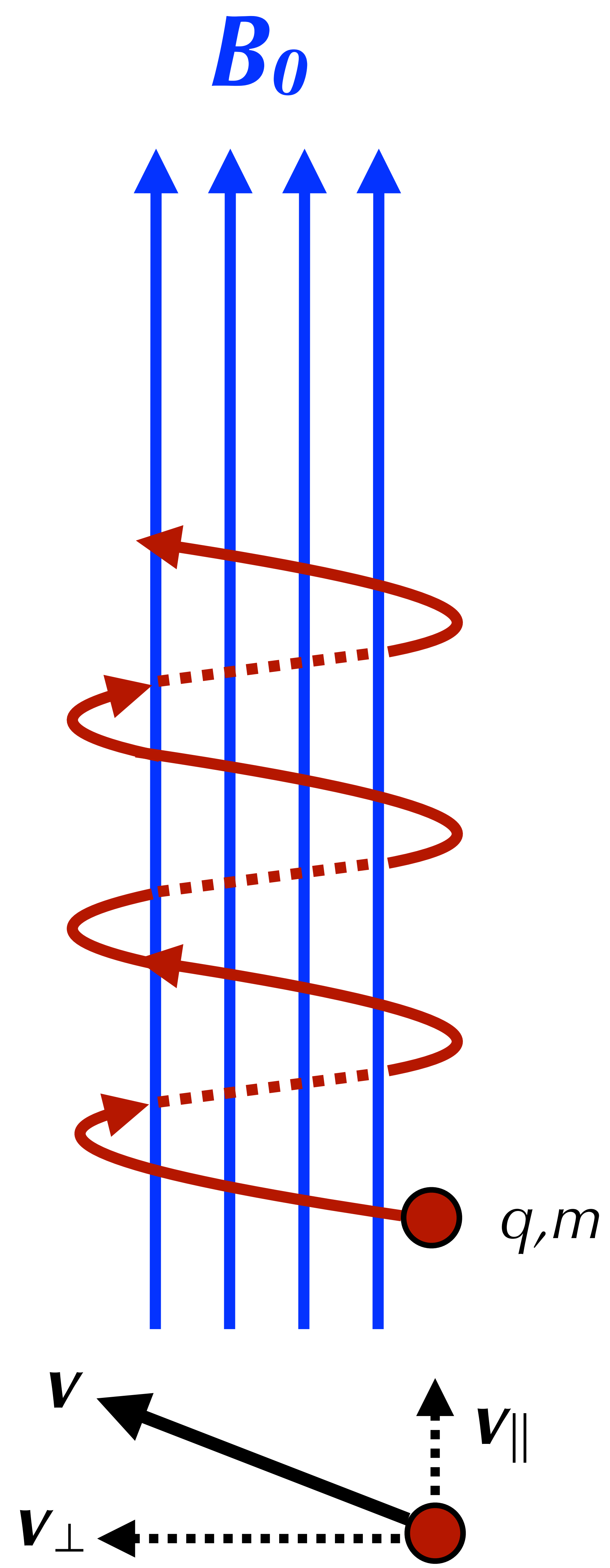

"**gyro-motion**" (helical trajectory of a charged particlein a magnetic field)

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \Omega\,(\boldsymbol{v}\times\boldsymbol{b})$$

$$\Omega \doteq \frac{q|\boldsymbol{B}|}{m\gamma c}$$ gyro-frequency

$$\boldsymbol{b} \doteq \frac{\boldsymbol{B}}{|\boldsymbol{B}|}$$ magnetic-field direction

$$\Rightarrow \begin{aligned} v_{\perp,1}(t) &= v_\perp\cos(\Omega t+\phi)\\ v_{\perp,2}(t) &= -v_\perp\sin(\Omega t+\phi)\\ v_\parallel(t) &= v_\parallel(0) = |\boldsymbol{v}|\mu = \text{const.}\end{aligned}$$

$$\mu \doteq \frac{v_\parallel}{|\boldsymbol{v}|} = \cos(\theta^{vb})$$ "***pitch angle***"

$$r_{\mathrm{L}} \doteq \frac{v_\perp}{\Omega} = \frac{v}{\Omega}\sqrt{1-\mu^2}$$ gyro-radius

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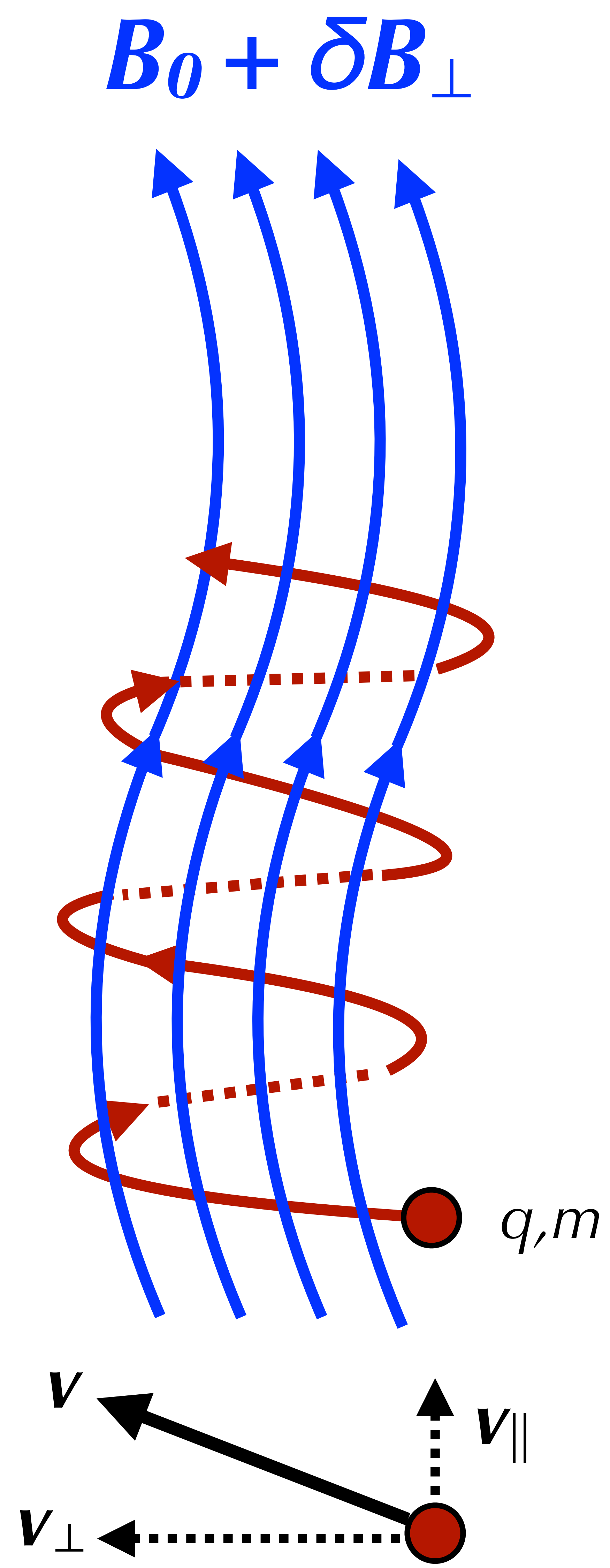

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☞ consider a ***magnetic-field perturbation*** of the type: $\delta\boldsymbol{B}_\perp \sim \delta B_{\perp,1}\boldsymbol{e}_{\perp,1}\sin(k_\parallel x_\parallel)$

☞ estimate time derivative of pitch angle: $\dfrac{\mathrm{d}\mu}{\mathrm{d}t} \sim \Omega\,\dfrac{\delta B_\perp}{B_0}\sqrt{1-\mu^2}\,\sin(\Omega t+\phi)\,\sin(k_\parallel v_\parallel t)$

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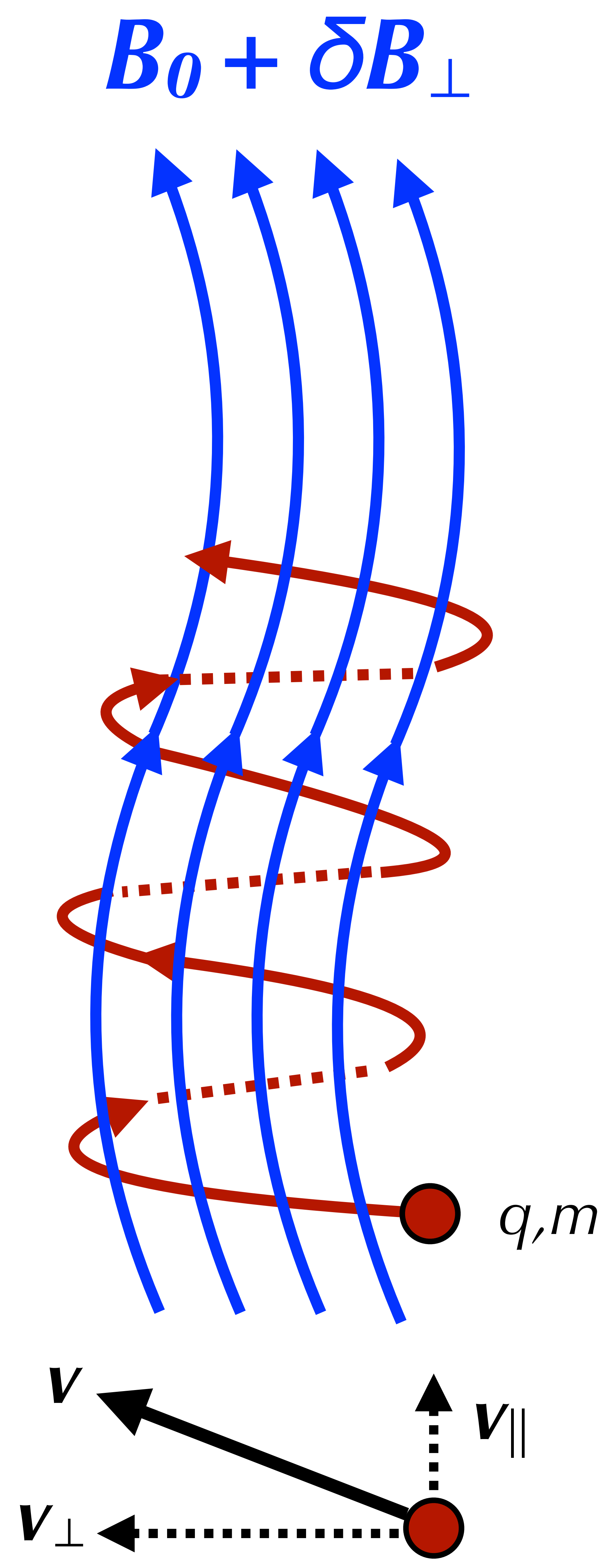

☞ mean-square variation of pitch angle: $\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle_\phi \propto (1-\mu^2)\left(\frac{\delta B_\perp}{B_0}\right)^2 \delta(k_\parallel v_\parallel - \Omega)$

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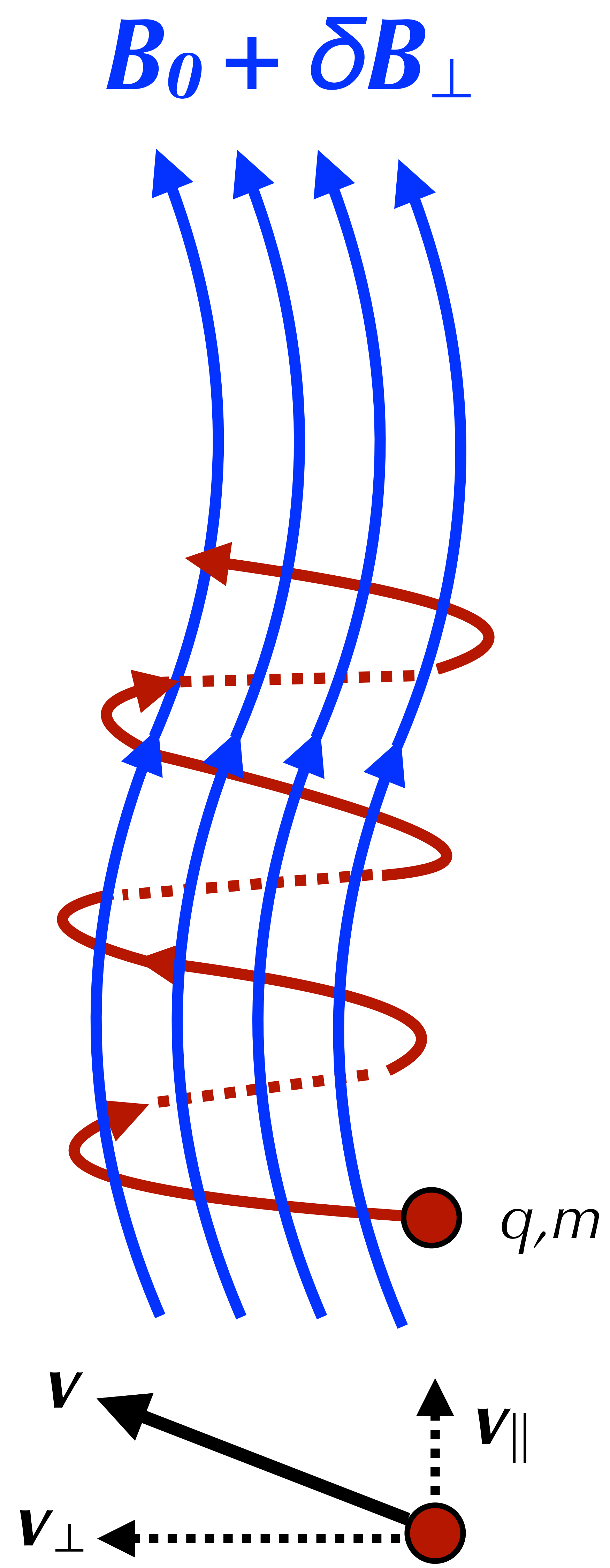

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⇒ a CR with Larmor radius r_L would only scatter on ***resonant modes***:

$$k_\parallel^{\rm res} = \frac{\Omega}{v_\parallel} \quad \Rightarrow \quad k_\parallel^{\rm res} r_{\rm L} \sim \frac{\sqrt{1-\mu^2}}{\mu}$$

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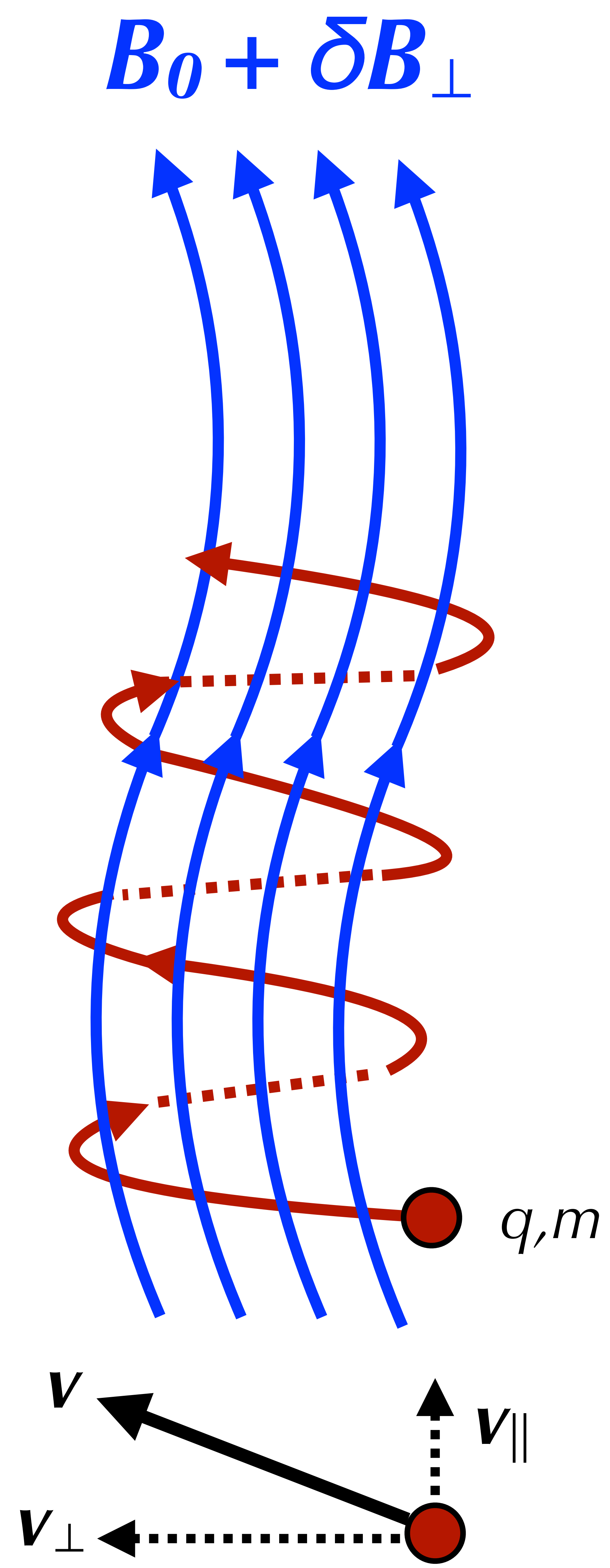

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⚠️ <u>first problem</u>: as $\mu \to 0$, the resonant wavenumber $k^{\mathrm{res}} \to \infty$.

In a turbulent cascade, the power in the fluctuations vanishes ($\delta B \to 0$ for $k \to \infty$),

so scattering efficiency vanishes for very low pitch angles (and no crossing through $\mu = 0$).

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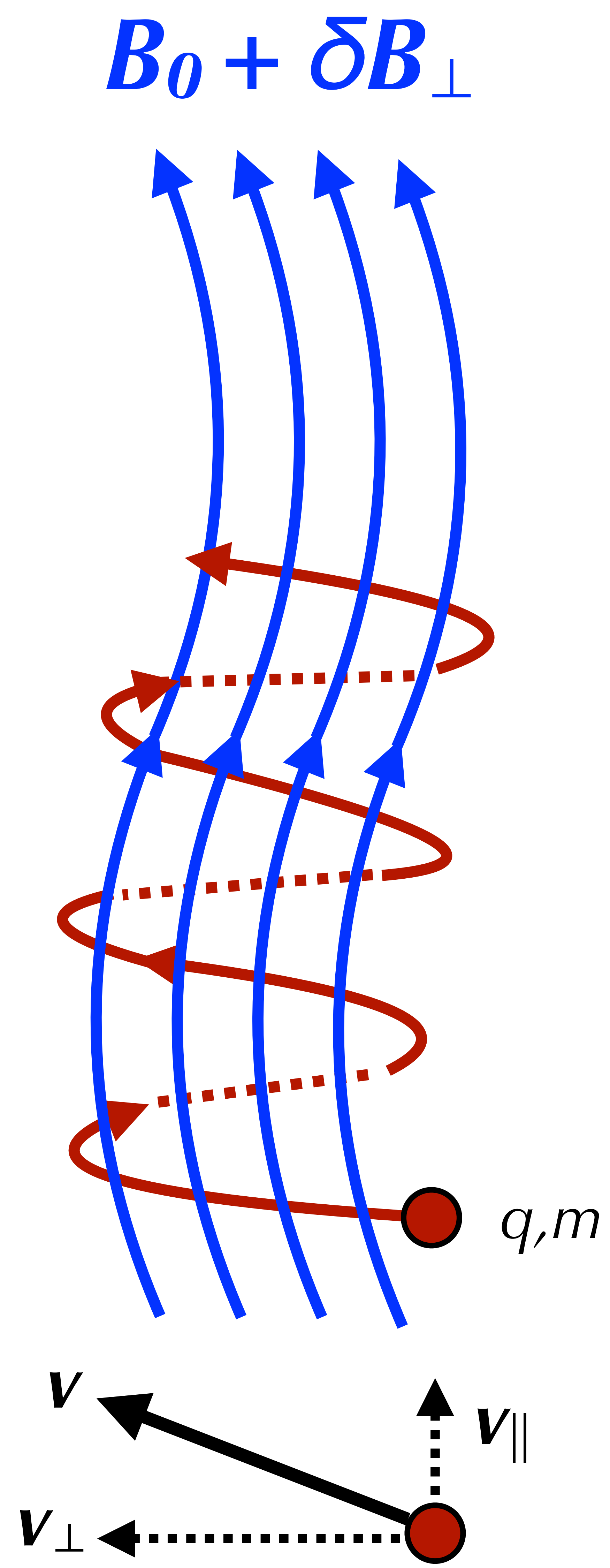

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☞ Consider spectrum of turbulent fluctuations $\mathcal{E}(k_\parallel) \sim k_\parallel^{-\alpha}$

☞ estimate scattering rate and spatial diffusion coefficient ($v \sim c$):

$$\nu_{\rm scatt} \sim \left(\frac{k_\parallel^{\rm res}\mathcal{E}(k_\parallel^{\rm res})}{B_0^2}\right)\Omega \sim \tau_{\rm scatt}^{-1} \quad \Rightarrow \quad D \approx \frac{1}{3}v\lambda_{\rm mfp} \sim v^2\tau_{\rm scatt} \propto \frac{c\, r_{\rm L}}{r_{\rm L}^{-1}\mathcal{E}(r_{\rm L}^{-1})} \sim E^{2-\alpha}$$

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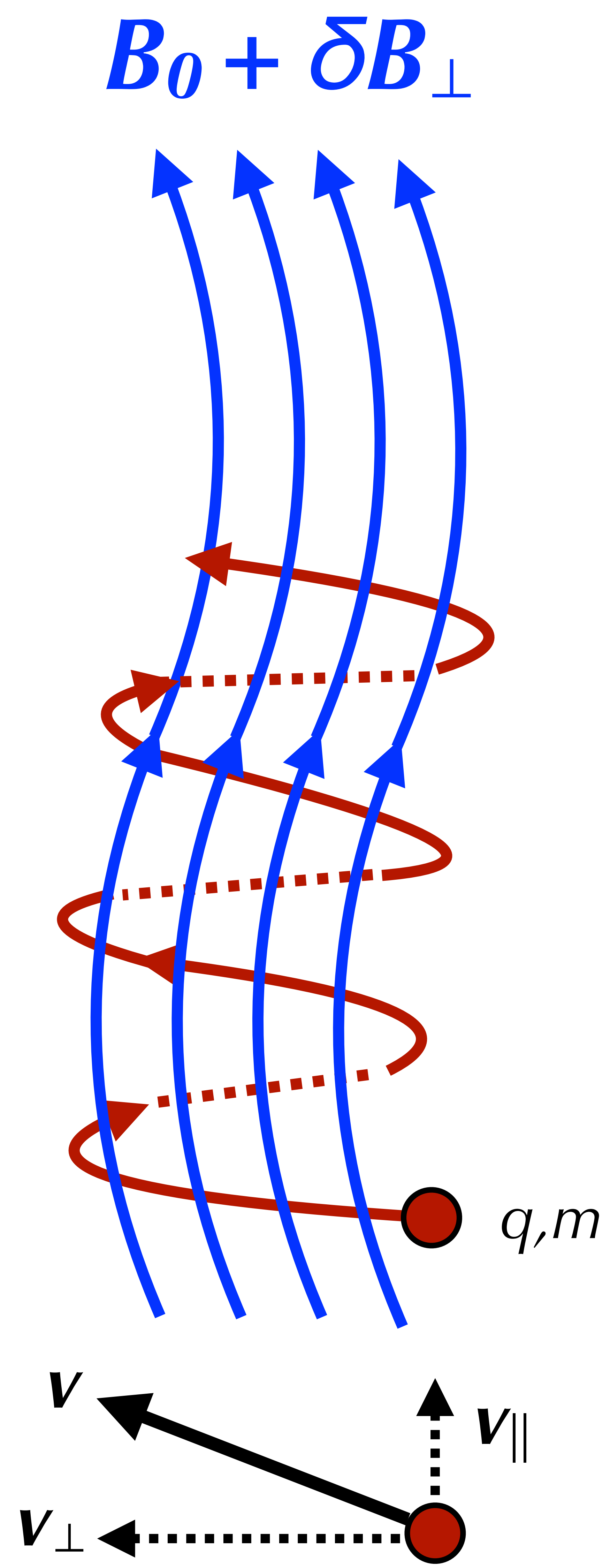

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$$\boxed{D(E) \propto E^\delta}$$

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CR diffusion

A more rigorous quasi-linear calculation for Alfvénic fluctuations would give:

[e.g., Kulsrud & Pearce, ApJ (1969); Völk, ASpSci (1973)]

$$D_{\mu\mu} = \Omega^2(1-\mu^2)\int \mathrm{d}\boldsymbol{k} \sum_{n=-\infty}^{\infty} \delta(\underbrace{k_\| v_\| - \omega_\mathrm{A} + n\Omega}_{\approx k_\| v_\| + n\Omega}) \frac{n^2 J_n^2(z)}{z^2}\,\mathcal{E}^\mathrm{A}(\boldsymbol{k}) \qquad z \doteq \frac{k_\perp v_\perp}{\Omega} = k_\perp r_\mathrm{L}$$

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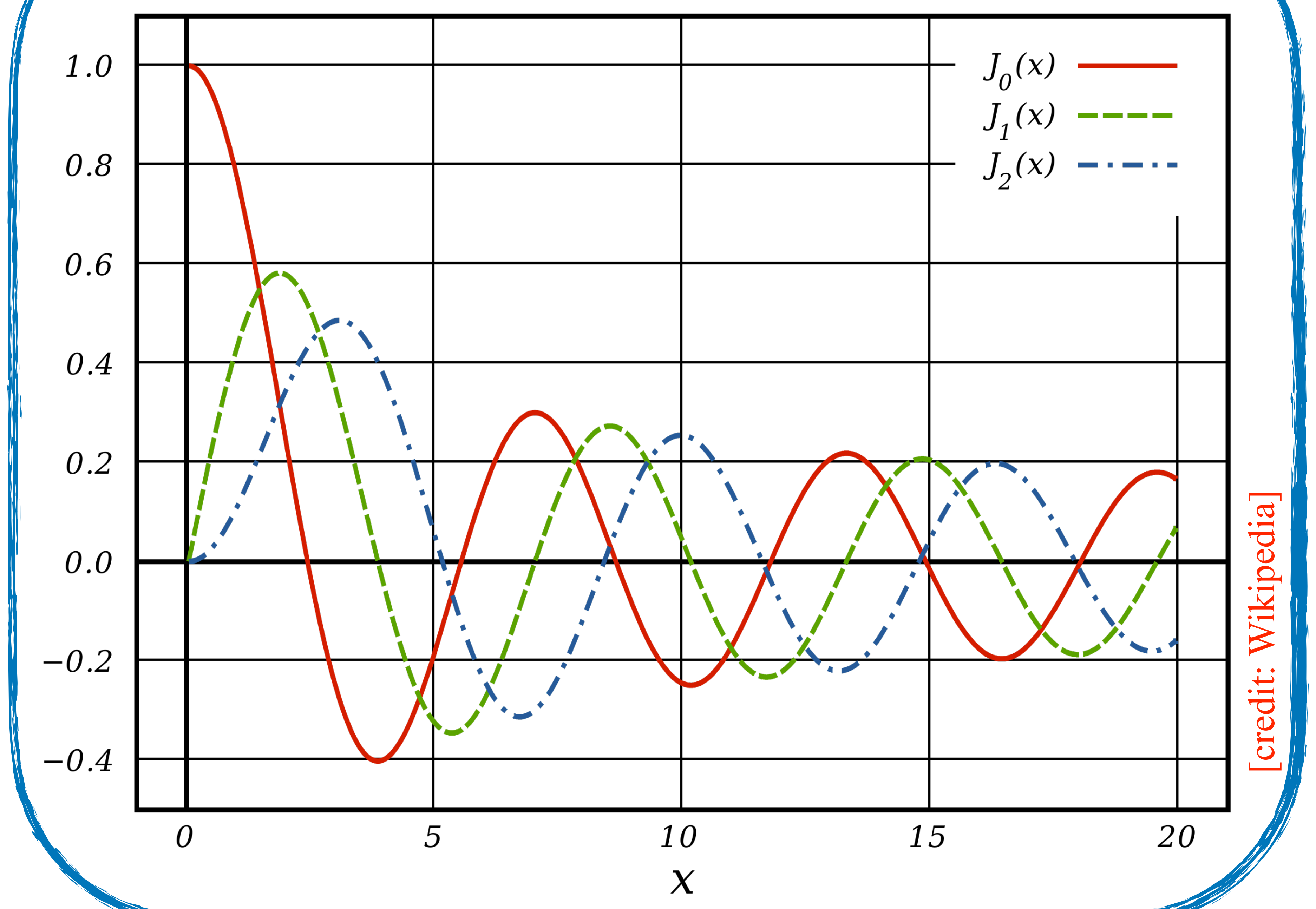
$$\frac{\omega_\mathrm{A}}{k_\parallel v_\parallel} = \frac{v_\mathrm{A}}{v_\parallel} \sim \frac{v_\mathrm{A}}{c} \ll 1$$

Bessel functions of the 1st kind



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⚠️ <u>second problem</u>:

[Chandran, PRL (2000)]

In anisotropic turbulence, $k_\perp >> k_\parallel$, like for the Alfvénic cascade, the gyro-resonant scattering is strongly suppressed

e.g., for a Goldreich-Sridhar cascade the fluctuations' power spectrum is confined by

$$k_\perp \gtrsim k_\parallel^{3/2}\ell_0^{-1/2} \quad \Rightarrow \quad z = k_\perp r_\mathrm{L} \gtrsim \left(n\frac{\sqrt{1-\mu^2}}{\mu}\right)^{3/2}\left(\frac{r_\mathrm{L}}{\ell_0}\right)^{-1/2} \gg 1$$

so a CR crosses many uncorrelated fluctuations of the required $k_\parallel$ during its gyro-orbit,
and such contributions tend to cancel out (cf. $J^2(z)/z^2$ behaviour at $z >> 1$)

CR diffusion

So, CR scattering by Alfvénic turbulence is very inefficient at CR-energies less than ~ 10-100 TeV

[this is true even from a very precise calculation of D, then included in global CR-transport simulations: see Fornieri et al., MNRAS (2021)]

what now?!

CR diffusion

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what now?!

well, MHD does not have just the Alfvén mode in a compressible world!

CR diffusion

☞ Dispersion relations from linearized ***compressible MHD***:

$$\begin{pmatrix} \omega^2 - k^2 v_A^2 - k_\perp^2 c_s^2 & 0 & -k_\perp k_\parallel c_s^2 \\ 0 & \omega^2 - k_\parallel^2 v_A^2 & 0 \\ -k_\perp k_\parallel c_s^2 & 0 & \omega^2 - k_\parallel^2 c_s^2 \end{pmatrix} \begin{pmatrix} \delta u_x \\ \delta u_y \\ \delta u_z \end{pmatrix} = 0$$

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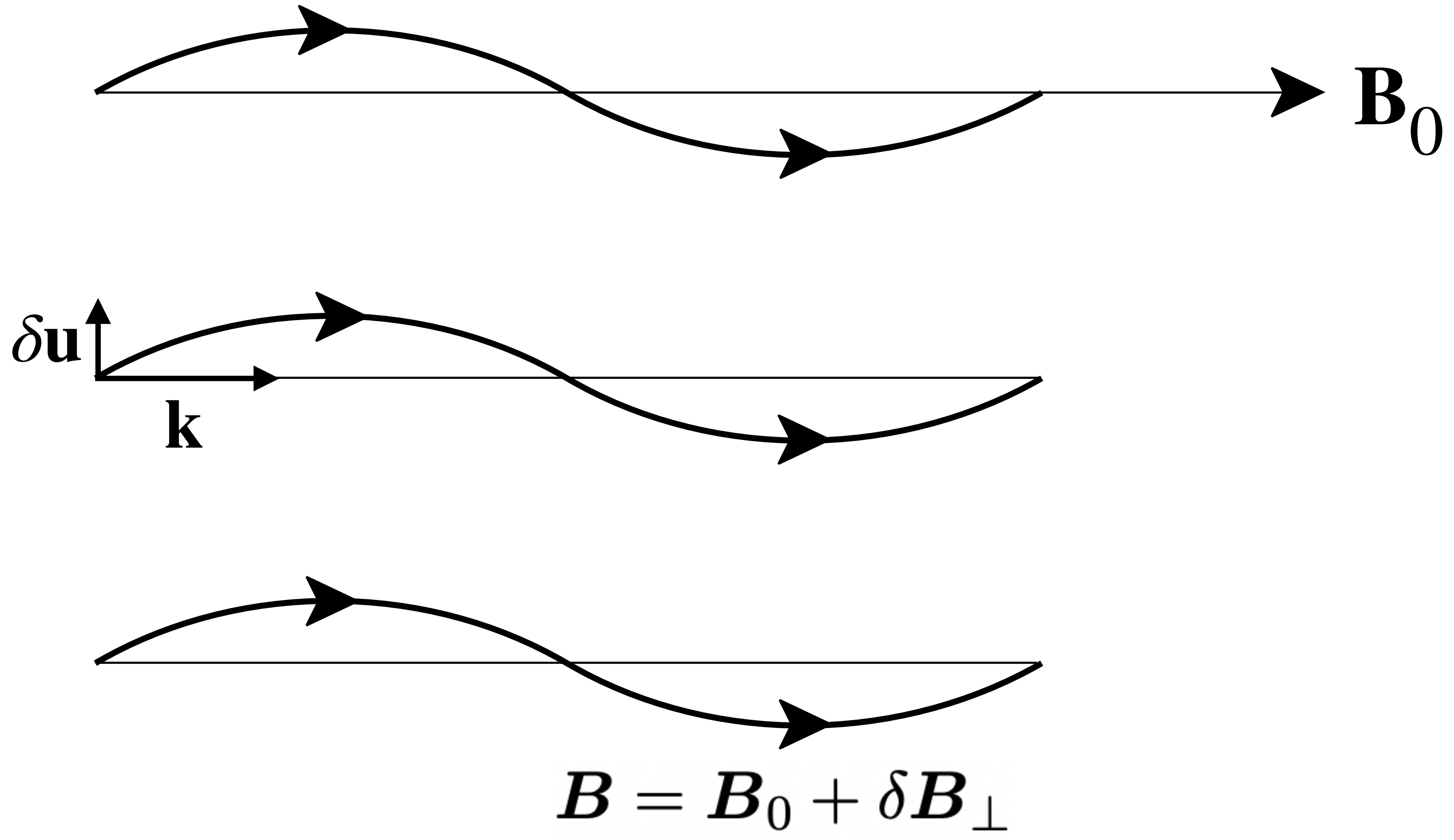
Alfvén mode: *transverse* oscillation

$$\boldsymbol{k} \cdot \delta \boldsymbol{u} = 0 \quad \text{and} \quad \delta \boldsymbol{u} \cdot \boldsymbol{B}_0 = 0$$



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Two additional roots: "***fast***" and "***slow***" magnetosonic waves

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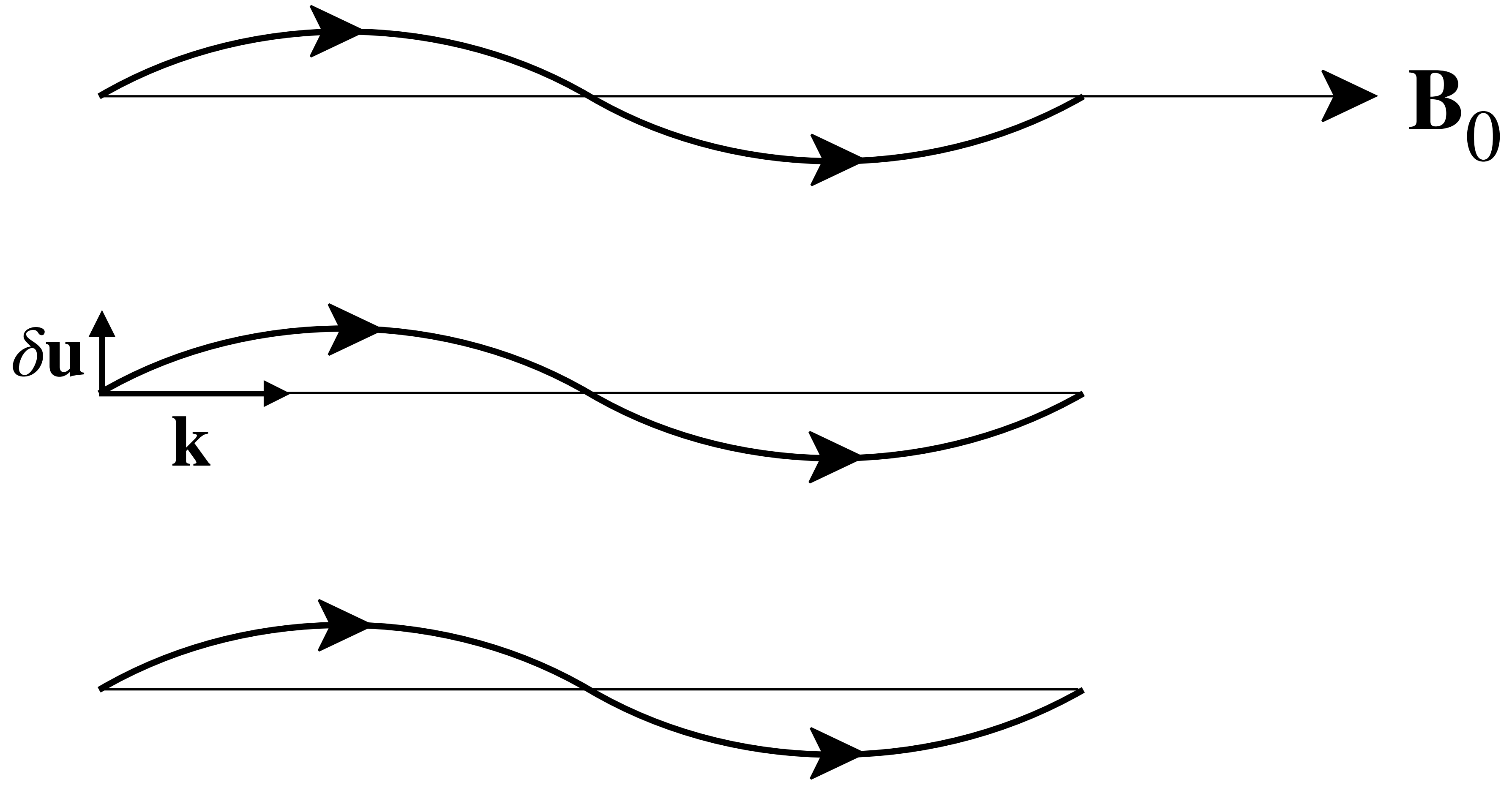

$$\boldsymbol{B} = \boldsymbol{B}_0 + \delta \boldsymbol{B}_\perp$$

CR diffusion

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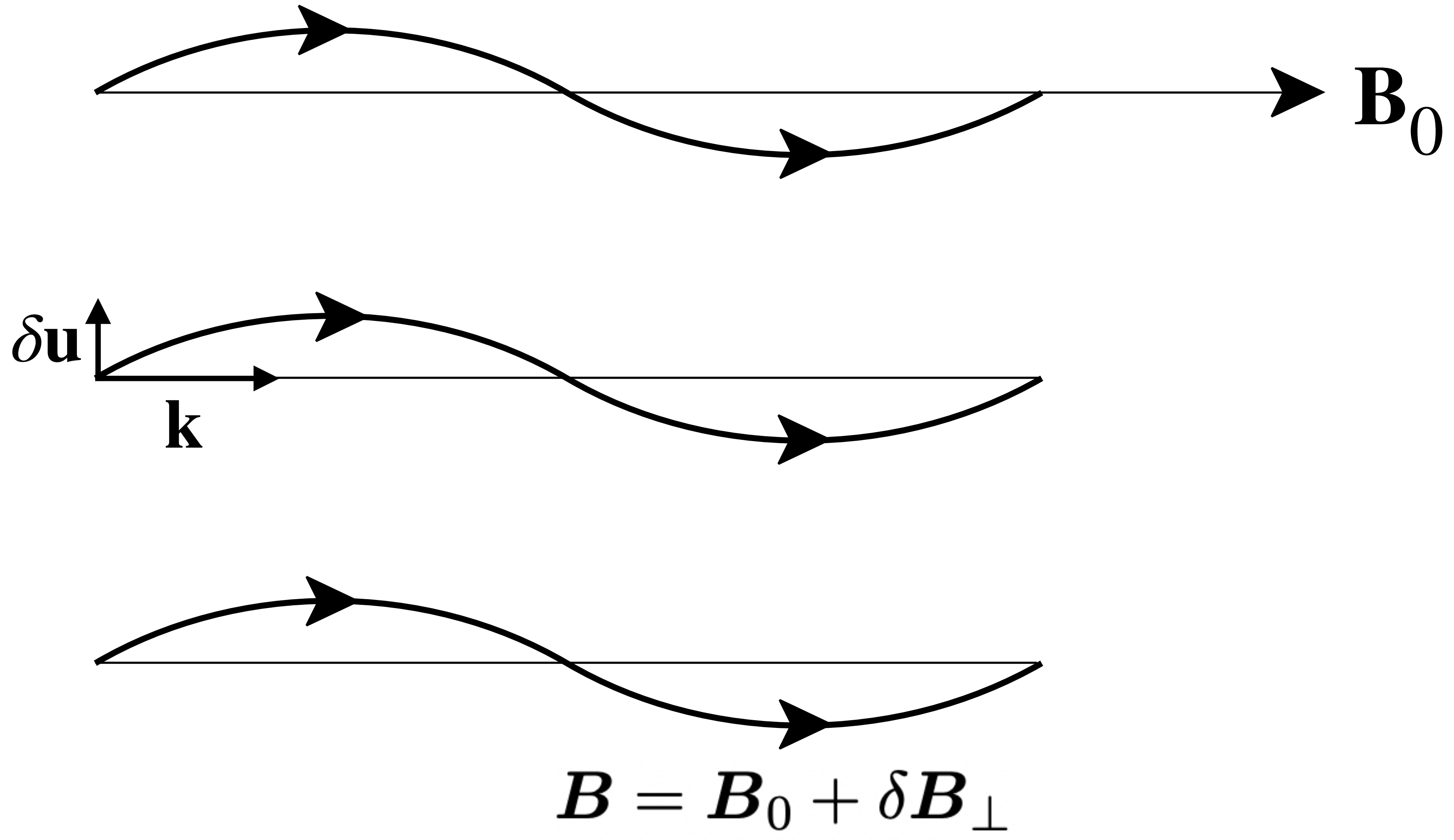
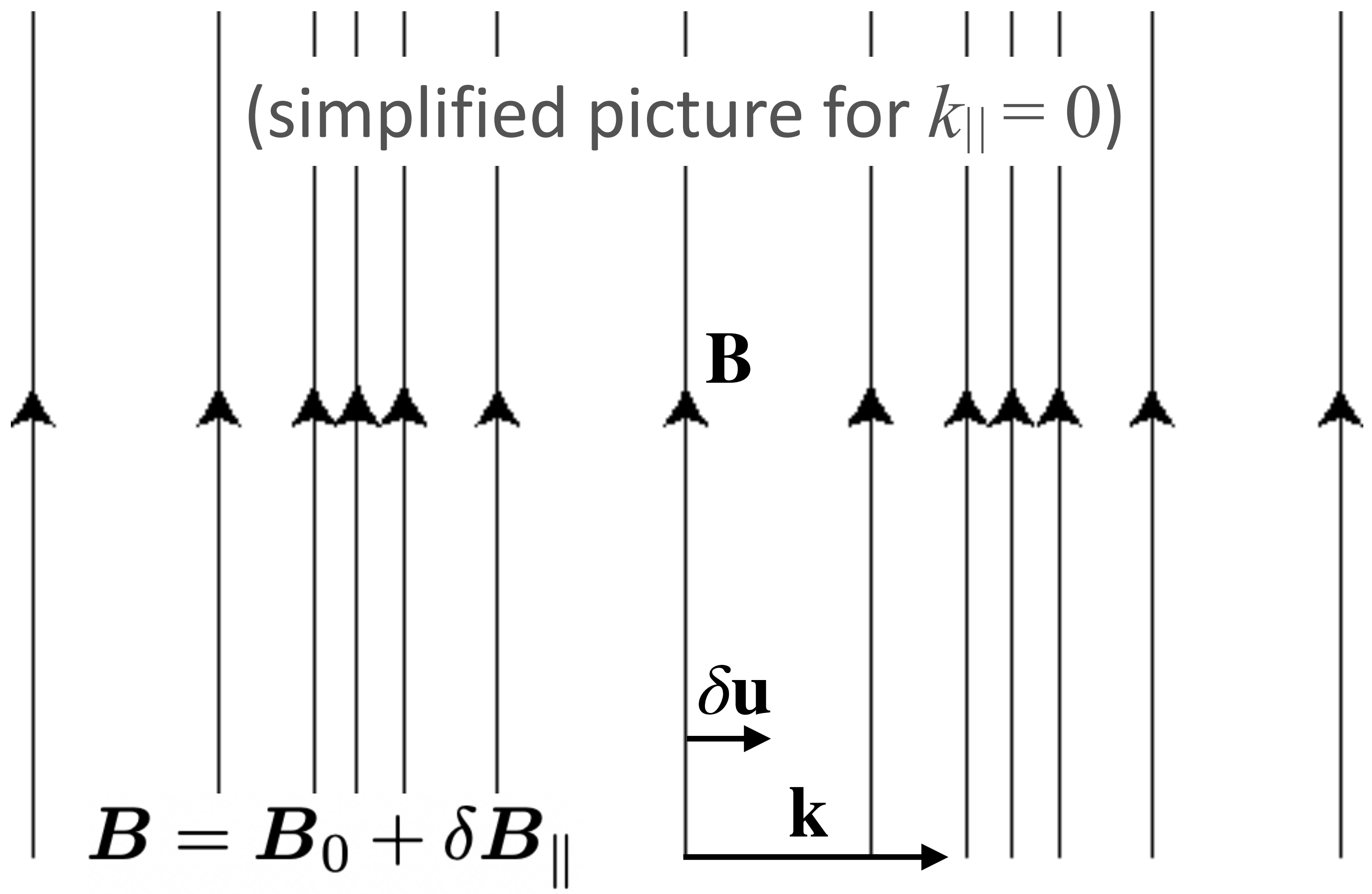


Magnetosonic modes: *longitudinal* oscillations

$$\boldsymbol{k} \cdot \delta \boldsymbol{u} \neq 0 \quad \text{and} \quad \delta \boldsymbol{u} \cdot \boldsymbol{B}_0 \neq 0$$



CR diffusion

Why magnetosonic modes are important for CR scattering?

CR diffusion

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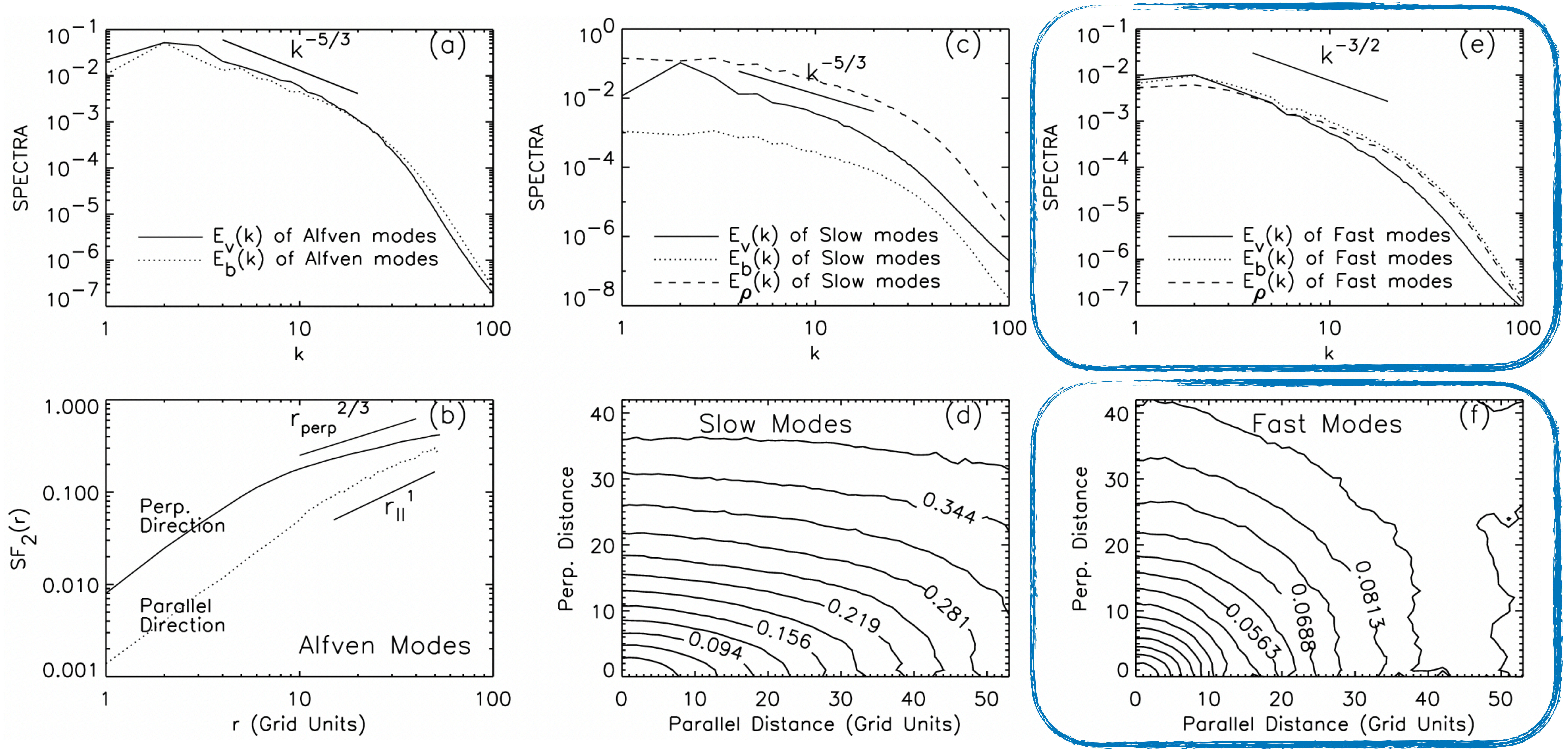
☞ Isotropy of the fast-magnetosonic cascade (from simulations)



[Cho & Lazarian, PRL (2002)]

CR diffusion

Why magnetosonic modes are important for CR scattering?

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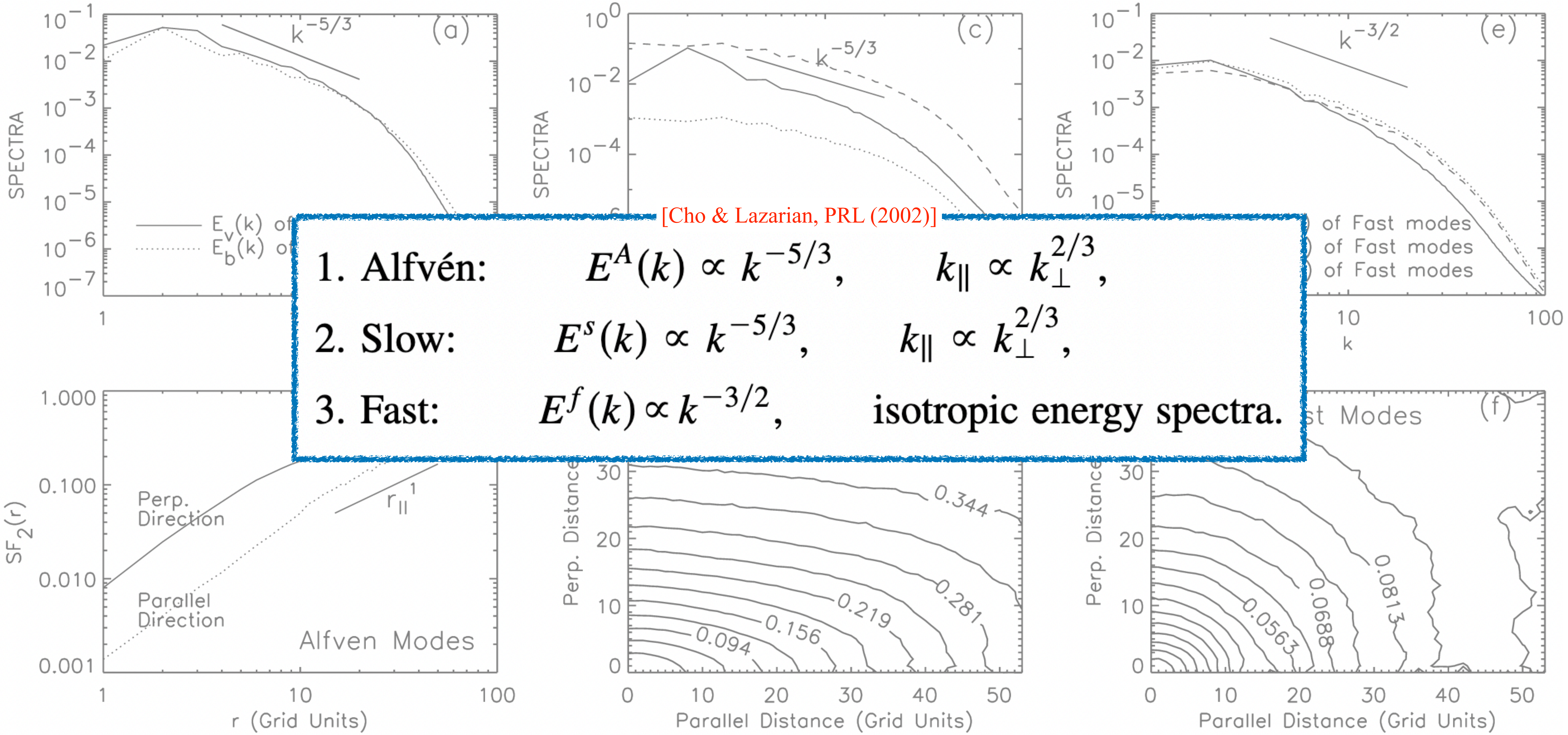


[Cho & Lazarian, PRL (2002)]

1. Alfvén: $E^A(k) \propto k^{-5/3}$, $k_{\parallel} \propto k_{\perp}^{2/3}$,
2. Slow: $E^s(k) \propto k^{-5/3}$, $k_{\parallel} \propto k_{\perp}^{2/3}$,
3. Fast: $E^f(k) \propto k^{-3/2}$, isotropic energy spectra.

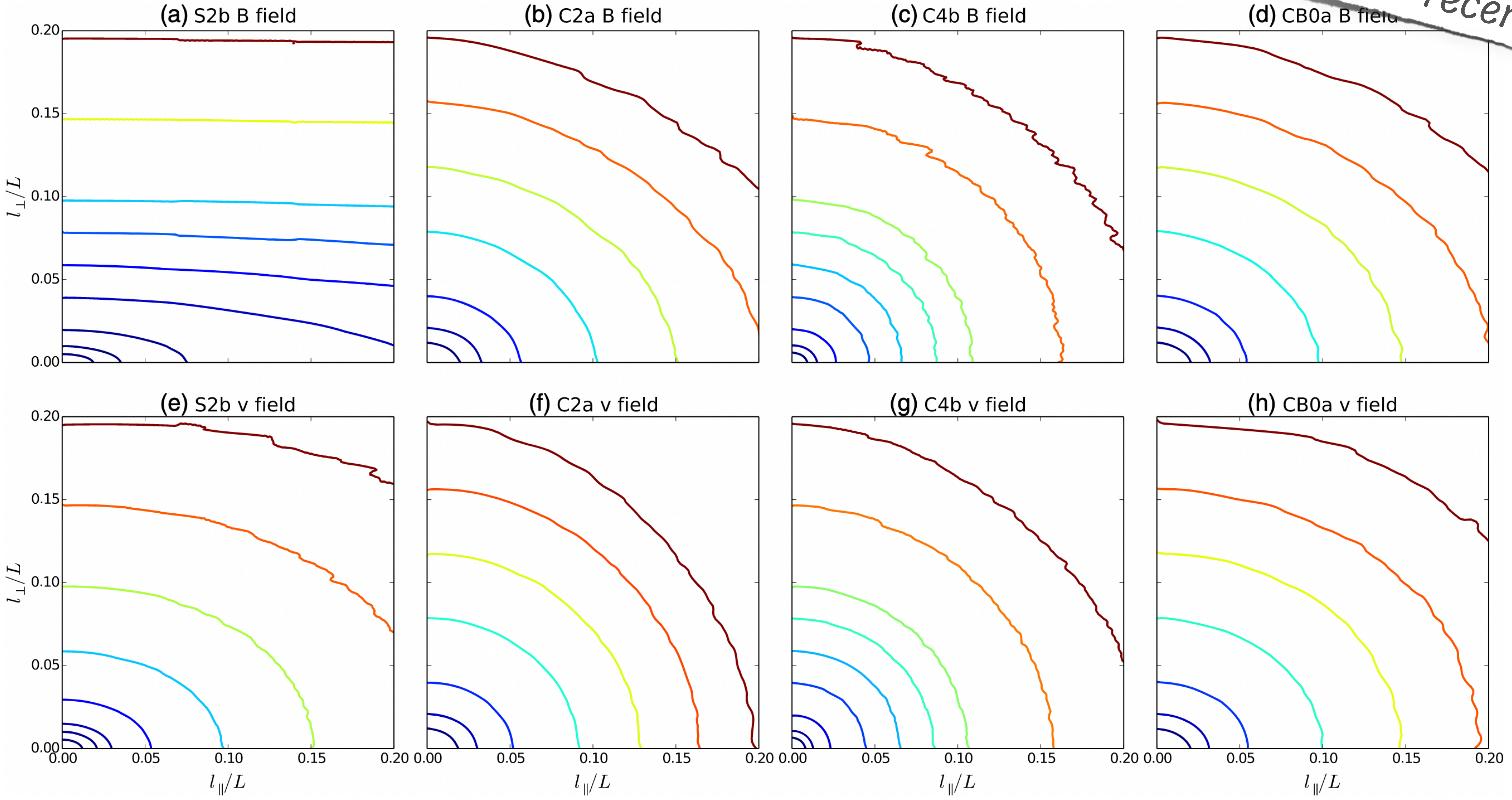
[Cho & Lazarian, PRL (2002)]

CR diffusion

Why magnetosonic modes are important for CR scattering?

☞ Isotropy of the fast-magnetosonic cascade (from simulations)

also more recent simulations



[Makwana & Yan, PRX (2020)]

CR diffusion

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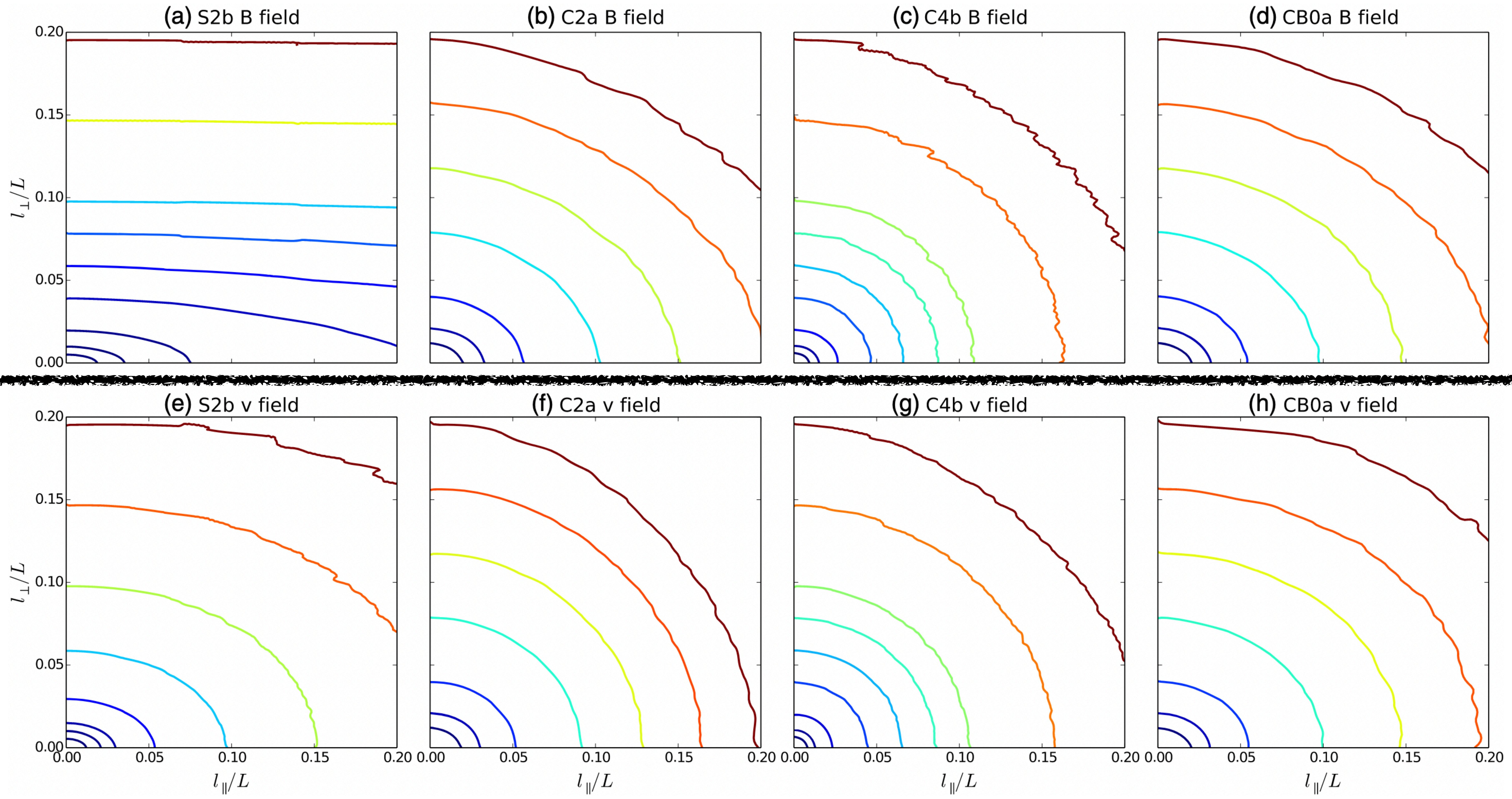

purely solenoidal injection

purely compressional injection

[Makwana & Yan, PRX (2020)]

CR diffusion

Why magnetosonic modes are important for CR scattering?

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so, the **fast-magnetosonic cascade** gets us past the problem of the anisotropy…

$$D_{\mu\mu} = \Omega^2(1-\mu^2)\int \mathrm{d}\boldsymbol{k}\sum_{n=-\infty}^{\infty}\delta(k_\| v_\| - \omega + n\Omega)\left\{\frac{n^2 J_n^2(z)}{z^2}\left[\mathcal{E}^{\mathrm{A}}(\boldsymbol{k}) + \mathcal{E}^{\mathrm{S}}(\boldsymbol{k})\right] + \boxed{\frac{k_\|}{k}\, J_n'^2(z)\,\mathcal{E}^{\mathrm{F}}(\boldsymbol{k})}\right\}$$

CR diffusion

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…but what about the μ→0 issue due to the resonance condition?

CR diffusion

Why magnetosonic modes are important for CR scattering?

☞ Isotropy of the fast-magnetosonic cascade (from simulations)

☞ Resonance broadening due to local $|\mathbf{B}|$ variations (from energy + magnetic-moment conservation)

CR diffusion

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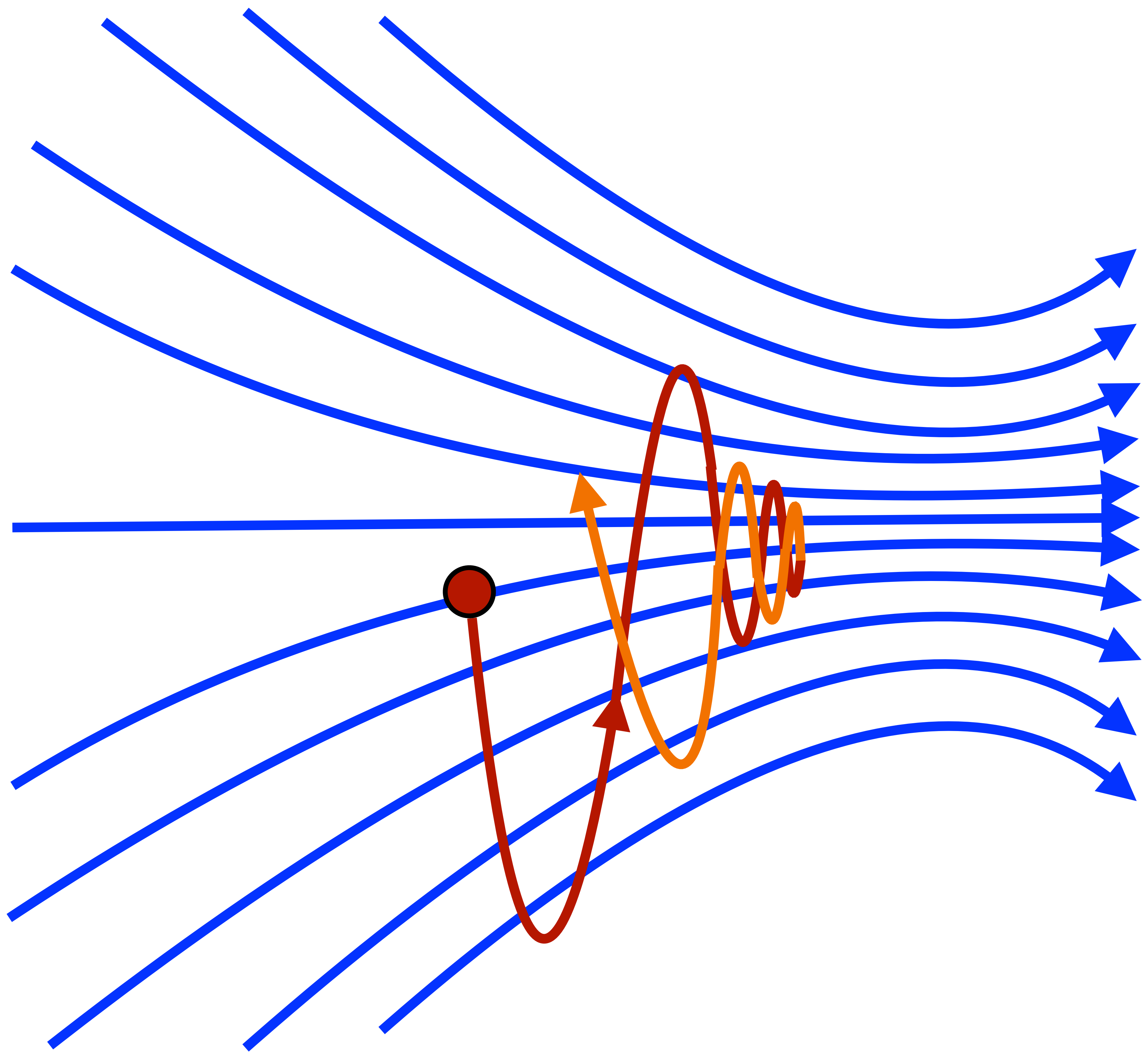
$$v_\perp^2 + v_\parallel^2 = v^2 = \text{const.} \qquad + \qquad \frac{v_\perp^2}{|\boldsymbol{B}|} = \text{const.}$$

$$\Rightarrow \quad \langle \Delta v_\parallel^2 \rangle = -(1-\mu^2)v^2 \frac{\langle \Delta |\boldsymbol{B}| \rangle}{B_0} \approx -(1-\mu^2)v^2 \frac{\delta B_\parallel^{\mathrm{rms}}}{B_0} + \mathcal{O}(\delta B^2)$$



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[Yan & Lazarian, ApJ (2008)]

$$\delta(k_\parallel v_\parallel - \omega + n\Omega) \quad \longrightarrow \quad \boxed{R(k_\parallel v_\parallel - \omega + n\Omega) = \frac{\sqrt{\pi}}{|k_\parallel| v \sqrt{1-\mu^2}\, M_\mathrm{A}^{1/2}} \exp\left(- \frac{(k_\parallel v_\parallel - \omega + n\Omega)^2}{k_\parallel^2 v^2 (1-\mu^2) M_\mathrm{A}} \right)}$$

with Alfvénic Mach number $\quad M_\mathrm{A} \sim (\delta B / B_0)_\mathrm{inj}$

Numerical Framework & Recent Results

[Fornieri et al., MNRAS (2021)]

$$D_{\mu\mu} = \Omega^2(1-\mu^2)\int d^3\boldsymbol{k} \sum_{n=-\infty}^{+\infty} R_n(k_\parallel v_\parallel - \omega + n\Omega)\left[\frac{n^2 J_n^2(z)}{z^2} I^{\mathrm{A}}(\boldsymbol{k}) + \frac{k_\parallel^2}{k^2} J_n'^2(z) I^{\mathrm{M}}(\boldsymbol{k})\right]$$

$$R_n(k_\parallel v_\parallel - \omega + n\Omega) = \frac{\sqrt{\pi}}{|k_\parallel| v_\perp M_{\mathrm{A}}^{1/2}} \cdot \exp\left(-\frac{(k_\parallel v\mu - \omega + n\Omega)^2}{k_\parallel^2 v^2(1-\mu^2) M_{\mathrm{A}}}\right)$$

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Advance Access publication 2021 February 9

The theory of cosmic ray scattering on pre-existing MHD modes meets data

Ottavio Fornieri[1,2,3★], Daniele Gaggero[3], Silvio Sergio Cerri[4], Pedro De La Torre Luque[5,6] and Stefano Gabici[7]

Computing CR diffusion coefficients

[Fornieri et al., MNRAS (2021)]

$$D_{\mu\mu} = \Omega^2(1-\mu^2)\int d^3\boldsymbol{k} \sum_{n=-\infty}^{+\infty} R_n(k_\parallel v_\parallel - \omega + n\Omega)\left[\frac{n^2 J_n^2(z)}{z^2} I^{\mathrm{A}}(\boldsymbol{k}) + \frac{k_\parallel^2}{k^2} J_n'^2(z) I^{\mathrm{M}}(\boldsymbol{k})\right]$$

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$$R_n(k_\parallel v_\parallel - \omega + n\Omega) = \frac{\sqrt{\pi}}{|k_\parallel| v_\perp M_{\mathrm{A}}^{1/2}} \cdot \exp\left(-\frac{(k_\parallel v\mu - \omega + n\Omega)^2}{k_\parallel^2 v^2 (1-\mu^2) M_{\mathrm{A}}}\right)$$

$$\langle \delta B_i(\boldsymbol{k}) \cdot \delta B_j^*(\boldsymbol{k}')\rangle / B_0^2 = \delta^3(\boldsymbol{k} - \boldsymbol{k}')\, \mathcal{M}_{ij}(\boldsymbol{k})$$

$$\mathcal{M}_{ij}^{\mathrm{A(S),sub}} = C_a^{\mathrm{A(S),sub}} I_{ij} k_\perp^{-10/3} \cdot \exp\left(-\frac{L^{1/3}|k_\parallel|}{M_{\mathrm{A}}^{4/3} k_\perp^{2/3}}\right) \qquad (M_{\mathrm{A}} \leq 1)$$

$$\mathcal{M}_{ij}^{\mathrm{A(S),super}} = C_a^{\mathrm{A(S),super}} I_{ij} k_\perp^{-10/3} \cdot \exp\left(-\frac{L^{1/3}|k_\parallel|}{M_{\mathrm{A}} k_\perp^{2/3}}\right) \qquad (M_{\mathrm{A}} > 1)$$

$$\mathcal{M}_{ij}^{\mathrm{F}} = C_a^{\mathrm{F}} J_{ij} k^{-7/2},$$

$$\textstyle\sum_{i=j} \mathcal{M}_{ij} = I^{\mathrm{A,S,F}}$$

normalization constant C_a from:

$$\langle \delta B(\boldsymbol{x})^2 \rangle \equiv B_0^2 \sum_{i,j} \int d^3\boldsymbol{k}\, \mathcal{M}_{ij}(\boldsymbol{k}) \overset{!}{=} \delta B_{\mathrm{rms}}^2 \equiv \langle \delta B^2 \rangle_L$$

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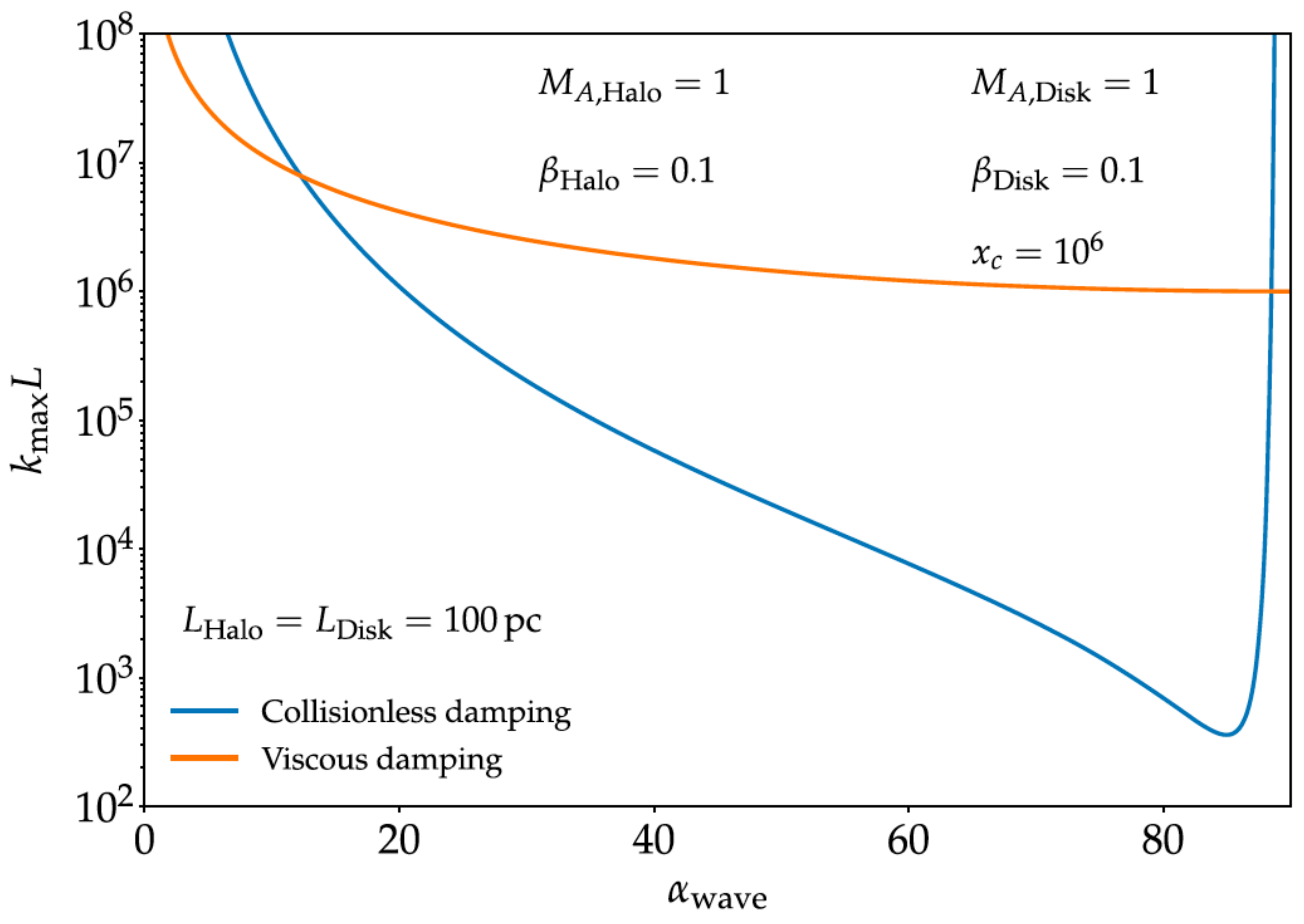
The integral in k has to be performed up to a "***truncation scale***" (dissipation scale)

depends on several environmental/plasma/wave parameters

$$\beta = \frac{P_g}{P_B}$$

$$x_c = \left(\frac{6\,\rho\,\delta V^2\, L}{\eta_0\, v_A}\right)^{2/3}$$

$$k_\parallel \equiv |\boldsymbol{k}| \cos\alpha_{\mathrm{wave}}$$



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☞ Now, having computed all modes' contributions to the ptich-angle scattering, compute the ***spatial diffusion coefficient***

$$D(R) = \frac{1}{4}\int_0^{\mu^*} d\mu \, \frac{v^2\,(1-\mu^2)^2}{D_{\mu\mu}^{\mathrm{M,T}}(R) + D_{\mu\mu}^{\mathrm{M,G}}(R) + D_{\mu\mu}^{\mathrm{A,G}}(R)}$$

$$R = L^{-1}|\boldsymbol{v}|/\Omega$$

particle's (dimensionless) rigidity

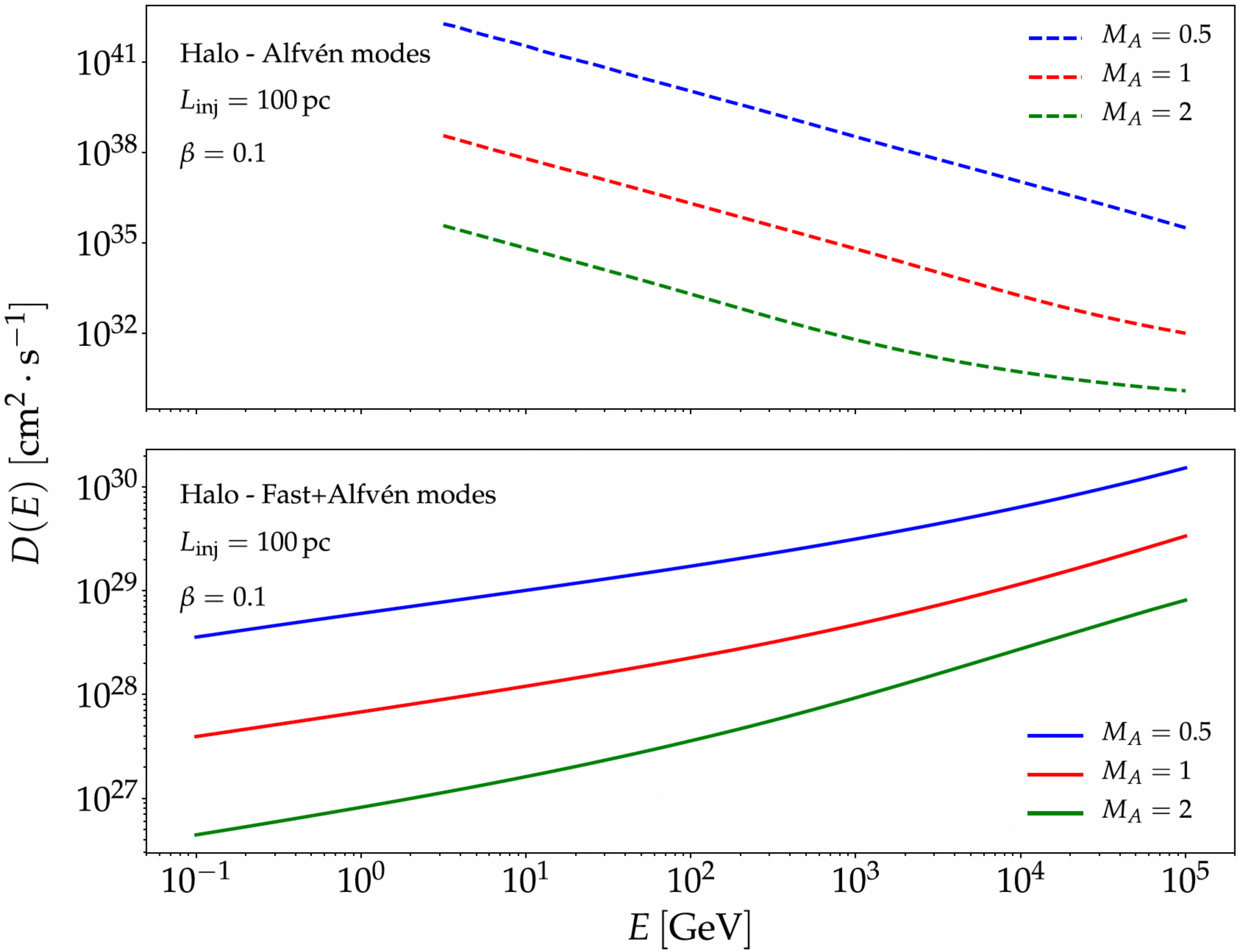
where μ^* is the largest pitch angle for which a cosmic ray with rigidity R can be considered confined by turbulence (i.e., in the ***diffusion regime***):

$$\frac{\tau_{\mu\mu}}{\tau_{\mathrm{stream}}} \sim \frac{v}{L'_{\mathrm{H,D}}} \frac{(1-\mu^2)}{D_{\mu\mu}} \ll 1$$

Computing CR diffusion coefficients

[Fornieri et al., MNRAS (2021)]

☞ **just Alfvén modes** are indeed **inefficient for CR confinement in the Galactic Halo**:



Computing CR diffusion coefficients

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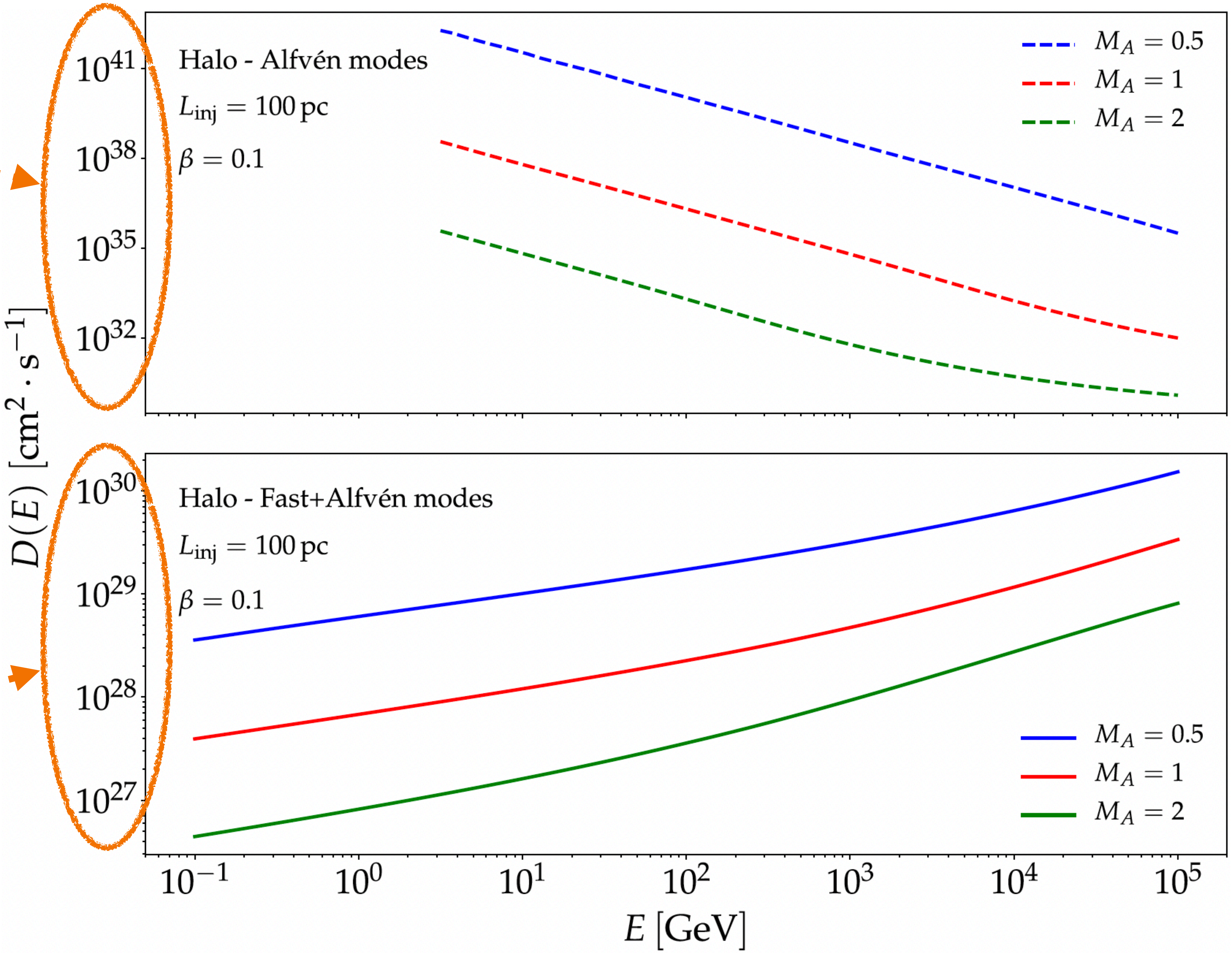
note different range!



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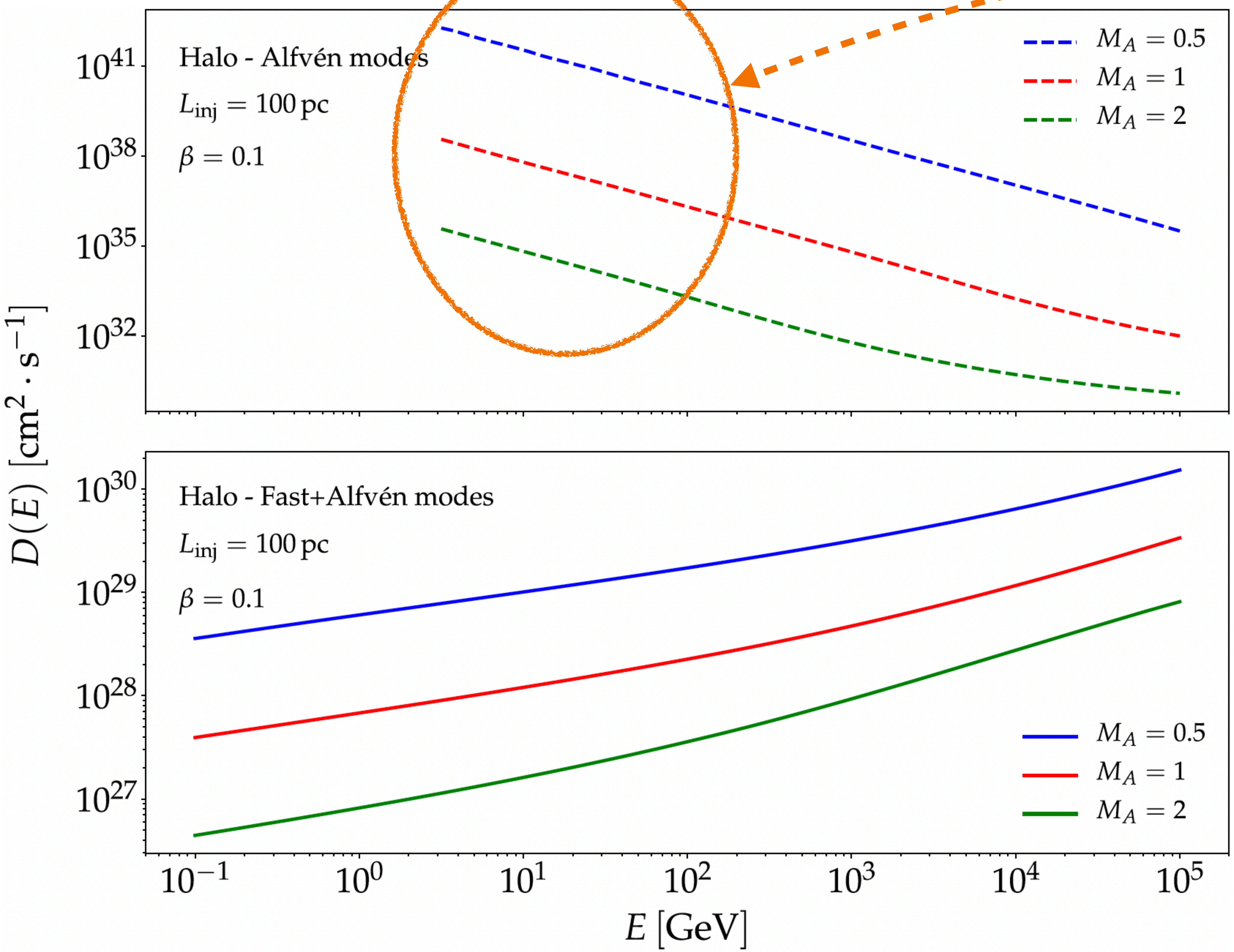


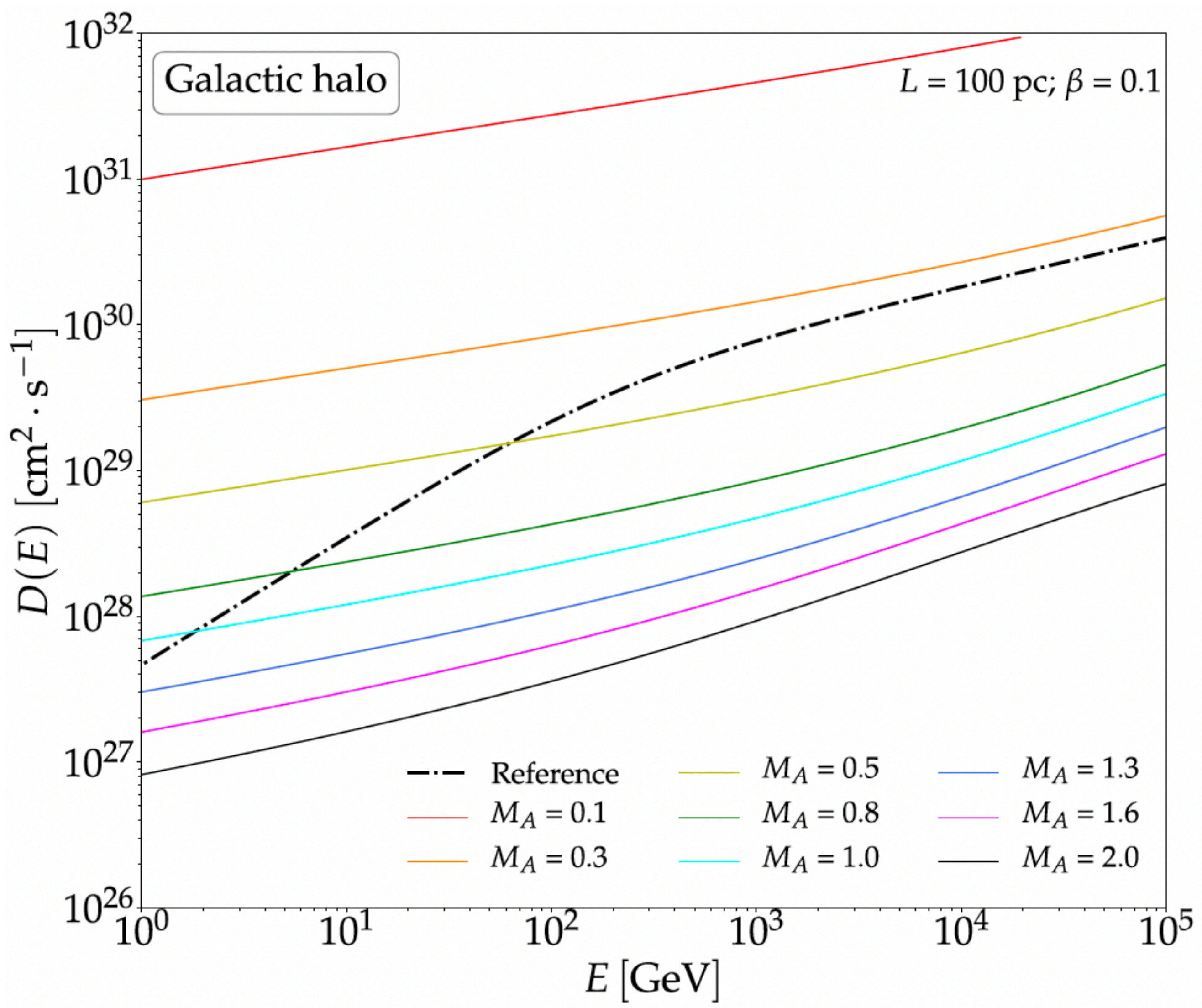
These CR energies corresponds to small spatial scales, where the ***anisotropy of the Alfvénic cascade is highly developed***

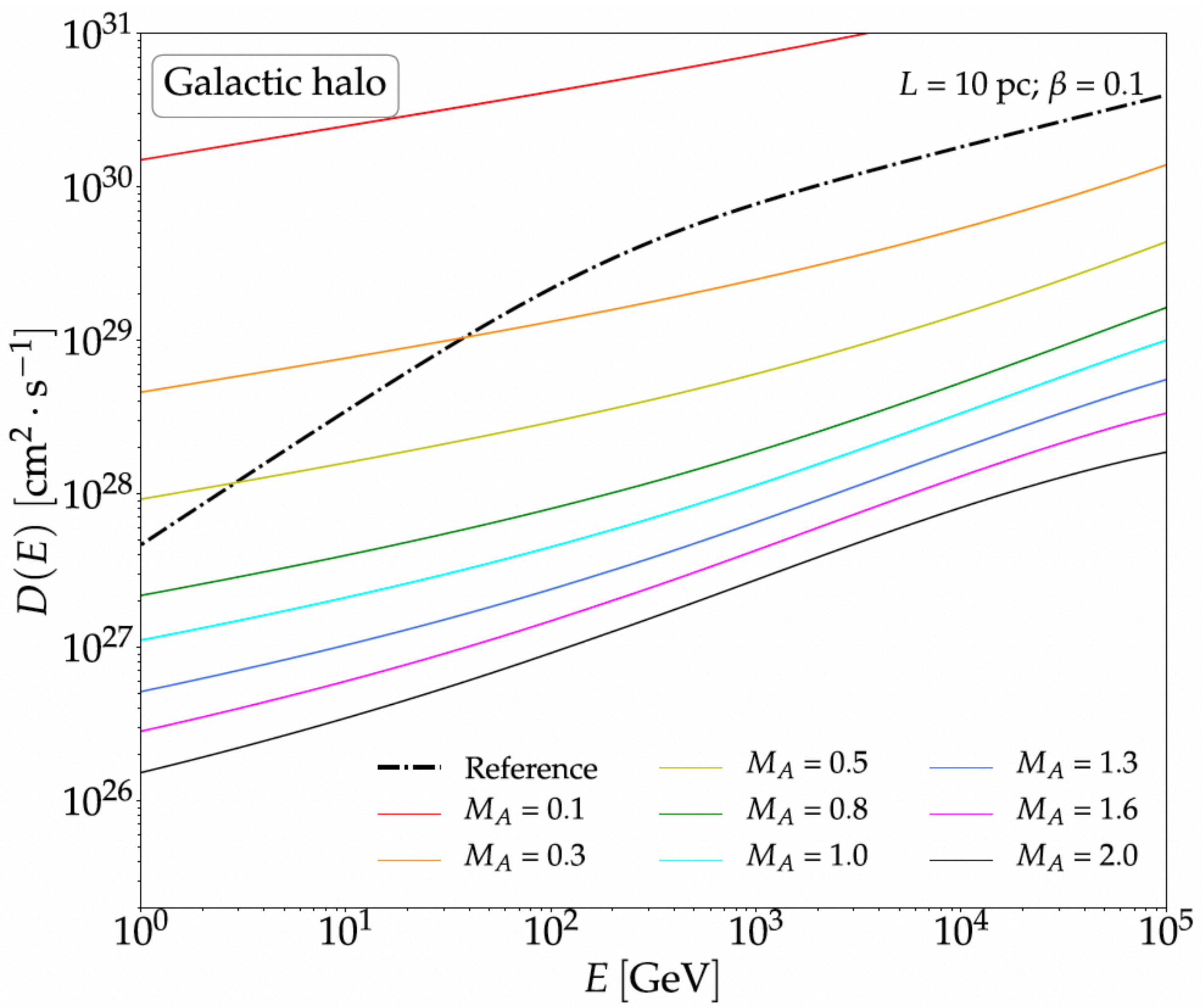
Computing CR diffusion coefficients

[Fornieri et al., MNRAS (2021)]

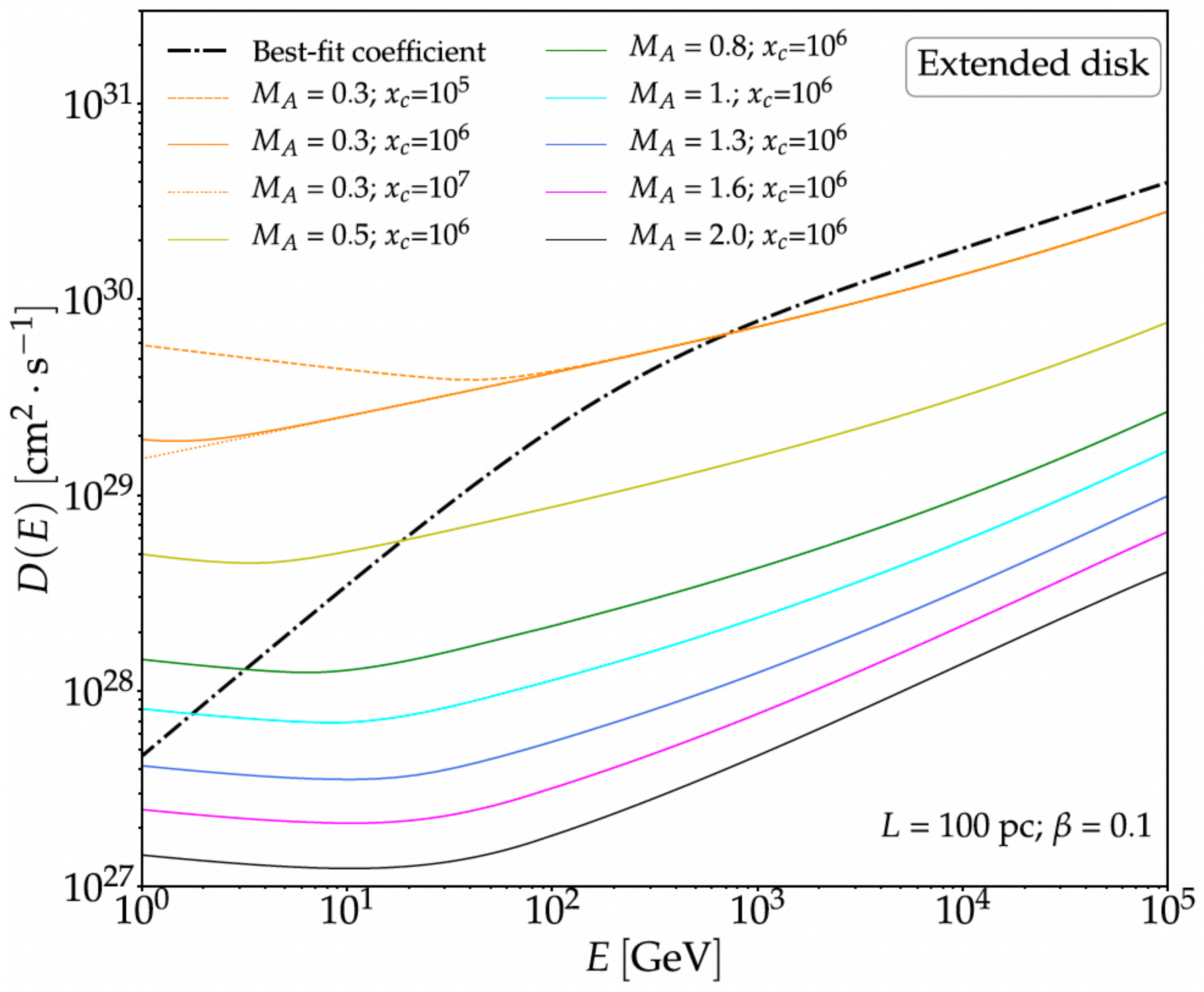
☞ parameter scan to see if we can match phenomenological best-fit models based on CR data:

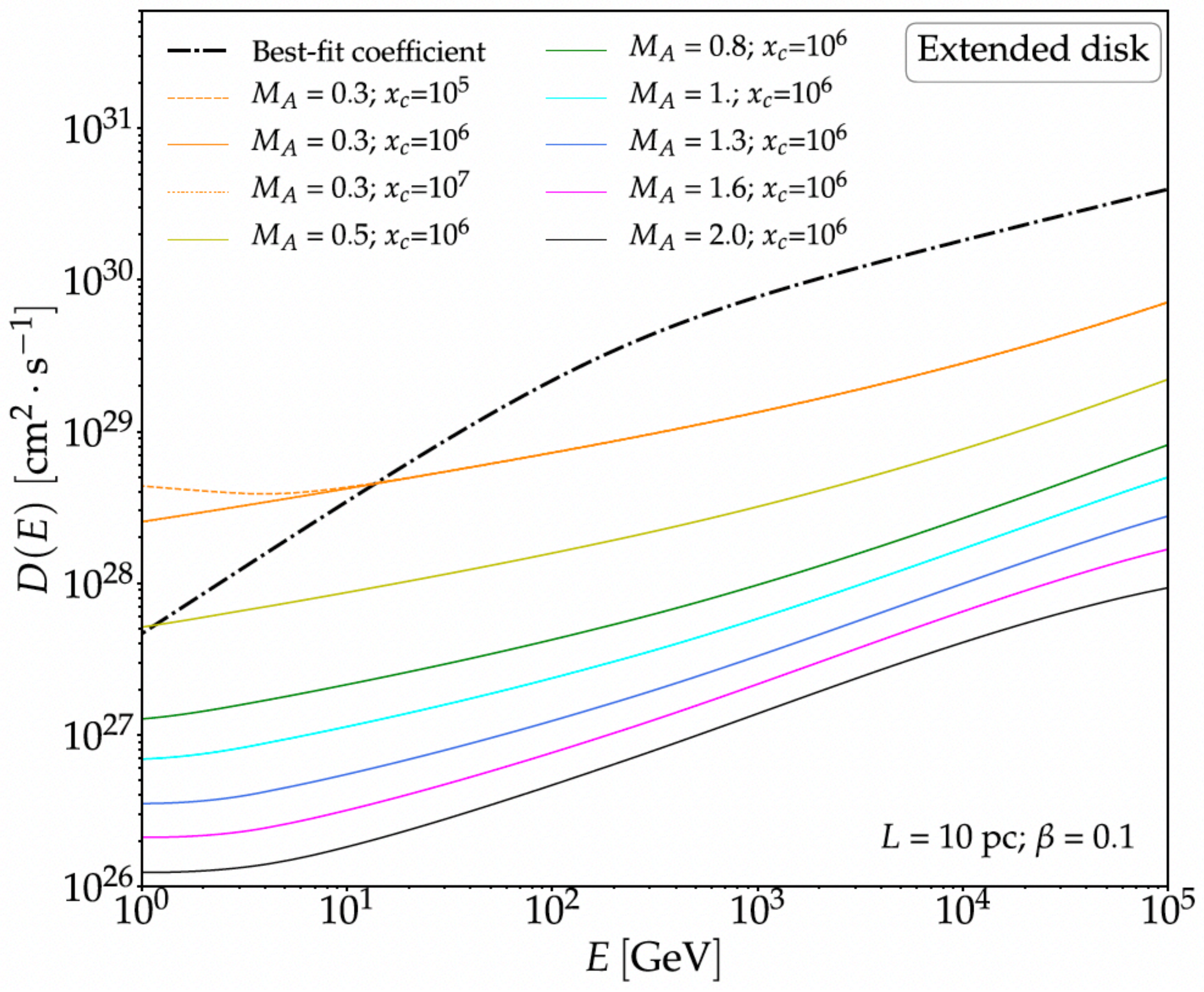
Galactic Halo:





Galactic Disk:





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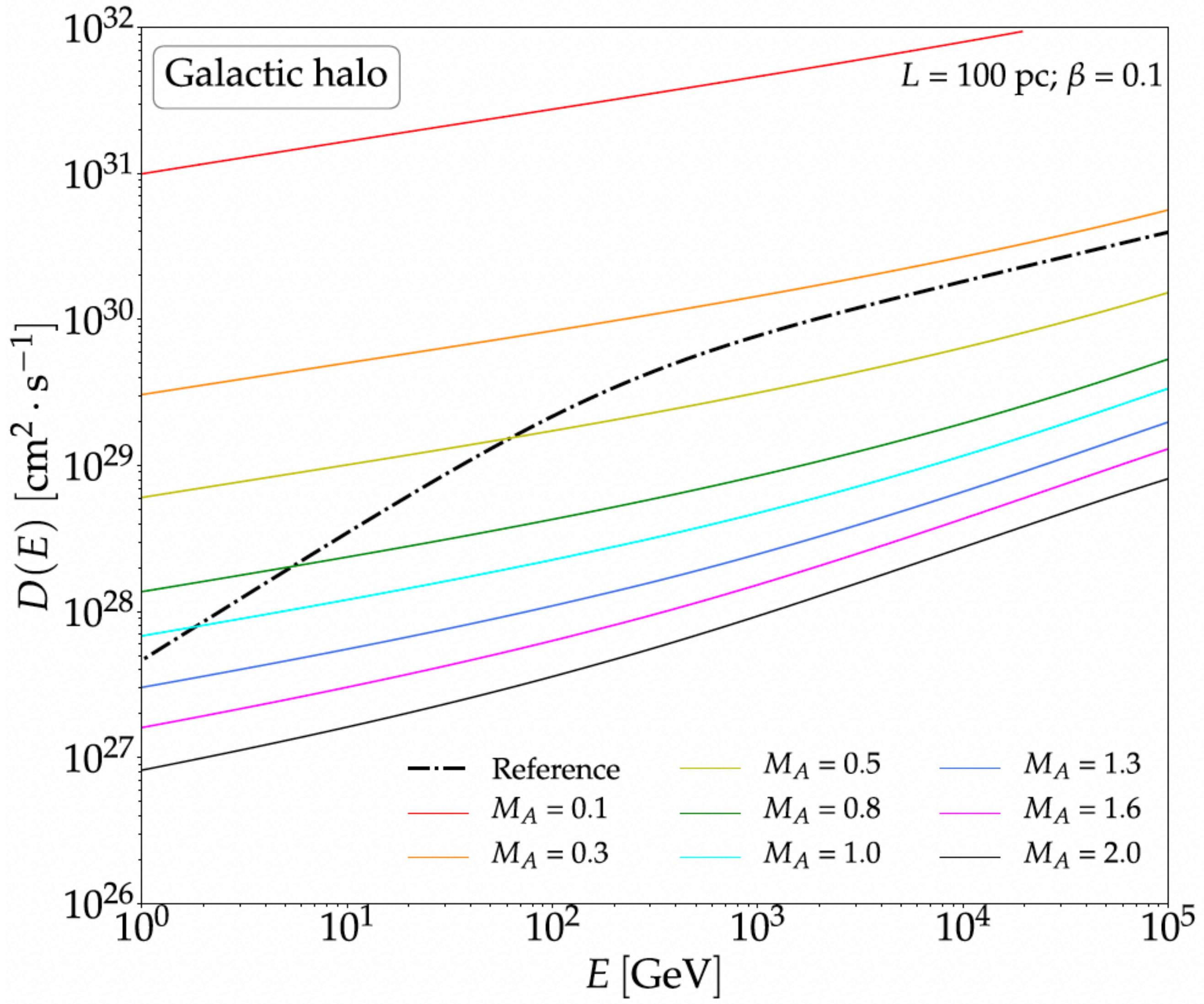

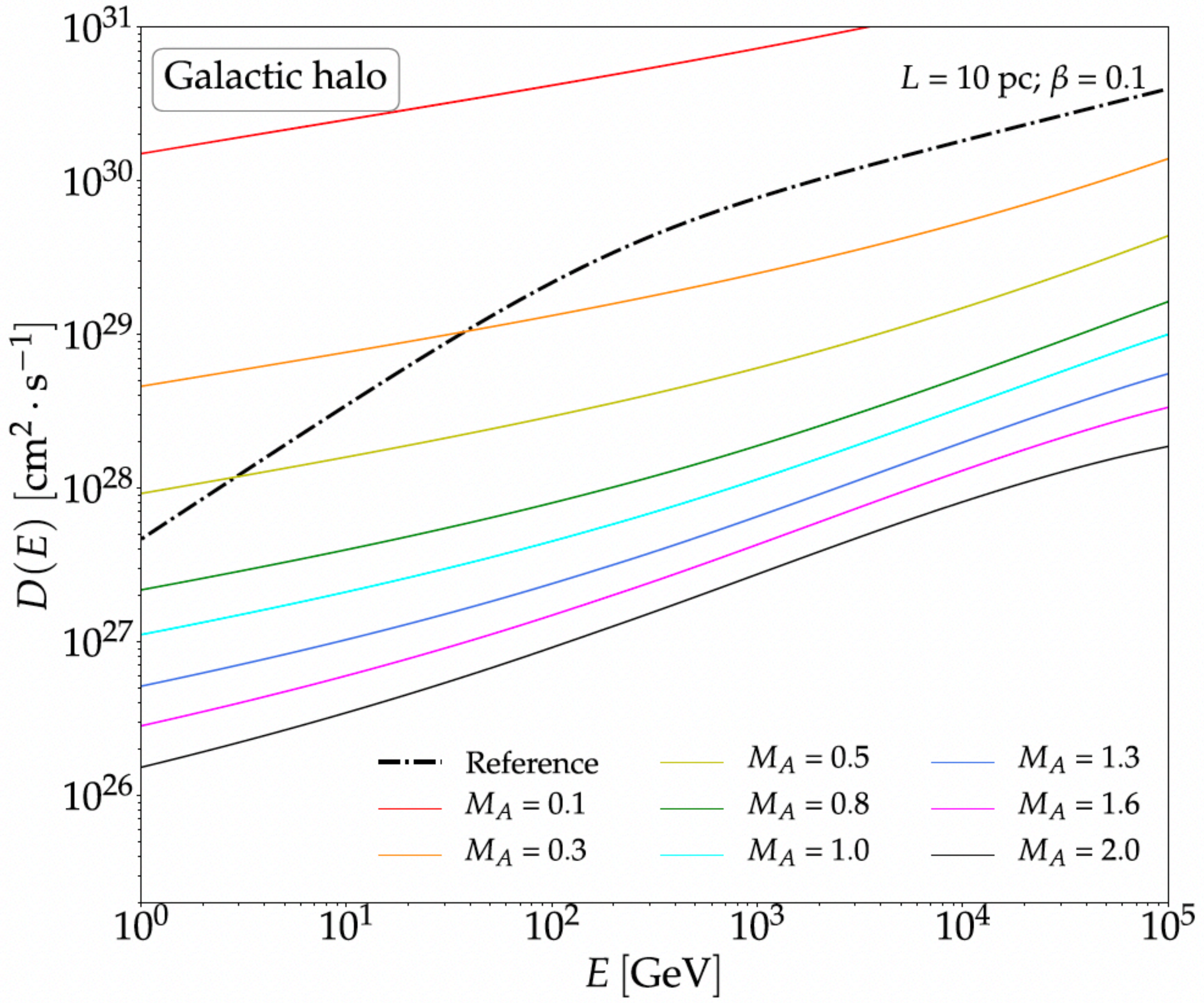

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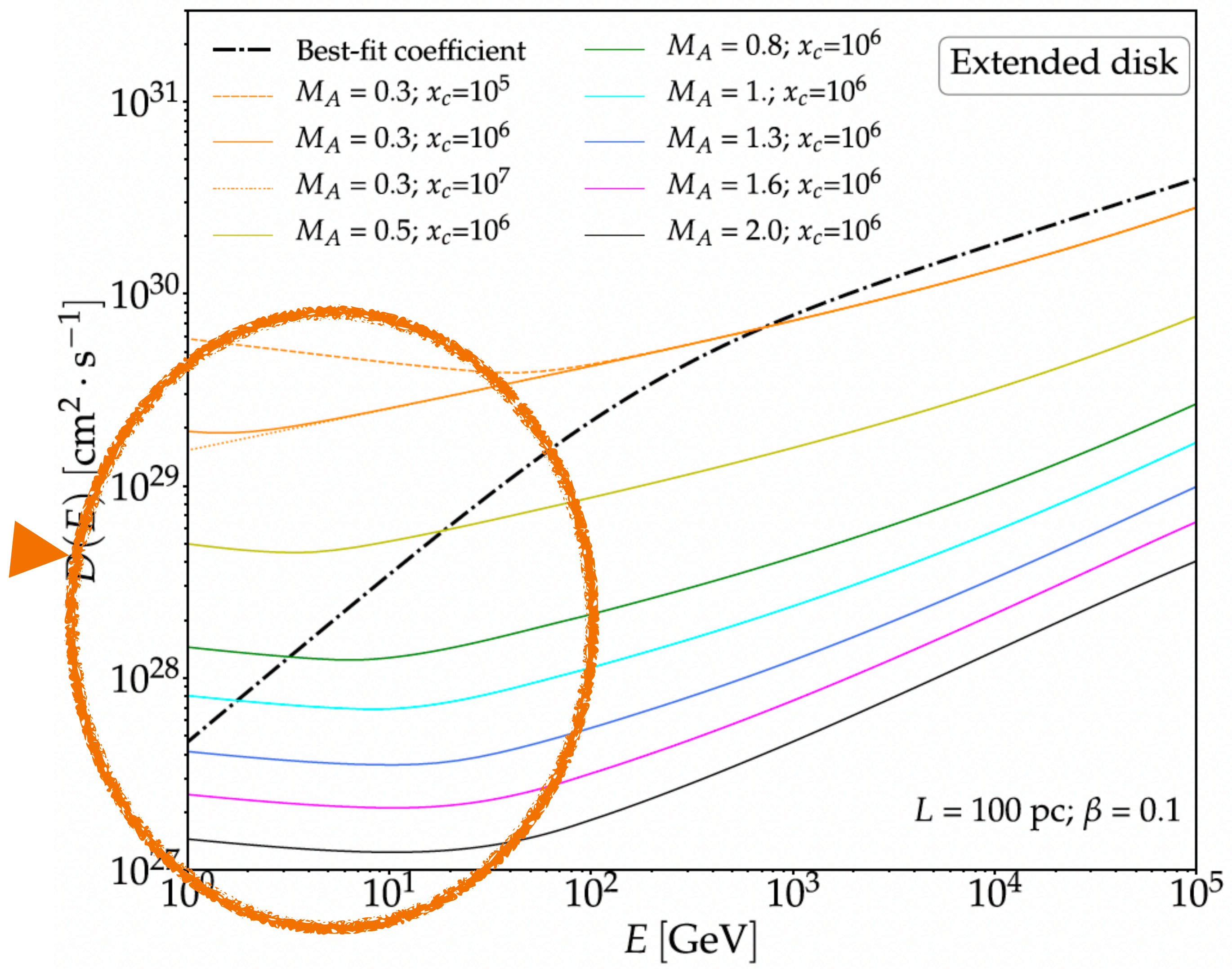

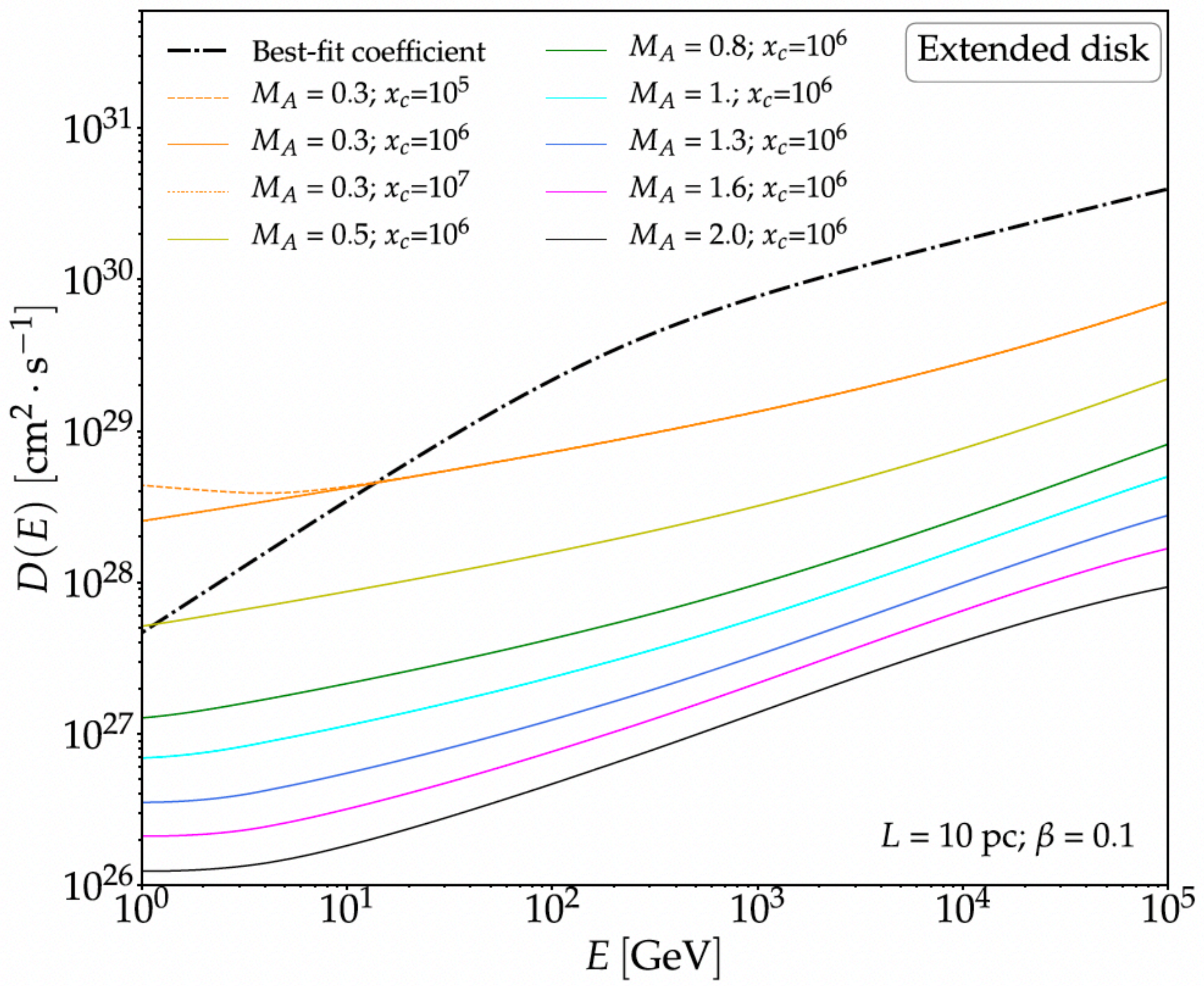

efficient damping of the fast modes in the disk

Computing CR diffusion coefficients

[Fornieri et al., MNRAS (2021)]

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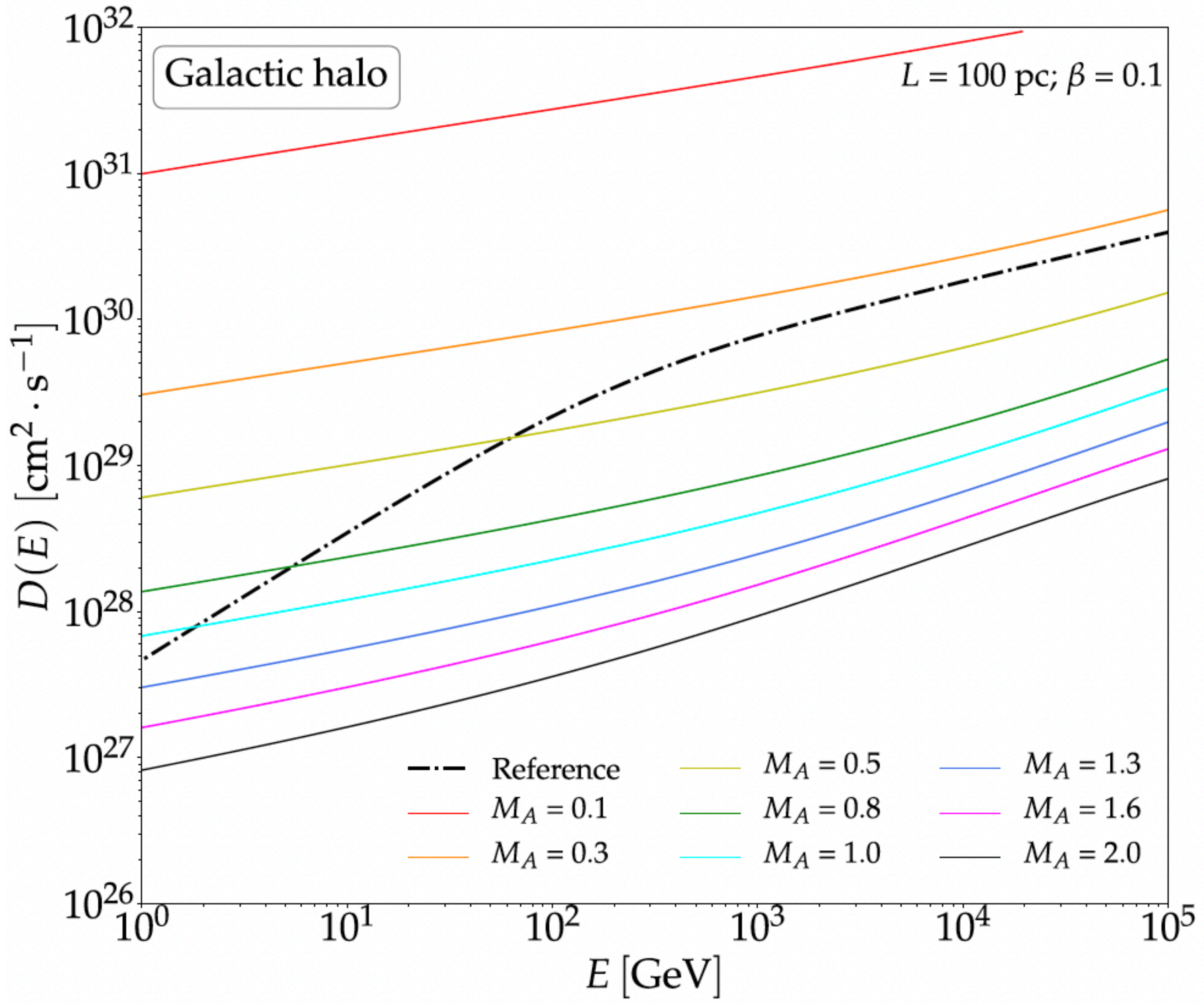

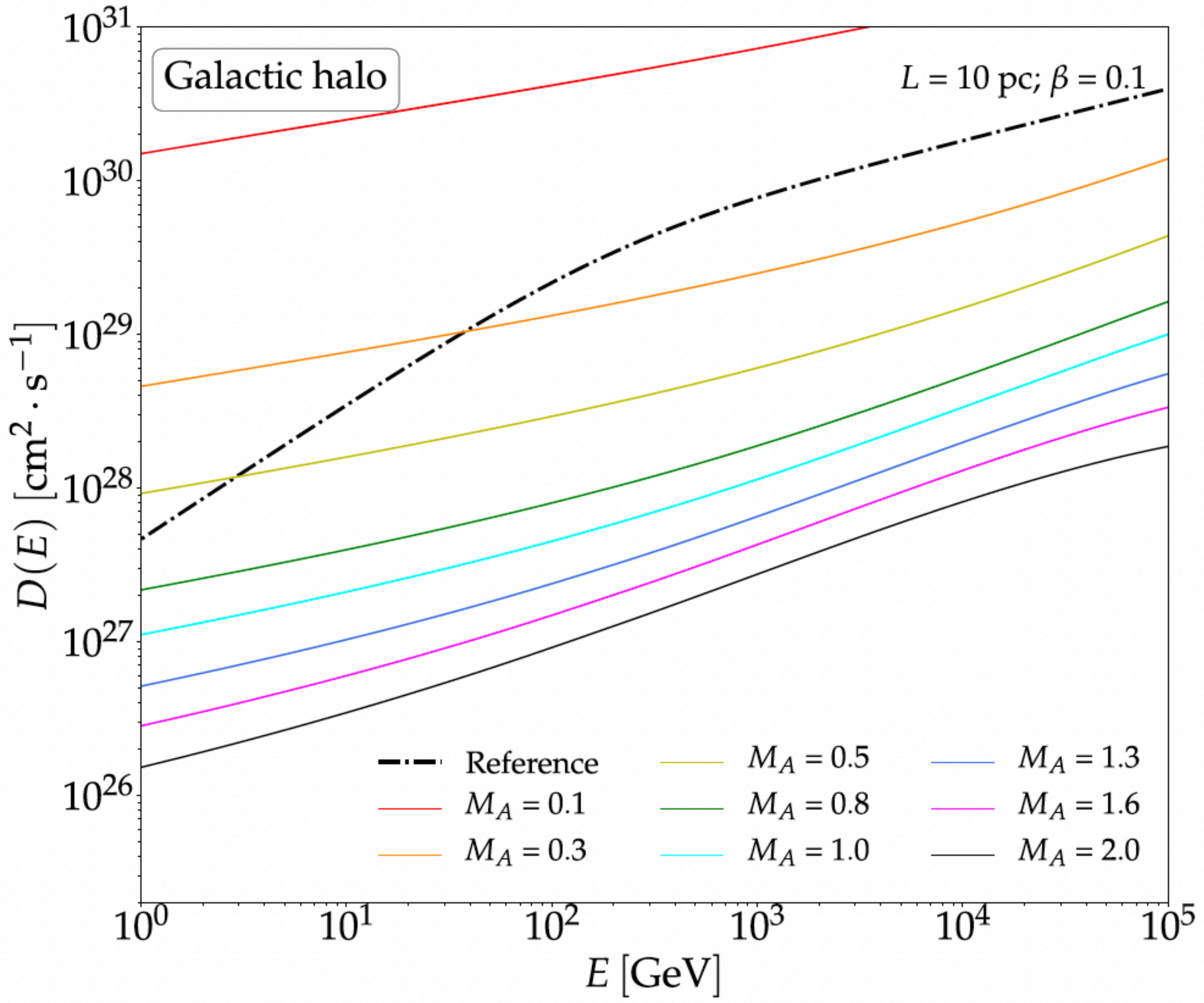

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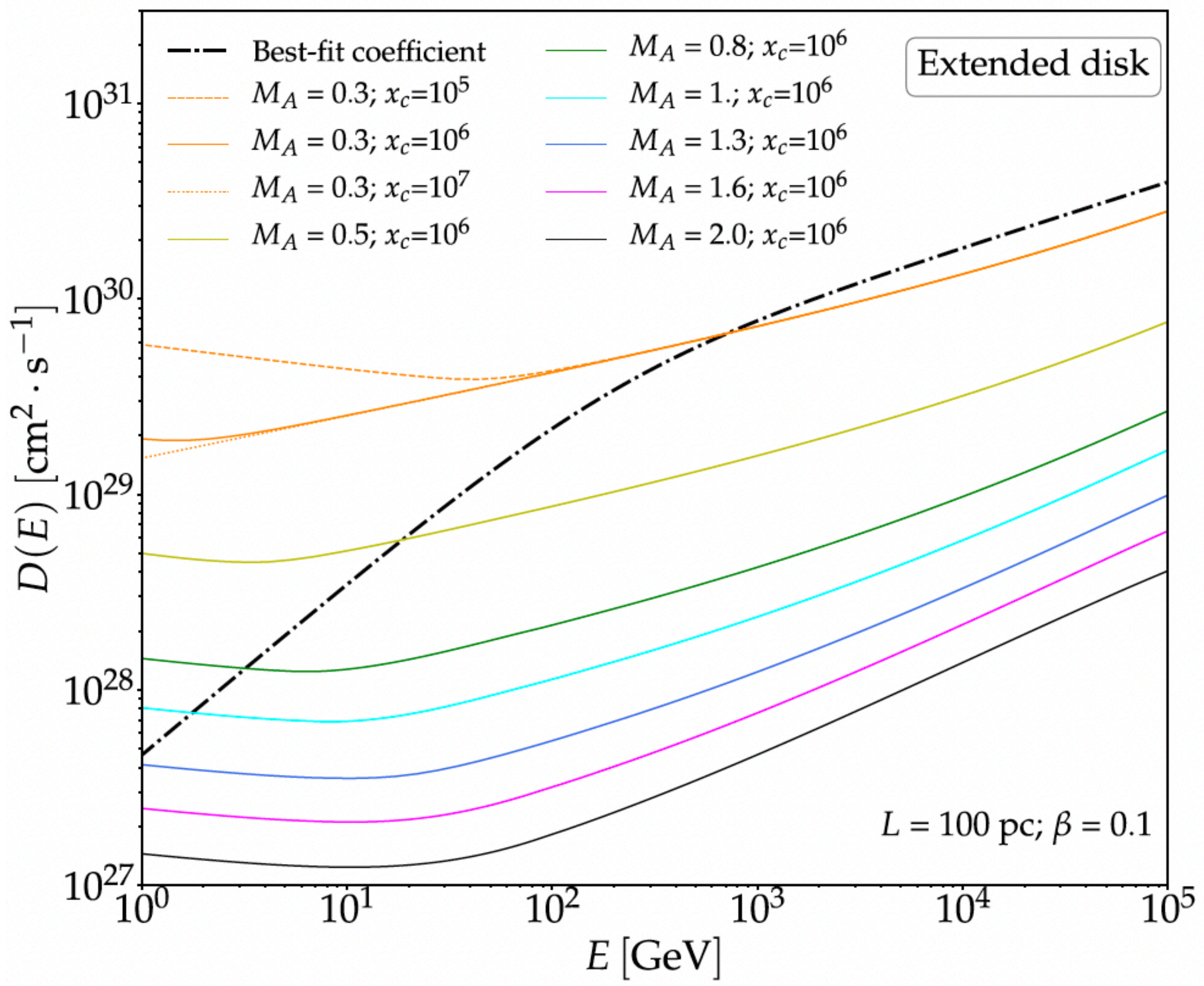

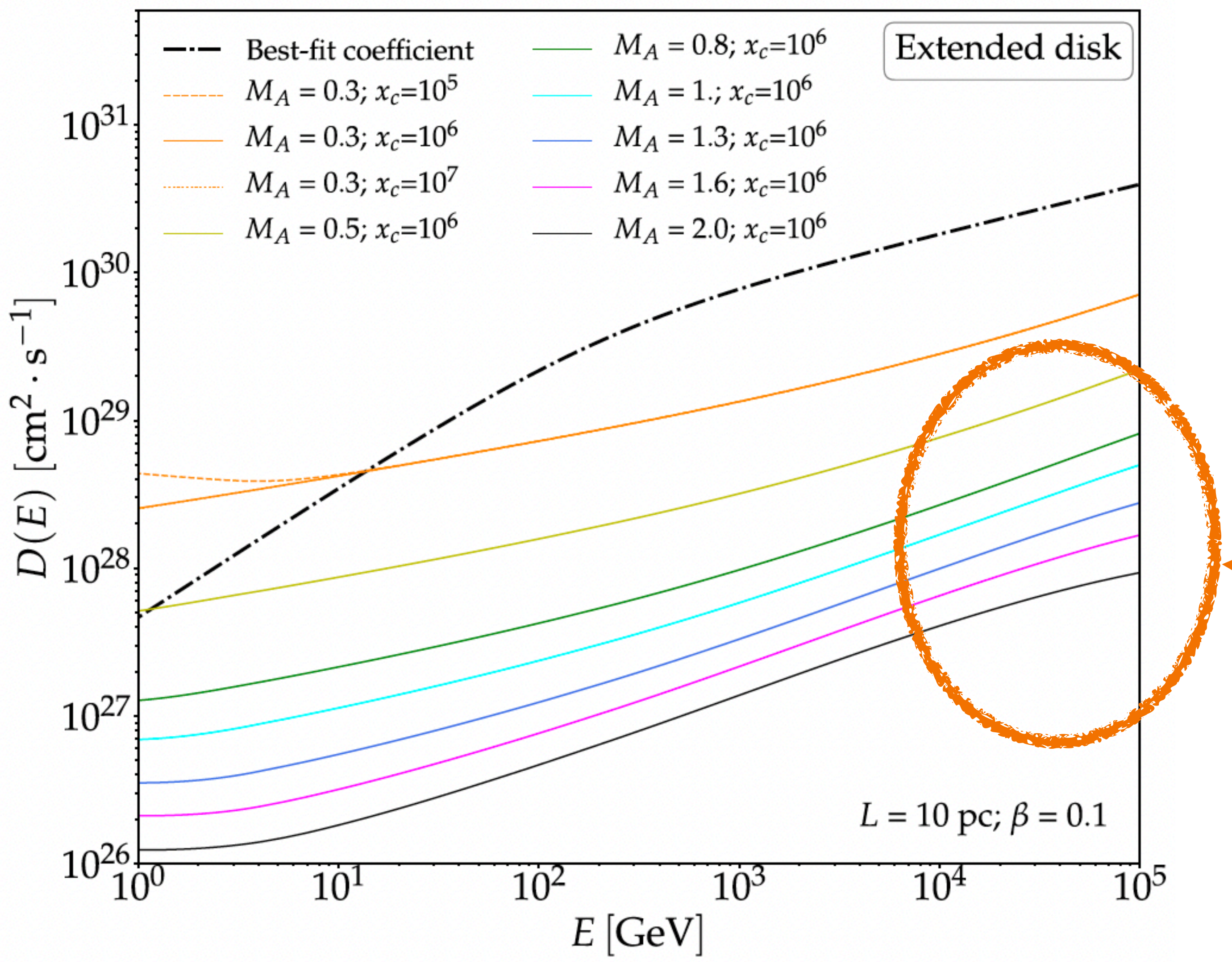

Alfvén modes may enter the picture again at high CR energies (E ≳ 10 TeV)

☞ interesting prediction for future CR observations!

The DRAGON code

[Fornieri et al., MNRAS (2021)]

☞ implement these coefficient in the **DRAGON code**:

(global CR-transport simulations within a 2-zone model of the Galaxy: disk + halo)

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for implementation details, see:

[Evoli et al., JCAP (2017)]

[Evoli et al., JCAP (2018)]

https://github.com/cosmicrays

DRAGON2

(**D**iffusion **R**eacceleration and **A**dvection of **G**alactic cosmic rays: an **O**pen **N**ew code)

some important physical ingredients:

✔ primary CRs (e.g., SNRs) → distribution of sources (including exotic sources)

✔ secondary CRs (spallation) → up-to-date nuclear cross sections

✔ CR transport → diffusion, re-acceletation, advection & losses

✔ large-scale structure → Galactic magnetic field & gas distribution

The DRAGON code

[Fornieri et al., MNRAS (2021)]

☞ implement these coefficient in the **DRAGON code**:

(global CR-transport simulations within a 2-zone model of the Galaxy: disk + halo)

testing some environmental/plasma parameters against B/C data

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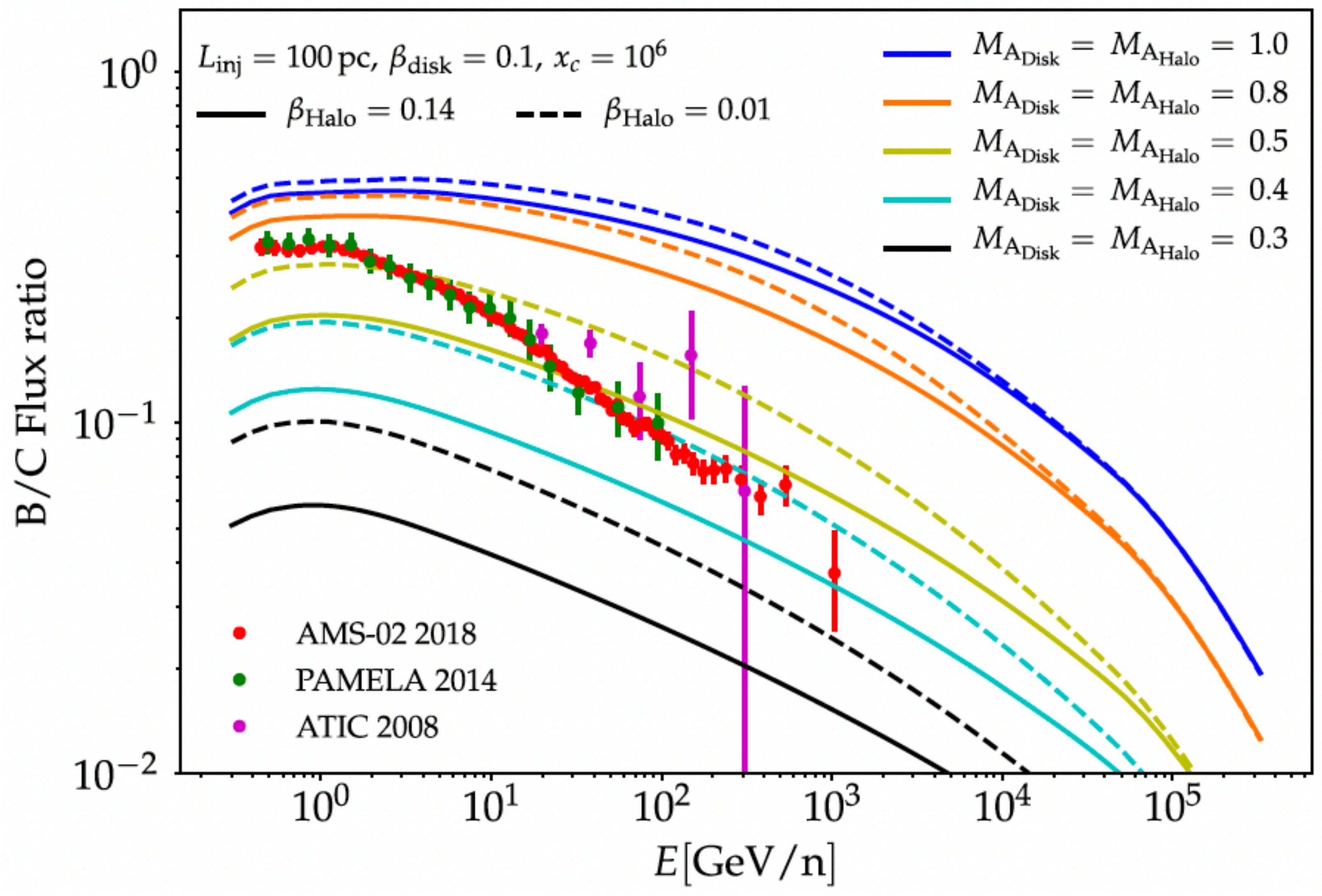
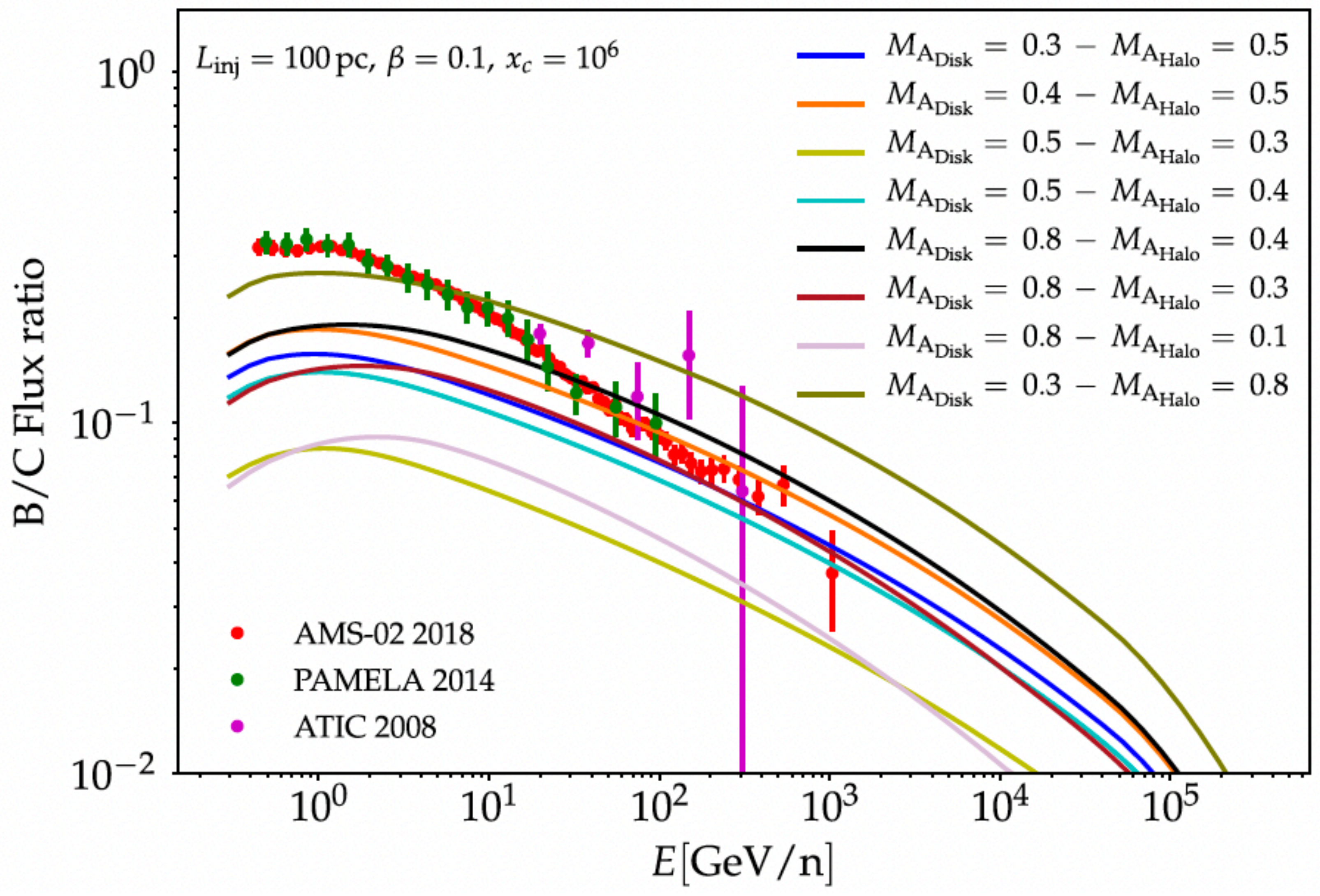
testing some environmental/plasma parameters against B/C data

[each line in these plots is a global simulation with a different D(E)]

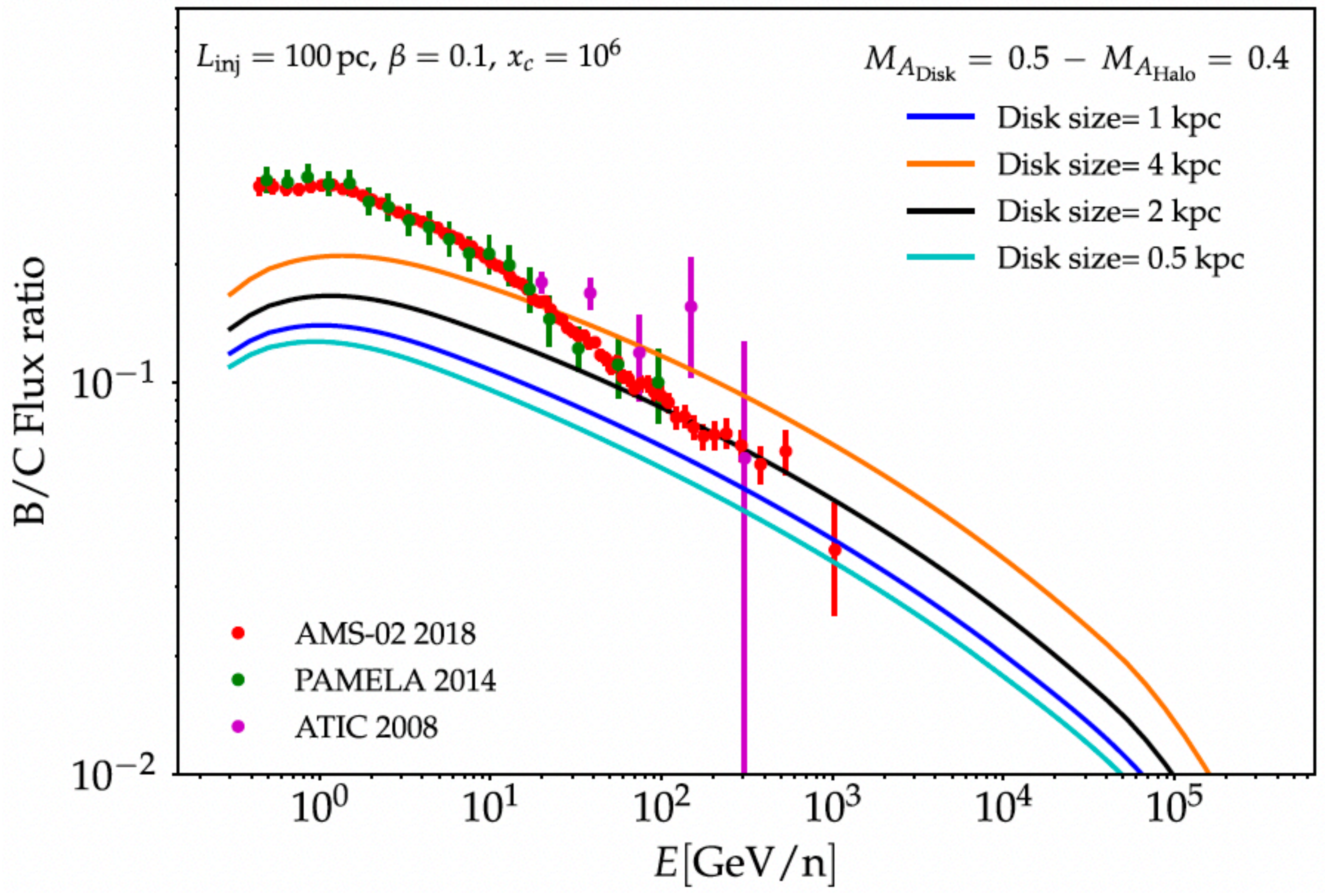


varying β_{halo} and M_A

(same M_A in disk and halo)



varying $M_{A,halo}$ and $M_{A,disk}$



varying disk size

The DRAGON code

[Fornieri et al., MNRAS (2021)]

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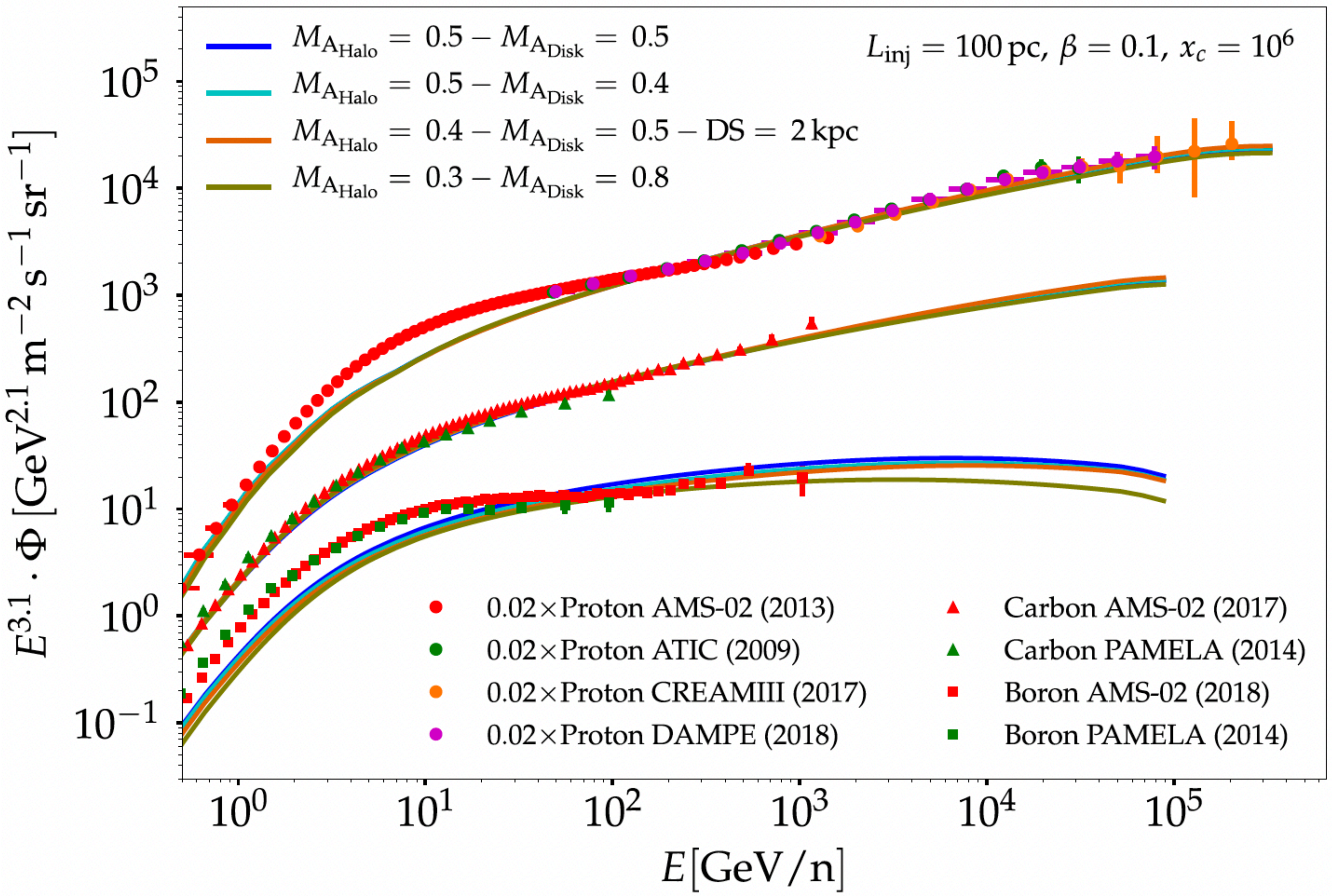
some "plausible" parameters vs single CR-species data



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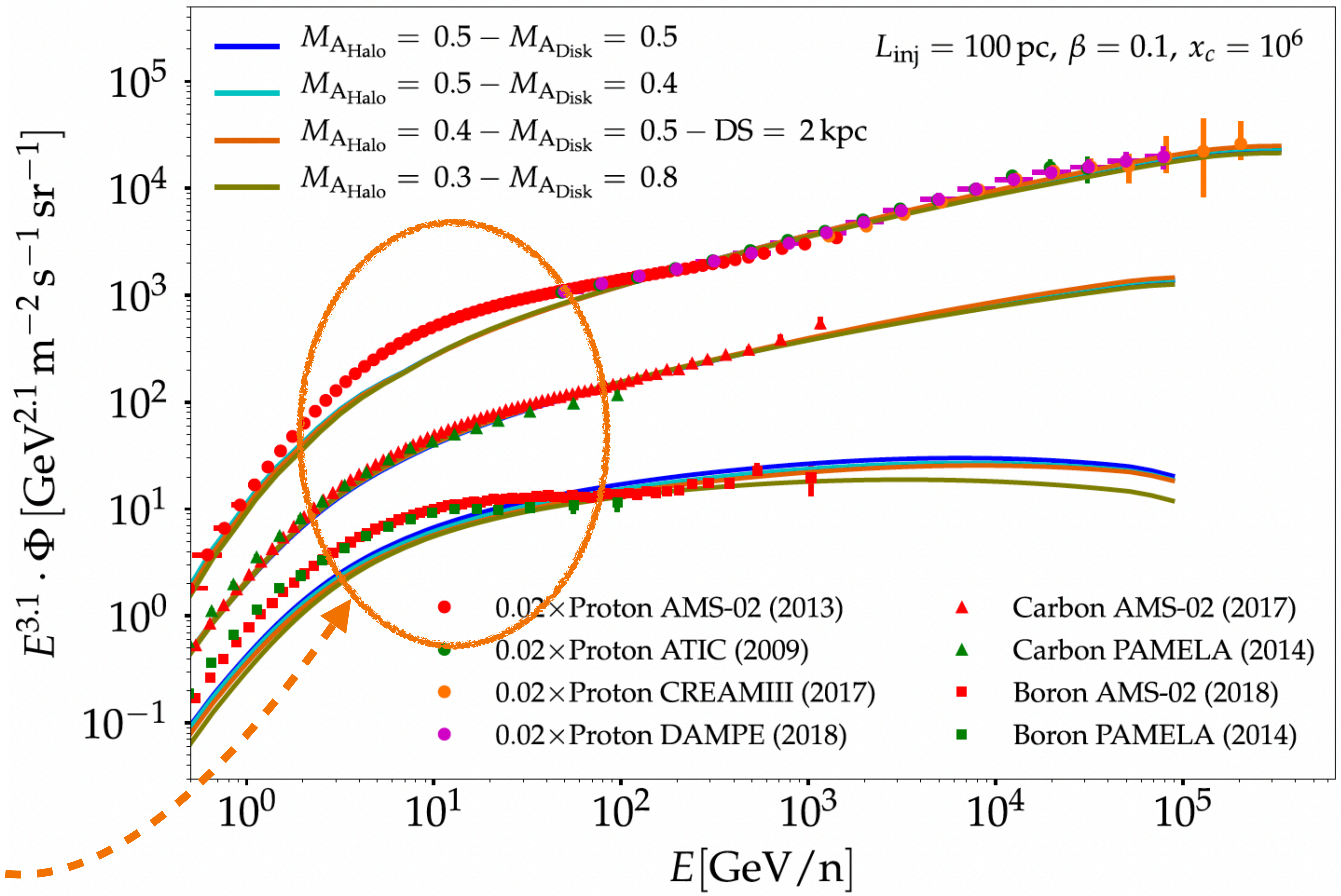


in this range of energies, confinement is likely due to **self-generated turbulence**

Take-home message

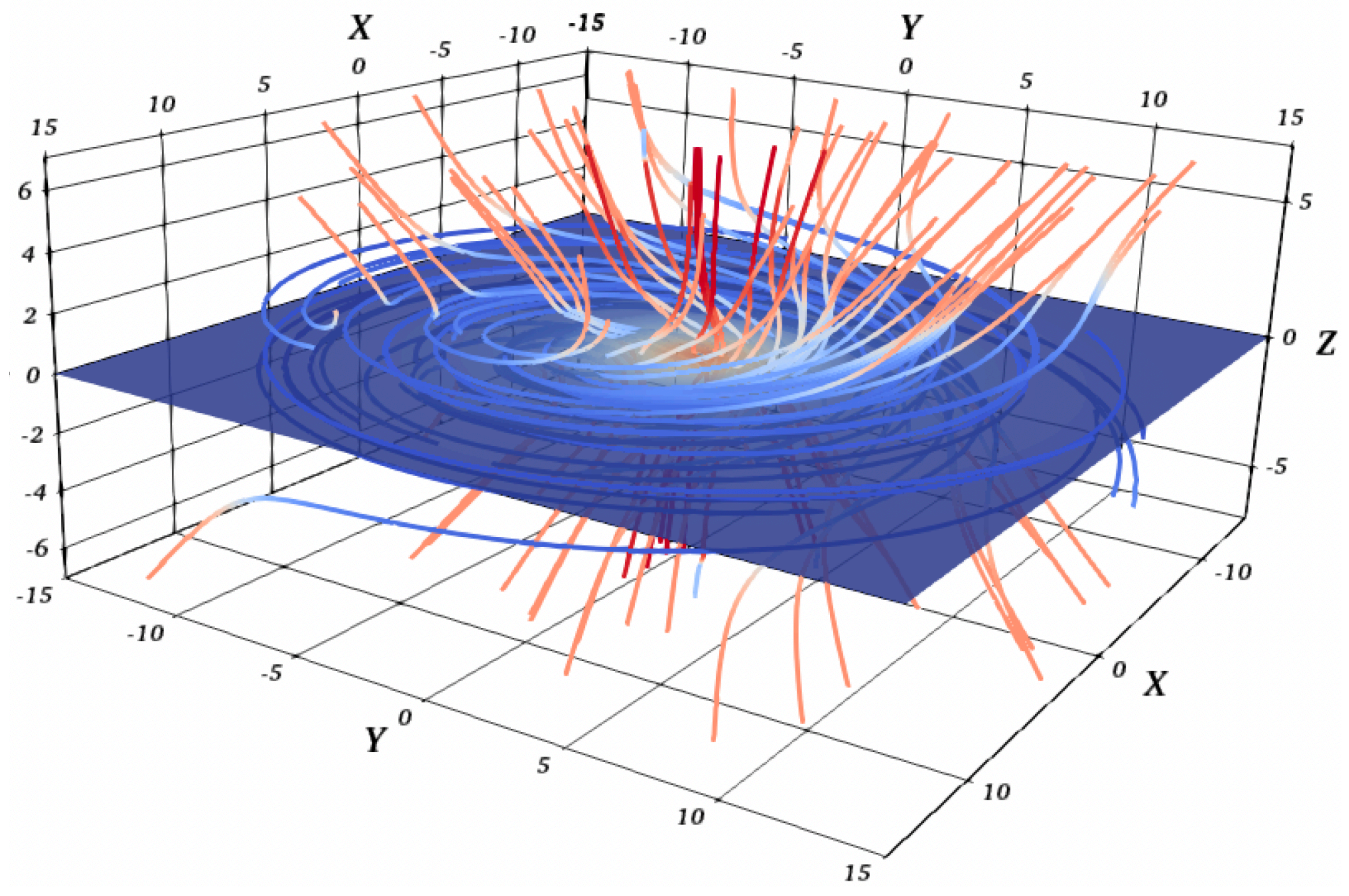
In order to interpret increasingly **accurate measurements**,
one needs to understand and model in great detail
the **micro-physics of cosmic-ray transport** in our Galaxy

Perspectives

☞ Beyond quasi-linear (or, weakly non-linear) theory of CR scattering on MHD modes

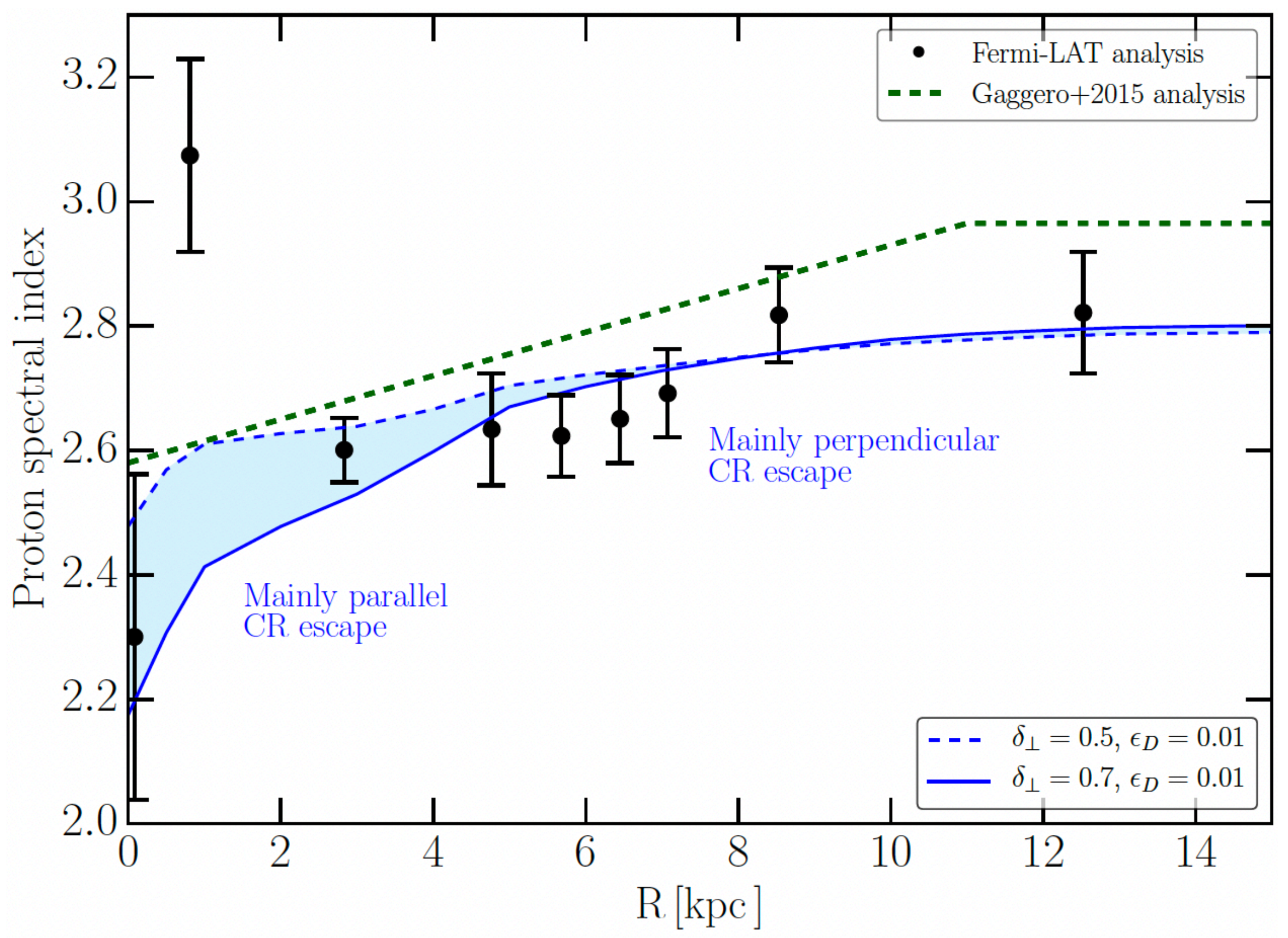
☞ Include the effect of self-generated turbulence

☞ Consider fully anisotropic CR transport in the large-scale Galactic magnetic field



$$D_{ij} \equiv D_\perp \delta_{ij} + (D_\parallel - D_\perp) b_i b_j$$

$$b_i \equiv \frac{B_i}{|\mathbf{B}|}$$



[Cerri et al., JCAP (2017)]

Thank you for your attention!